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MATLAB Based PCM Modeling and Simulation

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Abstract
PCM is the key technology of digital communication, and has especially been widely used in the optical fiber communication, digital microwave communication, satellite communication. Modeling PCM communication systems with the pulse code system by programming, and conduct computer simulation by MATLAB, to analysis performance of the linear PCM and logarithmic PCM.

Keywords: PCM modeling, SNR, quantitative, compression feature, simulation analysis

1. Introduction
Pulse Code Modulation, referred to as PCM. It is a kind of encoding that changes analog voice signal into digital signal [1]. In the late 1970 s, with the appearance of pulse code modulation encoder and decoder for super-large-scale integrated circuit as well as the development of optical fiber communication, digital microwave communications, satellite communications, PCM has been gradually widely used. At present, the PCM has become a key technology in digital communication [2-6].

2. Research Method
PCM modulation mainly includes sampling, quantization and encoding process. Sampling changes the continuous analog signals into the discrete time continuous amplitude sampling signals; Quantification changes the discrete time continuous amplitude sampling signals into the discrete time discrete amplitude digital signals; Coding makes the quantified signals into the output binary code groups. International standard PCM code groups (telephone voice) adopt eight-level codes represents a sampling value [1]. From the view of modulation concept in communication, it can be considered that, the PCM encoding process is analog signal modulating a binary pulse sequence, the carrier is pulse sequence, and modulation changes pulse sequence as none or "1", "0", therefore PCM is made into pulse code modulation. Pulse code modulation process as shown in Figure 1.

Encoded PCM code groups, via digital channels, can be directly transmitted by baseband or microwave, light wave carrier modulated pass band. At the receiving end, the binary code group inversely transforms into the reconstruction analog signal $\hat{x}(t)$. In the demodulation process, generally uses the sampling hold circuit, so the low pass filter adopts $\frac{x}{\sin x}$ type frequency to response to compensate for frequency distortion $\frac{\sin x}{x}$ introduced by sampling hold circuit is.

Pre-filtering is to limit the original speech signal frequency band within 300-3400 Hz standard long-distance analog telephone frequency band. Due to the original speech band is around 40-10000 Hz, so pre-filter can made out certain band distortion.

In the entire PCM system, the distortion of reconstruction signal $\hat{x}(t)$ mainly results from quantification as well as the channel transmission error, generally denotes as the signal and the quantization noise power ratio, namely is signal-to-noise ratio S/N.
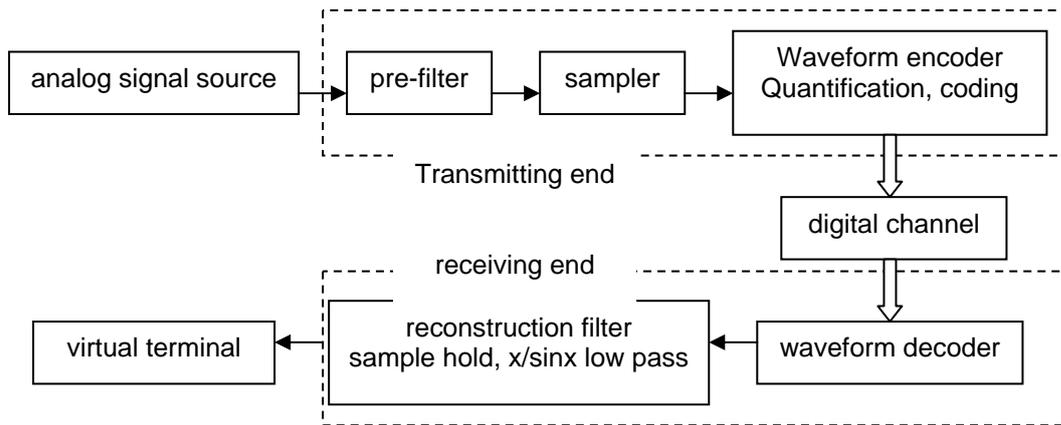
Figure 1. PCM modulation theory figure

3. Linear PCM and Logarithmic PCM Performance Analysis

Here takes sinusoidal signal as the example to analysis SNR features of linear PCM encoding and logarithmic PCM encoding.

3.1. Uniform Quantification

Based on the communication theory, when inputs sinusoidal signal $a = A_m * \sin(0.1 * pi * x)$, and the signal is not over loaded, if take the quantitative interval number as L, and $L = 2^n$, n is a positive integer, then $SNR \approx 4.77 + 20\log D + 6.02n$

It is measured in decibels (dB), $D = A_m / \sqrt{2}V$, V as the largest quantization level. Within the scope of the load range, SNR appears linearly increased with the increase of the input signal.

3.2. Non-uniform Quantification

1) Alaw Compression Features

Assuming that the input sinusoidal signal $a = \sin(0.1 * pi * x)$'s phase is random, and equal-probability distributed within the scope of $(-\pi, \pi)$. Then:

Quantization noise power

$$\sigma_q^2 = \frac{1}{3C^2 A^2 L^2}, 0 \le a \le 1/A$$

$$\sigma_q^2 = \frac{1}{3C^2 A^2 L^2}\{[2 - (aA)^2]\sin^{-1}(\frac{1}{aA}) + \frac{\pi(aA)^2}{2} + \sqrt{a^2 A^2 - 1}\}, 1/A \le a \le 1$$

$$C = 1/(1 + InA), A = 87.6$$

And Sinusoidal instantaneous power $S = \frac{a^2}{2}$

Based three formulas above, it can develop programs below, and come out the input arrays sample values x and SNR. And draw out the SNR curve.

```
x=0:0.01:20;
a=sin(0.1*pi*x);
a2=max(a);          % for the maximum amplitude
b=length(a);
```

```
a1=abs(a);        % for the absolute value of the input signal
X=20*log10(a1/a2);
n=8;
SNR1=6.02*n+4.77+X;    % uniform quantization SNR
plot(X,SNR1)
axis([-80 0 0 70]);
ylabel('SNR(dB)');xlabel('20logD');
grid on
text('Position',[-30,15],'String','L=256');
hold on
A=87.6;
C=1/(1+log(A));
S1=a.^2;
S=S1./2;      %sinusoidal signal power
for i=1:b
L=256;      %L=2^8(n=8)
if a1(i)<=1/A
q=1/(3*(C*A*L)^2);
else                                                    q=1/(3*pi*(C*A*L)^2)*((2-
(a1(i)*A)^2)*(asin(1/(a1(i)*A)))+pi*(a1(i)*A)^2/2+sqrt((a1(i)*A)^2-1));
end
Q(i)=q;       %Place the noise power at Array Q
end
S21=S./Q;
SNR21=10*log10(S21); Non-uniform quantization SNR when % n = 8 sinusoidal signal
input
plot(X,SNR21)
text('Position',[-30,40],'String','L=256');
hold on
for i=1:b
L=64;      %L=2^6(n=6)
if a1(i)<=1/A
q=1/(3*(C*A*L)^2);
else                                                    q=1/(3*pi*(C*A*L)^2)*((2-
(a1(i)*A)^2)*(asin(1/(a1(i)*A)))+pi*(a1(i)*A)^2/2+sqrt((a1(i)*A)^2-1));
end
Q(i)=q;
end
S22=S./Q;
SNR22=10*log10(S22); Non-uniform quantization SNR when % n = 6 sinusoidal signal
input
plot(X,SNR22)
text('Position',[-9,28],'String','L=64');
title ( 'logarithmic compression PCM, linear PCM snr features')
```

Figure 2 is the result of the program running. The straight line in Figure is the uniform quantization when n = 8 sinusoidal signal input, two curves respectively represents non-uniform quantization SNR when n = 8 sinusoidal signal input, and n = 6 sinusoidal signal input.

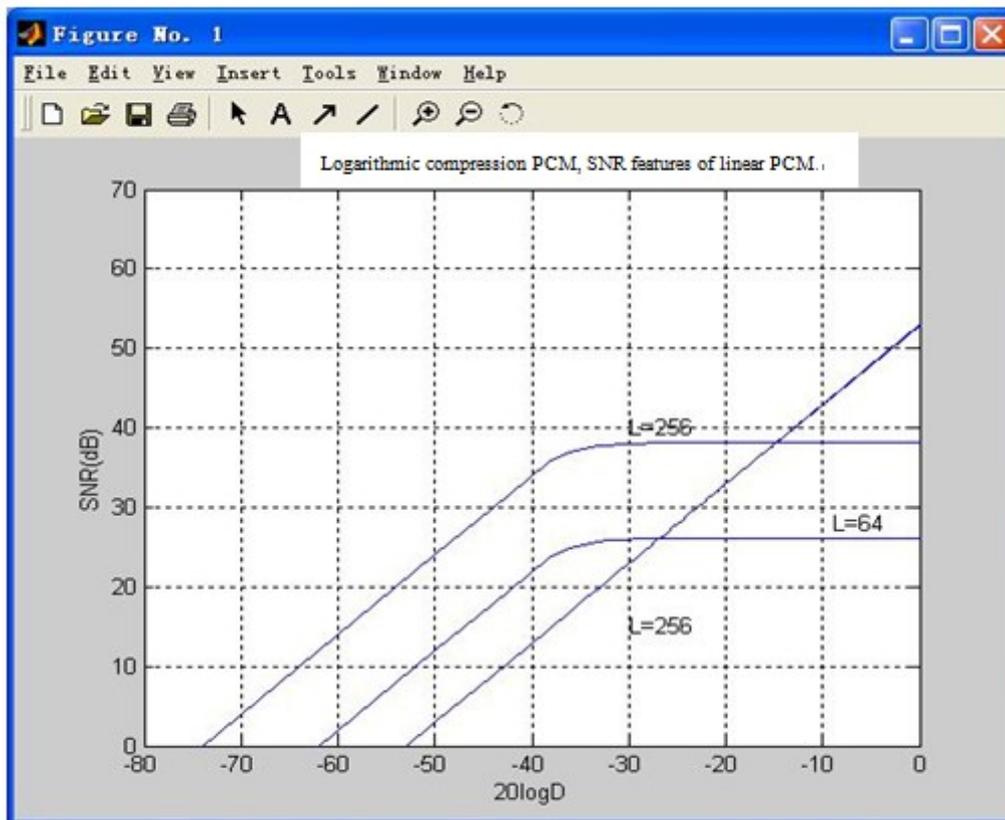Figure 2. Logarithmic compression PCM, linear PCM snr features

2) Alaw 13 Line Compression Features

There are two methods to calculate the Alaw 13 line SNR: First, the direct calculation method, that is directly using the Alaw 13 line compression & expansion characteristics to calculate the quantization noise and quantization SNR; Second, the indirect calculation method, that is, to firstly calculate the uniform quantization noise and uniform quantization SNR in accordance with the uniform quantization, then to calculate the SNR improvement quantities resulted from the compression & expansion, then the sum of them is the non-uniform quantization SNR. Here takes the second method to conduct simulation, that is:

In the formula

$$\left(\frac{S}{N_q} \right)_{dB} = \left[\frac{S}{N_q} \right]_{dB} + [Q]_{dB}$$

$\left(\frac{S}{N_q} \right)_{dB}$ is the non-uniform quantization SNR

$\left[\frac{S}{N_q} \right]_{dB}$ is the uniform quantization SNR

$[Q]_{dB}$ is the SNR improvement quantities.

In the above formula, there is no consideration with over loaded noise, because the calculation of overloaded noise, either by uniform quantization or non-uniform quantization, the calculation methods and formulas are all the same, the SNR improvement quantities in the formula should be

$$[Q]_{dB} = 10\log(1/\beta)$$

In this formula,

$$\beta = \int_{-1}^{1} \frac{1}{\left[\dfrac{dy}{da}\right]^2} p(a) da$$

is the 13 line Approximation method improvement factors

y —Quantitative output signal;

a —Input signal;

$p(a)$ —Probability density of normalized signal amplitude;

$\dfrac{dy}{da}$ —Segment of the slope of corresponding line.

If taking the audio signal as the input signal, then the SNP can be

$$\left(\frac{S}{N_q}\right)_{dB} = 6n + 4.77 + 20\log\frac{a}{u} + [Q]_{dB}$$

Because the sinusoidal signal's probability density is

$$P(a) = \frac{1}{\pi} \bullet \frac{1}{\sqrt{u^2 - a^2}}$$

In this formula,

a —Instantaneous value of sinusoidal signal;

u —Peak value of sinusoidal signal.

Then the improvement factor is

$$\beta = \frac{2}{\pi} \int_{0}^{1} \frac{1}{\left[\dfrac{dy}{da}\right]^2} \bullet \frac{1}{\sqrt{u^2 - a^2}} da$$

For the reason that there are differences in slope at each paragraphs of the 13 line, in the specific calculation, it is should according to the amount of the instantaneous value of input signal to integrate with each paragraphs, improvement factor and at each paragraphs calculated as:

The 1st, 2nd paragraph:
$$\beta_1 = \beta_2 = \frac{1}{256}$$

The 3rd paragraph:
$$\beta_3 = \frac{1}{64}\left(1 - \frac{3}{2\pi}\arcsin\left(\frac{1}{2}\right)\right)$$

The fourth paragraph:
$$\beta_4 = 4 \times \beta_3 - \frac{1}{64}\left(1 - \frac{3}{2\pi}\arcsin\left(\frac{1}{4}\right)\right)$$

The fifth paragraph:
$$\beta_5 = 4 \times \beta_4 - \frac{1}{64}\left(1 - \frac{3}{2\pi}\arcsin\left(\frac{1}{8}\right)\right)$$

The sixth paragraph:
$$\beta_6 = 4 \times \beta_5 - \frac{1}{64}\left(1 - \frac{3}{2\pi}\arcsin\left(\frac{1}{16}\right)\right)$$

The seventh paragraph:
$$\beta_7 = 4 \times \beta_6 - \frac{1}{64}\left(1 - \frac{3}{2\pi}\arcsin\left(\frac{1}{32}\right)\right)$$

$$\beta_8 = 4 \times \beta_7 - \frac{1}{64}(1 - \frac{3}{2\pi}\arcsin\left(\frac{1}{64}\right))$$

The eighth paragraph:

According to the above method, it can be designed into the following program and figured out the 13 line encoding SNR curve when the input is the sinusoidal signal:

```
x=0:0.01:20;
a=sin(0.1*pi*x);
a2=max(a);        % for the maximum amplitude
b=length(a);
a1=abs(a);        % for the absolute value of the input signal
X=20*log10(a1/a2);
n=8;
SNR11=6.02*n+4.77+X;        % Uniform quantization SNR
plot(X,SNR11)
axis([-80 0 0 60]);
ylabel('SNR(dB)');xlabel('20logD (Instantaneous value/peak value)');
grid on
text ('Position',[-35,15],'String', 'linear quantizing n=8');
hold on
B(1)=1/256;
B(2)=1/256;
B(3)=1/64*(1-3/(2*pi)*asin(1/2));        % for improvement factors
B(4)=B(3)*4-1/64*3/(2*pi)*asin(1/4);
B(5)=B(4)*4-1/64*3/(2*pi)*asin(1/8);
B(6)=B(5)*4-1/64*3/(2*pi)*asin(1/16);
B(7)=B(6)*4-1/64*3/(2*pi)*asin(1/32);
B(8)=B(7)*4-1/64*3/(2*pi)*asin(1/64);
n=8;
A=87.6;
for i=1:b
a1=a(i)*(2^11-1);
C=pcmad(8,a1);        % 13 line encoding
C1=C(2:4);        % take the 2nd and fourth number of the code
S=exdac(3,C1);        % for paragraph
s=B(S+1);        % for improvement factors with the corresponding paragraph

w=-10*log10(s);        % logarithmic of improvement factors
Q(i)=w;        % place in Array Q(i)
end
W=6*n+4.77;
SNR21=W+X+Q;
plot(X,SNR21)
text ('Position',[-70,32],'String', 'linear quantizing n=8');
title ('Sinusoidal input snr features (13 line approximation)')
```

Figure 3 is the result of the program running. Through the Figure can be seen that with the 13 line approximation of Alaw compression feature, SNR curve will appear fluctuations, and it is no longer the smooth curve, there are 6 troughs, 7peaks in total.
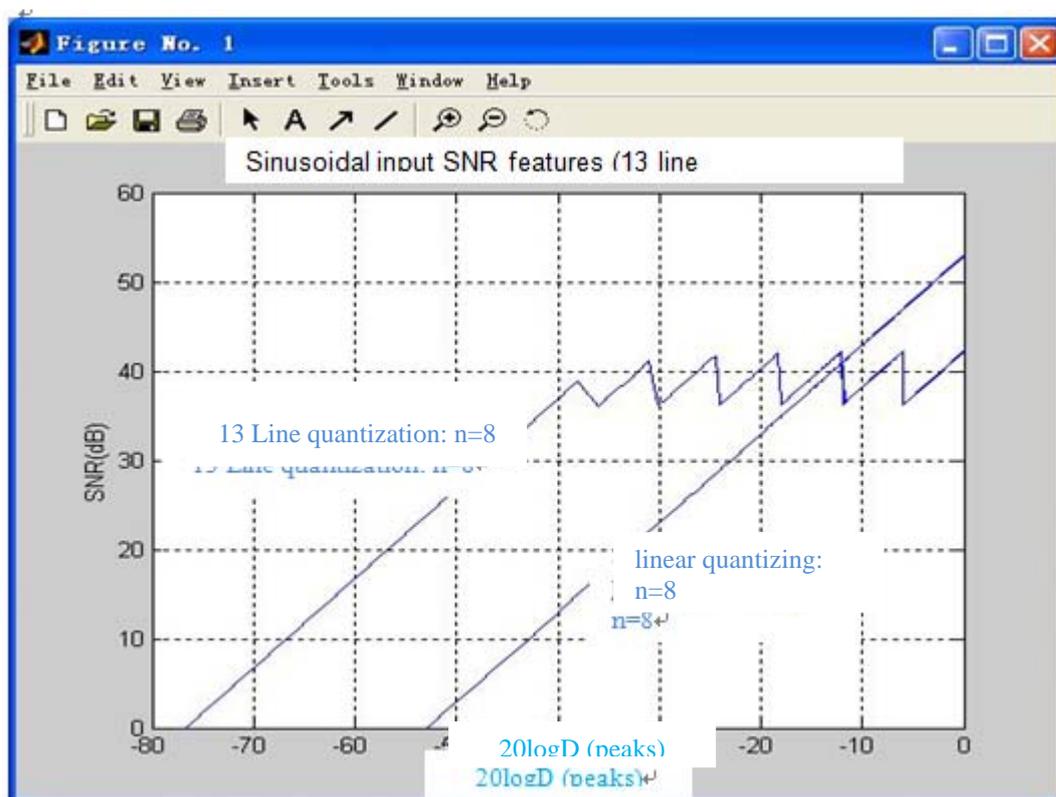
Figure 3. Sinusoidal input SNR features (13 line approximation).

4. Conclusion

PCM has been as the key technology in the modern digital communications, which occupies an important position in the communication engineering. In this paper PCM system simulation was established based on the Matlab and comparative analysis on the performance of the linear PCM and logarithmic PCM, and the given specific examples can provide reference for other communication system simulation analysis.

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