**TELKOMNIKA**, Vol.11, No.11, November 2013, pp. 6309~6314
e-ISSN: 2087-278X



# Hypersonic Vehicle Tracking based on Improved Current Statistical Model

**He Guangjun*, Lv Hang, Li Baoquan, Li Yanbin**
School of Air and Missile Defense, Air Force Engineering University, Xi'an, 710051, China
*Corresponding author, e-mail: guangjunhe@sina.com

***Abstract***

*A new method of tracking the near space hypersonic vehicle is put forward. According to hypersonic vehicles' characteristics, we improved current statistical model through online identification of the maneuvering frequency. A Monte Carlo simulation is used to analyze the performance of the method. The results show that the improved method exhibits very good tracking performance in comparison with the old method.*

***Keywords:*** *current statistical model, near space, hypersonic, target tracking, maneuvering frequency*



## 1. Introduction

In recent years, near space hypersonic vehicle has always been a hot area of research. X-37B and X-51A test vehicles in the US made a successful trial flight in April and May of 2010 separately [1, 2], marking that the US has won a completely new breakthrough in the area of hypersonic cruise vehicle. Meanwhile, the series of X-51B~X-51H etc are following the prescribed order. Russia, Japan and some other countries in Europe are also striving to develop their own near space hypersonic vehicles [3]. These vehicles are divided into two categories: low dynamic ones and high dynamic ones. For those high dynamic vehicles with the speed of more than 5 Mach, how to track them gradually turns to be a problem. In the paper, the near space hypersonic vehicles' trajectory is analyzed, then, according to its characters we studied how to track hypersonic target applying improved current statistical (CS) model. Simulations show that the improved CS model exhibits a very good performance.

## 2. Analysis of Near Space Hypersonic Vehicle Tracking

According to literature [4], near space hypersonic vehicles usually adopt scramjet combined engine and turbo rocket engine due to their flight environment. And in order to save fuel consumption and reduce the demands for the cooling system, a jumping trajectory is often used. A typical flight trajectory is described as Figure 1 [5].

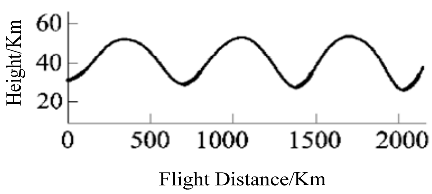


Figure 1. Jumping Trajectory

As Figure 1 shows, the vehicle starts to jump from the height of 30km, with the initial flight path angle of 0 degree. At that time it is at the lowest point of the orbit, just then the scramjet engine ignites to boost. When it reaches a certain height below 40km, the engine turns off. And the vehicle glides freely to the next lowest point. In the terminal part of the glide, the

*Received April 1, 2013; Revised June 9, 2013; Accepted July 1, 2013*

vehicle raises its head depending on aerodynamic force. Then the engine ignites to boost again and the process is repeated, leading a non-stop flight eventually to the scheduled destination.

In order to track the aforementioned jumping trajectory, a proper model is needed to be built. When we build a model of a maneuvering target, we mainly expect it not only to conform to reality but also to be convenient in mathematical treatment. By analyzing the whole jumping trajectory, we divide the idealized course of flight into constant velocity (CV) motion and constant acceleration (CA) motion, and we use CV model and CA model to track the trajectory. For the CV motion, we choose white noise process that conforms to the characteristics of the trajectory to make up the unpredictable error attached in the model. While for the CA motion, we choose the CS model. CS model sees the estimation of last moment as the current mean acceleration, and the acceleration in the model consists of the mean acceleration and a Singer acceleration process [6]. For the model hypothesis that the acceleration and the mean acceleration are identified online, it is even closer to the reality than Singer model and it has a wider range of applications. This model is acquired on the basis of doing research into general maneuvering targets, and it has good performance in tracking plane targets. But the maneuvering frequency in the model is an empirical value took by analyzing general targets, while the target's movement is mainly uniform, the estimation of the acceleration may have random change, resulting in error comparing with reality. By combining all the analysis results, we can use CV model and CS model to describe the tracking of hypersonic vehicles, but some parameters is needed to be identified and set properly according to the characters of the actual trajectory.

## 3. Tracking Model for Near Space Hypersonic Vehicle

In general, here we consider a typical linear time-varying discrete system, it can be represented as:

$$\begin{aligned} X(k) &= \Phi(k-1)X(k-1) + \omega(k) \\ Z(k) &= H(k)X(k) + v(k) \end{aligned} \tag{1}$$

Where $X(k)$ denoted by $X(k) \in R^n$ is the system state, k is denoted by $k \in [1, N]$ and $Z(k)$ denoted by $Z(k) \in R^q$ is the output measurement. $\Phi(k) \in R^{n \times n}$ is the state transition matrix. $H(k) \in R^{n \times q}$ is the measurement matrix. $\omega(k) \sim \mathrm{N}(0, \ Q(k))$ and $v(k) \sim \mathrm{N}(0, \ R(k))$ are the mutually independent noises that are used to describe the system disturbance and the measurement noise, respectively.

For CS model, the following formula can be acquired.

$$\ddot{x}(t) = a(t) = \bar{a} + \tilde{a}(t) \tag{2}$$

$$\dot{a}(t) = -\alpha a(t) + \alpha \bar{a}(t) + \omega(t) \quad \text{OR} \quad \dot{a}(t) = -\alpha \tilde{a}(t) + \omega_1(t) \tag{3}$$

In Formula (2), $\tilde{a}(t)$ is the zero-mean colored acceleration noise, and it is a Singer acceleration process. $\bar{a}$ is the mean value of maneuvering acceleration, and it is constant in each sampling period. $\alpha$ is the maneuvering frequency ( reciprocal of the maneuvering time constant). $\omega(t)$ is zero-mean white noise with the variance $\sigma_\omega^2 = 2\alpha\sigma_a^2$, in which $\sigma_a^2$ represents acceleration variance of the target. $\omega_1(t)$ is Gaussian noise with mean value of $\alpha\bar{a}$.

In Formula (3), there is a key hypothesis in CS model

$$\bar{a}_{k+1} \overset{def}{=} E[a_{k+1} / Z^k] = E[a_k / Z^k] \overset{def}{=} \hat{a}_k \tag{4}$$

Where $Z^k$ is all the output measurement till the moment k, and $\hat{a}_k$ is the estimation at moment k.

The variance $\sigma_{\omega}^{2}$ is obtained from the following formula.

$$\begin{cases} \sigma_{\omega}^{2} = \dfrac{2\alpha(4-\pi)}{\pi}\left[a_{\max} - \hat{a}(t)\right]^{2}, & \hat{a}(t) > 0 \\ \sigma_{\omega}^{2} = \dfrac{2\alpha(4-\pi)}{\pi}\left[a_{-\max} + \hat{a}(t)\right]^{2}, & \hat{a}(t) < 0 \end{cases} \tag{5}$$

Obviously, in CS model, it is needed to acquire $\overline{a}(t)$, $\alpha$, $\omega_1(t)$ and $\sigma_{\omega}^{2}$. The mean value of acceleration $\overline{a}(t)$ and the variance of noise $\sigma_{\omega}^{2}$ can be acquired from Formula (4) and Formula (5), through working out the estimation of last moment. $\alpha$ is a an empirical value set according to the flight characters of general airplane , when the target makes a slow turn the value is 1/60 and when it makes a escape maneuver the value is 1/20. But for near space hypersonic vehicles, we cannot simply choose these empirical values. Adaptive online identification algorithm is used to decide the value of $\alpha$ according to the flight conditions.

## 4. Online Identification of Maneuvering Frequency

A lot of literature on maneuvering target tracking model believe that the online identification of maneuvering frequency $\alpha$ is very important for hypersonic vehicle tracking [7, 8]. But there is no fine enough terms of settlement till now. We studied two methods of online identification of $\alpha$ as following.

### 4.1. Online Identification based on Kalman Filtering

Suppose the maneuvering frequency at moment k is denoted by $\alpha = \alpha(k)$. Then the target's motion model can be presented by Formula (1). The state equation is $X(k+1) = \Phi(k)X(k) + \omega(k)$, and the observation equation is $Z(k) = HX(k) + v(k)$.

When $\alpha = \alpha(k-1)$, the filtering process is

$$\begin{aligned} &\hat{X}(k|k-1) = \Phi(k-1)\hat{X}(k-1|k-1) \\ &P(k|k-1) = \Phi(k-1)P(k-1|k-1)\Phi^{T}(k-1) + Q(k-1) \\ &K(k) = P(k|k-1)H^{T}S^{-1}(k) \\ &\hat{X}(k|k) = \hat{X}(k|k-1) + K(k)\gamma(k) \\ &P(k|k) = [I - K(k)H]P(k|k-1) \end{aligned} \tag{6}$$

Where $\gamma(k) = Z(k) - H\hat{X}(k|k-1)$ and $S(k) = HP(k|k-1)H^{T} + R(k)$ are predication residual and its covariance matrix. $\hat{X}(k-1|k-1)$, $P(k-1|k-1)$ and $Z(k)$ are inputs.

After we get predication residual and its covariance matrix at moment k-1 from Formula (6), we can work out matching probability of the target's motion state between the moment k-1 and k. The calculation process is as following.

$$\Lambda(k) = \frac{1}{\sqrt{2\pi|S(k)|}}\exp\left\{-\frac{1}{2}\gamma^{T}(k)S^{-1}(k)\gamma(k)\right\} \tag{7}$$

Conduct normalization processing to the result of likelihood function $\Lambda(k)$, we will get the matching probability $\lambda(k)$ between the model being used and the actual model. We introduced hard threshold $\lambda_0$, if $\lambda(k) > \lambda_0$ we think $\alpha$ can reflect the target's motion state truthfully, if not we continue to find the value of $\alpha$ through iteration.

**4.2. Online Identification based on $H_\infty$ Filtering**

When filtering model and target motion mode match accurately, and the statistical character of system disturbance is known already, Kalman filtering based online identification is the most optimal solution. But for the high speed and high maneuvering targets, the actual noise covariance is usually greater than the assumed noise covariance, thus leading to low precision and even the diverge of the filter. Then $H_\infty$ filtering based online identification is put forward.

Add $Y(k)=L(k)x(k)$ to Formula (1), therein $L(k)$ is state combination matrix and it is related to covariance updating of $H_\infty$ filtering. Here we do not make any assumptions, but for the energy is limited.

When $\alpha=\alpha(k-1)$, the filtering input are $\hat{X}(k-1|k-1)$ $P(k-1|k-1)$ and $Y(k)$. Set noise suppression parameter $\gamma>0$, then judge if $P(k-1|k-1)$ meet the condition

$$P(k-1)+H^T H-\gamma^{-2}L^T L>0 \tag{8}$$

Where $L=I$. If Formula (8) cannot be met, increase the value of $\gamma$ and judge once again. Else if Formula (8) is true, predict filtering covariance matrix of the next moment by:

$$P(k)=\Phi P(k-1)\Phi^T+Q-\Phi P(k-1)\begin{bmatrix}H^T & L^T\end{bmatrix}R_{e,k}^{-1}\begin{bmatrix}H\\ L\end{bmatrix}P(k-1)\Phi^T \tag{9}$$

Where:

$$R_{e,k}=\begin{bmatrix}R & 0\\ 0 & -\gamma^2 I\end{bmatrix}+\begin{bmatrix}H\\ L\end{bmatrix}P(k-1)\begin{bmatrix}H^T & L^T\end{bmatrix} \tag{10}$$

Then the state estimation of the target based on $H_\infty$ filtering is:

$$\hat{X}(k)=\Phi\hat{X}(k-1)+K(k)\left[Y(k)-H\Phi(k)\hat{X}(k-1)\right] \tag{11}$$

Where $K(k)$ is filtering gain denoted by:

$$K(k)=P(k)H^T\left[R+HP(k)H^T\right]^{-1} \tag{12}$$

Where predication residual and its covariance matrix are denoted by:

$$\gamma(k)=Y(k)-H\hat{X}(k/k-1),\qquad S(k)=HP(k/k-1)H^T+R \tag{13}$$

It should be noted that the parameters Q and R used in $H_\infty$ filtering is different from those in Kalman filtering. In fact, they are weight coefficient set by us according to target's motion mode and actual noise background. They are written as above is to make it easy to compare with Kalman filtering.

Then solving likelihood function through Formula (5) and solving maneuvering frequency are similar to methods in chapter 4.1.

**5. Simulations and Analysis**

Apply the aforementioned algorithm to identify maneuvering frequency $\alpha$ online. The simulation scenario is a high speed and high maneuvering target moves as Figure 1 shows: first CA, second CV and third constant retarded acceleration then repeat from CA. In the simulation, the initial value of $X(k)$ is denoted by $X_0 = [x \quad \dot{x} \quad \ddot{x}] = [300000 \quad 2640 \quad 30]$, (units are m, $m/s$ and $m/s^2$). The Monte Carlo simulation time is 100.

The measurement noise is modeled as:

$$v(k) = \left( \beta \begin{bmatrix} x(k) \\ \dot{x}(k) \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta \dot{x} \\ 0 \end{bmatrix} \right) e(k) \tag{14}$$

Where $\beta$ denotes relative error coefficient, $\Delta X$ and $\Delta \dot{X}$ denotes fixed measurement error, and $e(k)$ is a zero-mean normal pseudo-random number with variance of 1. The observation noise covariance is as following:

$$R(k) = \left( \beta \begin{bmatrix} x(k) \\ \dot{x}(k) \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta \dot{x} \\ \xi \end{bmatrix} \right)^2 E\left[e^2(k)\right] \tag{15}$$

Where $\xi$ denotes arbitrarily small real. Simulation results are as Figure 2 and Figure 3 show.

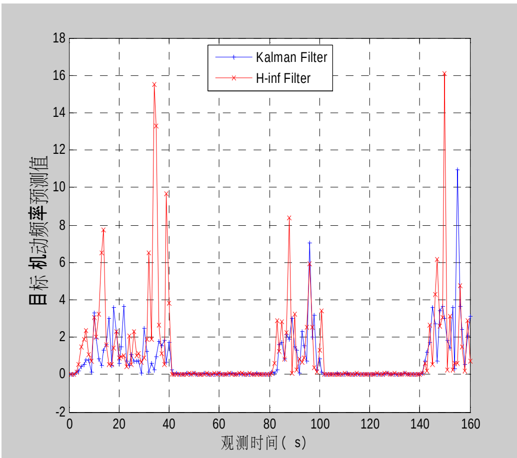


Figure 2. Prediction of Target's Maneuvering Frequency $\alpha$

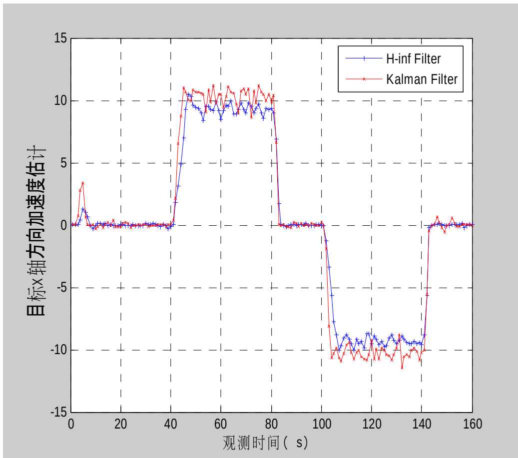


Figure 3. Prediction of Target's Maneuvering Acceleration

It can be concluded from the figures that when the target moves uniformly and in a straight line, the prediction of its maneuvering frequency is greater, and the filtering model is similar to CV model. When the target moves with constant acceleration, the prediction of its maneuvering frequency is very small (in fact the order of magnitude is 10-3), and the filtering model is similar to CA model. Conclusion can be drawn that the improved algorithm put forward in the paper can identify the maneuvering frequency of near space hypersonic vehicles adaptively online, thus making the filtering model match the actual conditions more accurately. Both Kalman and $H_\infty$ filtering based online identification can identify maneuvering frequency relatively authentically, but the latter method gives a maneuvering frequency with a wider range, that is because in $H_\infty$ filtering the character of interference noise is unknown.

**References**


[1] Xiao Fugen. Background of X-37B Space Vehicle Development. *Spacecraft Environment Engineering*. 2010; 27(5): 558-565.

[2] William Hennigan. Hypersonic X-51 WaveRider Shatters Record at 3,500 Mph. *Stars and Strips*. 2010; 69(42): 3.

[3] Huang Wei. Research Status of Near Space Hypersonic Vehicles. *Winged Missiles Journal*. 2007; (10): 28-31.

[4] Xue Yongjiang, Li Tifang. The Development of Near Space Vehicles and Analysis of Key Technology. *Aerodynamic Missile Journal*. 2011; (2): 32-36.

[5] Zhao Jun, Meng Lingsai. Jumping Trajectory Optimization of the Near Space Vehicles With Hypersonic Speed. *Tactical Missile Technology*. 2010; (5): 32-35.

[6] Zhou Hongren, Jing Zhongde, Wang Peide. Maneuvering Target Tracking. Beijing: National Defense Industry Press. 1991.

[7] Han Chongzhao, Zhu Hongyan, Duan Zhansheng, etc. Multi-Source Information Fusion. Bejing: Tsinghua University Press. 2006: 124-294.

[8] Xu Jingshuo, Qin Yongyuan, Peng Rong. New Method for Selecting Adaptive Kalman Filter Fading Factor. *Systems Engineering and Electronics*. 2004; (26): 1552-1554.

[9] Chen Jiajun, Liuaofeng, Xin Jinsheng, Gu Xuefeng. Research on Maneuvering Frequency Fuzzy Adaptive Target Tracking Algorithm. *Journal of Projectiles Rockets Missiles and Guidance*. 2010; (30): 259-262.

[10] Dai Hong-de, Dai Shao-wu, Cong Yuan-cai, Wu Guang-bin. Performance Comparision of EKF/UKF/CKF for the Tracking of Ballistic Target. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2012; 10(7):1537-1542.

[11] Kalman RE. A New Approach to Linear Filtering and Prediction Problem. *IEEE Transactions ASME.* 1960; (20): 34-45.

[12] Xiuling He, Yan Shi, Jiang Yunfang. Improved Based on “Self-Adaptive Turning Rate” Model Algorithm. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2013; 11(6).