Tracking control for planar nonminimum-phase bilinear control system with disturbance using backstepping

Khozin Mu'tamar, Janson Naiborhu, Roberd Saragih, Dewi Handayani
Department of Mathematics, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Bandung, Indonesia

Article Info

Article history:

Received Jan 13, 2022
Revised Mar 17, 2022
Accepted Mar 29, 2022

Keywords:

Backstepping method
Bilinear control system
Internal dynamic
Nonminimum-phase
Uncertain system

ABSTRACT

This article presents the design control of a tracking problem for a non-minimum phase bilinear control system containing disturbance. The bilinear control system is assumed to have a relative degree one and non-minimum phase, which means it has unstable internal dynamics. The disturbance exists only in state variables corresponding to the control function in external dynamics. The control design was carried out using the backstepping method, which was applied to the normal form of the bilinear control system. Internal dynamics will be stabilised using virtual control to overcome unstable internal dynamics. The last step will stabilise the external dynamics and disturbance using the original control function. The simulation results show that the proposed control method can rapidly drive the output to the given trajectory. Control performance varies depending on the control parameter setting. The higher the control parameter, the better the control performance, evaluated using integral absolute error.

Corresponding Author:

Khozin Mu'tamar
Department of Mathematics, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung
Jl. Ganesha No. 10, Bandung 40132, Jawa Barat, Indonesia
Email: mutamar.khozin@students.itb.ac.id

1. INTRODUCTION

The dynamical system is widely used in modelling and analysing natural phenomena, both physical and social behaviour. The accuracy in the analysis process is highly dependent on the modelling process, including the selection of the type of model used [1], [2]. Models with nonlinear equations are the best form of modelling, but nonlinear systems are complicated to analyse, so approximations are often made to interpret them. The bilinear control system is one of the classes and is the best approximation of the nonlinear control system [3], [4]. The approximation of a nonlinear control system using a bilinear control system can be made using the Carleman transformation [3] or the Jacobian transformation [5]. The advantages of a bilinear control system in approaching a nonlinear control system compared to a linear control system in terms of performance, optimisation and modelling processes [6]–[8]. Intensive studies on the use of bilinear control systems in several fields in economics and chemistry can be referred to [1], [9], [10].

Some examples of the use of other bilinear control systems can be seen in the boost converter problem in [11], [12], on controlling the spread of disease in [13], [14], on directing ship motion in [15], on regulating hot water storage in [16], on the diesel engine fuel collector in [17]. The application of control to the Hilbert chamber using finite feedback has also been carried out [18]. The model's accuracy is also determined from the modelling process carried out. According to Amato [19] the dynamical system contains an uncertainty factor due to simplifying the modelling and parameter estimation processes. The first uncertainty factor occurs in the modelling process because the phenomenon's complexity cannot be fully expressed in a mathematical model and natural phenomena contain a constant disturbance function that cannot always be modelled [2].

Inaccurate estimation of model parameters also results in an uncertainty system. The dynamical system, which represents the phenomenon, will contain parameters whose values cannot always be determined precisely or whose values change at certain time intervals. The uncertainty factor in a mathematical model can be expressed in the form of a function that represents a parameter whose value is constant but unknown, a time-varying parameter, or a function that defines an external disturbance [21]. One of the problems arising from a disturbance function is not achieving asymptotic stability from the system output [22].

One of the methods used in the nonlinear control system is the backstepping method. This method was developed in the 1990s based on the stability of Lyapunov which the process is carried out iteratively [23]. Using the backstepping method can be referred to in [24]–[29]. Generally, the backstepping method is applied to systems with a strict feedback form [30]–[32], which can be obtained using the transformation in [33]. This transformation is a nonlinear transformation that can make the control design more complex.

This article presents the control design of a non-minimum phase bilinear control system containing a disturbance function. The main difference with previous studies in [12]–[14], [16], [18] is in terms of the presence of a disturbance function in the system and the type of system and in [15], [34]–[36] is in terms of the use of the control design method used. The method used in this article is the backstepping method, which develops the previous results in [37]. In [37], it is assumed that the system can be linearised exactly so that it has no internal dynamics, and the control design is only a matter of stabilisation. Compared with the work in [27]–[29], the system used is not in a non-minimum phase. Even though the system contains uncertainty, the absence of internal dynamics makes it possible to design controls directly on the system. We applies backstepping to the normal form obtained through the input-output linearisation transformation using the procedure in [38], [39]. Since there are internal dynamics, they will be stabilised first using virtual controls. The whole system and the disturbance function will be stabilised in the last iterative process using the actual control variables. The novelty aspect of this article includes theories and methods in solving a non-minimum phase system containing a disturbance function. To solve the non-minimum phase system, it is no longer necessary to redefine the system output to become a minimum phase. In addition, the use of backstepping ensures that the internal dynamics can be stabilized to zero or bounded using virtual control.

This article is structured as: After this sections, a research method explains the input and output linearisation transformations applied to a bilinear control system. It also describes the basic backstepping procedure to stabilise the bilinear control system and the method for calculating control performance using integral absolute error (IAE). In the results section, the control design for the non-minimum-phase bilinear control system containing disturbance is described and analysed for its stability. Some examples are provided to simulate control design and to see the performance of the proposed method using several different control parameters.

2. RESEARCH METHOD

Given a bilinear control system with a single input and output:

$$\begin{cases} \dot{x}(t) &= Ax(t) + u(t)Bx(t) \\ y(t) &= h\big(x(t)\big) \end{cases} \tag{1}$$

where $A, B \in \mathbb{R}^{2\times 2}$, $x(t) \in \mathbb{R}^2$ is the state variable, $u(t) \in \mathbb{R}$ is the control variable, and $y(t)$ is the system output. It is assumed that the system output can be expressed as a linear combination of state variables, i.e., $h\big(x(t)\big) = hx(t)$ for $h^T \in \mathbb{R}^2$.

Linearisation of the bilinear control system is done based on input-output feedback using the Lie derivative defined by $L_f h\big(x(t)\big) = \nabla h\big(x(t)\big) f\big(x(t)\big)$. The relative degree of a bilinear control system in (1) is the natural number $\rho \in \mathbb{N}$ that satisfies $L_f^{i-1} h\big(x(t)\big) = hA^{i-1}Bx(t) = 0$ for each $i = 1,2,\dots,\rho-1$ and $L_g L_f^{\rho-1} h\big(x(t)\big) = hA^{\rho-1}Bx(t) \neq 0$. If $\rho < n$, then a bilinear control system has internal dynamics independent of the control variables and if $\rho = n$ is satisfied, then a bilinear control system in (1) is a system that can be linearly exactly. If the system's internal dynamics are unstable, then the system is called a non-minimum phase system.

Assuming the bilinear control system in (1) has a relative degree two which satisfies $L_g h\big(x(t)\big) = 0$. The input and output linearization transformation (IOFL) transformation are given by (2).

$$z = T(x) = \begin{bmatrix} h(x) \\ L_f h(x) \end{bmatrix} \quad (2)$$

The derivative of (2) with respect to t and using its inverse gives:

$$\begin{cases} \dot{z}_1(t) &= z_2(t) \\ \dot{z}_2(t) &= \nu(t) \end{cases} \quad (3)$$

where $\nu(t) = hA^2T^{-1}z(t) + u(t)hABT^{-1}z(t)$ is the new control function that applies to the linear control system in (3). Choose Lyapunov function $V(t) = \frac{1}{2}z_1^2 + \frac{1}{2}(z_2 + r_1 z_1)^2$. The linear control system in (3) can be stabilised using the backstepping method with the control function $\nu(t) = (r_1^2 - 1)\frac{\partial V}{\partial z_1} - (r_1 + r_2)\frac{\partial V}{\partial z_2}$ for $r_1, r_2 \in \mathbb{R}^+$.

It is assumed that the bilinear control system in (1) has a relative degree one to satisfy $L_g h(x(t)) \neq 0$. Defined coordinate transformation using input and output linearization transformation (IOFL) [38], [39].

$$z = T(x) = \begin{bmatrix} h(x) \\ \phi(x) \end{bmatrix} \quad (4)$$

Where $\phi(x)$ is a function that satisfies $L_g\phi(x) = 0$ with $g(x) = Bx(t)$. The derivative of (4) with respect to t and using the inverse transformation of (4) produces:

$$\begin{cases} \dot{z}_1(t) &= \nu(t) \\ \dot{z}_2(t) &= L_f\phi(x) = c_1 z_1(t) + c_2 z_2(t) \end{cases} \quad (5)$$

where $\nu(t) = hAT^{-1}z(t) + u(t)hBT^{-1}z(t)$ is the new control function that applies to the linear control system in (5). In contrast to the system that can be exactly linearised in (3), the control value does not affect internal dynamics. If the controlled output value reaches the path, then the dynamic zero on the internal dynamic is $\dot{z}_2(t) = c_2 z_2(t)$. If $c_2 \in \mathbb{R}^+$, then dynamic zero is unstable so that the system in (1) is the non-minimum phase.

The control function parameter $\{r_1, r_2\}$ in the backstepping method determines the control performance to bring the system output to follow the given path. Performance values are evaluated using the integral absolute error (IAE), which is defined by (6).

$$\text{IAE} = \int_0^{t_f} |y(t) - y_d(t)|\, dt \quad (6)$$

The integral in (6) needs the numerical solution of the system of (1) so that the value of (6) is evaluated using Simpson's method given by;

$$S_n = \frac{h}{3}\left(f_0 + f_n + 4\sum_{i=0}^{\frac{n}{2}} f_{2i+1} + 2\sum_{i=0}^{\frac{n}{2}} f_{2i+2}\right)$$

where n is the number of partitions used, $h = \frac{t_f - t_0}{n}$ is the width of the partition, $t_i = t_0 + ih$ is the partition point for the t domain and $f_i = |y(t_i) - y_d(t_i)|$ for $i = 0,1,2, \dots, n$.

3. RESULTS AND DISCUSSION

This section presents a control design for tracking problems in a bilinear control system with a disturbance function using the backstepping method for a non-minimum phase system. Simulation examples are also given to see the performance of the control in bringing the output along the given trajectory.

Consider a planar bilinear control system with a disturbance function.

$$\begin{cases} \dot{x}(t) &= Ax(t) + u(t)Bx(t) + N\omega(t) \\ y(t) &= h(x(t)) \end{cases} \quad (7)$$

where $A, B \in \mathbb{R}^{2\times 2}$, $x(t) \in \mathbb{R}^2$ is the state variable, $u(t) \in \mathbb{R}$ is the control variable, and $y(t) = h(x(t))$ is the system output assumed to be expressed as a linear combination of the state variables, so that $y(t) = hx(t)$ and $\omega(t) \in \mathbb{R}$ are disturbance functions.

- Assumption 1. The tracking path $y_d(t)$ and its derivatives are smooth and bounded.
- Assumption 2. The disturbance function $\omega(t)$ is unknown but bounded.

3.1. Control design for the system with exact linearisation

Theorem 1, given a bilinear system with disturbance in (7). If the system has a relative degree of two, the control function that stabilises the system globally is $u(t) = \frac{v(t)-hA^2x(t)}{hABx(t)}$, where $v(t)$ is given by:

$$v(t) = \ddot{y}_d(t) - (1 - r_1^2)e_1(t) - (r_1 + r_w)w(t) - \operatorname{sign}(w)\max(d)$$

Proof, it is known that the system has a relative degree of two to be linearised exactly. Using the transformation of (2), we get (8):

$$\begin{cases} \dot{z}_1(t) &= z_2(t) \\ \dot{z}_2(t) &= v(t) + d(t) \end{cases} \tag{8}$$

defined the difference between the output and the given path (9),

$$\begin{cases} e_1(t) &= z_1(t) - y_d(t) \\ e_2(t) &= z_2(t) - \dot{y}_d(t) \end{cases} \tag{9}$$

the derivative of (9) with respect to t and by using (8) gives (10),

$$\begin{cases} \dot{e}_1(t) &= e_2(t) \\ \dot{e}_2(t) &= v(t) + d(t) - \ddot{y}_d(t) \end{cases} \tag{10}$$

where $d(t) = hN\omega(t)$ is the disturbance function and $v(t)$ is the new control variable used in the linearised system of (10), and its relationship to the control variable $u(t)$ is:

$$v(t) = hA^2x(t) + u(t)hABx(t) \tag{11}$$

The first step is to stabilise the state variable $e_1(t)$ using $e_2(t)$ as a virtual control. Define the Lyapunov function $V_1(t) = \frac{1}{2}e_1^2(t)$. The derivative of $V_1(t)$ with respect to t gives:

$$\dot{V}_1(t) = e_1(t)e_2(t)$$

For the state variable $e_1(t)$ to be asymptotically stable, $\dot{V}_1(t) < 0$ must be valid for every t. Select virtual control $e_2(t) = -r_1e_1(t)$ with $r_1 \in \mathbb{R}^+$. Define a new state variable (12):

$$w(t) = r_1e_1(t) + e_2(t) \tag{12}$$

based on (12), we get $e_2(t) = w(t) - r_1e_1(t)$, and by substituting in (10), we get (13).

$$\begin{cases} \dot{e}_1(t) &= w(t) - r_1e_1(t) \\ \dot{w}(t) &= r_1\big(w(t) - r_1e_1(t)\big) + v(t) + d(t) - \ddot{y}_d(t) \end{cases} \tag{13}$$

The final step is to stabilise the entire system in (13) using the variable control $v(t)$. Define the Lyapunov function $V_2(t) = \frac{1}{2}[e_1^2(t) + w^2(t)]$. The derivative of $V_2(t)$ with respect to t will results,

$$\dot{V}_2(t) = -r_1e_1^2(t) + w(t)[r_1w(t) + (1 - r_1^2)e_1(t) + v(t) - \ddot{y}_d(t)] + w(t)d(t) \tag{14}$$

It is assumed that there are $r_w \in \mathbb{R}^+$ and $\Phi \in \mathbb{R}^-$ such that $\dot{V}_2(t) = -r_1e_1^2(t) - r_ww^2(t) - \Phi$. Using this assumption and using (14), it obtained in (15):

$$r_1 w(t) + (1 - r_1^2)e_1(t) + v(t) - \ddot{y}_d(t) = -r_w w(t) + \theta. \tag{15}$$

Substitute (15) into the Lyapunov function in (14) to get (16).

$$\dot{V}_2(t) = -r_1 e_1^2(t) - r_w w^2(t) + w(t)(\theta + d(t)) \tag{16}$$

To satisfy $\dot{V}_2(t) < 0$, select $\theta = -\text{sign}(w)\max(d)$. Substitute $\theta = -\text{sign}(w)\max(d)$ into (16) so that the final form of the derivative of the Lyapunov function is obtained that is (17).

$$\dot{V}_2(t) = -r_1 e_1^2(t) - r_w w^2(t) + |w|(\text{sign}(w)d(t) - \max(d)) \tag{17}$$

Because $\max(d) > d(t)$ then $\text{sign}(w)d(t) - \max(d) < 0$ and $\dot{V}_2(t) < 0$ apply to every $t \geq 0$. Furthermore, the variable control $v(t)$ is obtained from (15) and using the relation between the state variables $\{e_1(t), e_2(t), w(t)\}$ in (12), the following control function is obtained (18).

$$v(t) = \ddot{y}_d(t) - (1 - r_1^2)e_1(t) - (r_1 + r_w)w(t) - \text{sign}(w)\max(d) \tag{18}$$

Using the Lyapunov function $V(e_1, w)$ and its derivatives in (17), we found that the value $\{e_1(t), w(t)\}$ is bounded. Use Assumption 1, and because the value of $y_d(t)$ is bounded, based on (9), the value of $z_1(t)$ is also bounded. Because $\{e_1(t), w(t)\}$ are bounded, from (12), the value of $e_2(t)$ is bounded. As a result, from (9), we obtained $z_2(t)$ is also bounded. Based on the LaSalle-Yoshizawa theorem, we get $\lim_{t\to\infty}(e(t)) = 0$, so $\lim_{t\to\infty}(y(t) - y_d(t)) = 0$ applies.

3.2. Control design for non-minimum phase system

Theorem 2. Consider a bilinear control system in (7) with relative degree $\rho = 1$ and satisfies Assumption 1 and 2. Let $[k_1 \quad k_2] = cAT^{-1}$ with $T = \langle h, c \rangle$ and c is an internal dynamic equation matrix that satisfies $cBx(t) = 0$. The control function that takes the output to the path $y_d(t)$ is $u(t) = \frac{v(t) - hAx(t)}{hBx(t)}$, where the value of $v(t)$ is given by:

$$v(t) = \dot{y}_d(t) + (k_2 + r_2 + r_w)y_d(t) - \frac{M^T Tx(t)}{k_1} + \frac{1}{k_1}M\phi(t) - \frac{|k_1|}{k_1}\text{sign}(w)\max(d)$$

where $M = \begin{bmatrix} k_1(k_2 + r_2 + r_w) \\ 1 + (k_2 + r_w)(k_2 + r_2) \end{bmatrix}$ and the function $\phi(t)$ are bounded solutions of the first-order differential:

$$\dot{\phi}(t) - k_2\phi(t) - k_1 y_d(t) = 0$$

Proof, it is known that the bilinear control system of (7) has a relative degree $\rho = 1$ so that it has internal dynamic and internal dynamic stabilisation will be performed first. Using the transformation in (4), we get (19).

$$\begin{cases} \dot{z}_1(t) &= v(t) + d(t) \\ \dot{z}_2(t) &= k_1 z_1(t) + k_2 z_2(t) \end{cases} \tag{19}$$

Define the difference between the output and the given path,

$$\begin{cases} e_1(t) &= z_1(t) - y_d(t) \\ e_2(t) &= z_2(t) - \phi(t) \end{cases} \tag{20}$$

where $\phi(t)$ is a function to be defined later. The derivative of (20) with respect to t and using (19) produces (21).

$$\begin{cases} \dot{e}_2(t) &= k_1\big(e_1(t) + y_d(t)\big) + k_2\big(e_2(t) + \phi(t)\big) - \dot{\phi}(t) \\ \dot{e}_1(t) &= v(t) + d(t) - \dot{y}_d(t) \end{cases} \tag{21}$$

The first step is to stabilise the state variable $e_2(t)$ using $e_1(t)$ as a virtual control. Define the Lyapunov function $V(e_2) = \frac{1}{2}e_2^2(t)$. The derivative of $V(e_2)$ with respect to t produce:

$$\dot{V}(e_2) = e_2(t)(k_1(e_1(t) + y_d(t)) + k_2(e_2(t) + \phi(t)) - \dot{\phi}(t))$$

the state variable $e_2(t)$ is asymptotically stable if it satisfies $\dot{V}(e_2) < 0$ for every $t \geq 0$. Choose virtual control $k_1(e_1(t) + y_d(t)) + k_2(e_2(t) + \phi(t)) - \dot{\phi}(t) = -r_2 e_2(t)$ with $r_1 \in \mathbb{R}^+$. Define a new state variable (22).

$$w(t) = k_1 e_1(t) + (k_2 + r_2) e_2(t) + k_1 y_d(t) + k_2 \phi(t) - \dot{\phi}(t) \tag{22}$$

Substitute new state variable in (22) and its derivative into (21) to produce (23).

$$\begin{cases} \dot{e}_2(t) &= w(t) - r_2 e_2(t) \\ \dot{w}(t) &= k_1(v(t) + d(t) - \dot{y}_d(t)) + (k_2 + r_2)(w(t) - r_2 e_2(t)) + k_1 \dot{y}_d(t) + k_2 \dot{\phi}(t) - \ddot{\phi}(t) \end{cases} \tag{23}$$

The final step is stabilising the entire system in (23) using the variable control $v(t)$. Define the Lyapunov function $V(e_2, w) = \frac{1}{2}(e_2^2(t) + w^2(t))$. The derivative of $V(e_2, w)$ with respect to t will results (24).

$$\dot{V}(e_2, w) = -r_2 e_2^2(t) + w(t)(e_2(t) + \dot{w}(t)) \tag{24}$$

Substituting (23) into (24) and assuming there are $r_w \in \mathbb{R}^+$ and $\Phi \in \mathbb{R}^-$ such that $\dot{V}(e_2, w) = -r_2 e_2^2(t) - r_w w^2(t) - \Phi$ to get (25).

$$\begin{aligned} &e_2(t) + k_1(v(t) - \dot{y}_d(t)) + (k_2 + r_2)(w(t) - r_2 e_2(t)) + k_1 \dot{y}_d(t) + k_2 \dot{\phi}(t) - \ddot{\phi}(t) \\ &= -r_w w(t) + \theta \end{aligned} \tag{25}$$

Using (25), (24) became (26):

$$\dot{V}(e_2, w) = -r_2 e_2^2(t) - r_2 w^2(t) + w(t)(k_1 d(t) + \theta) \tag{26}$$

To satisfy $\dot{V}(e_2, w) < 0$, choose the value $\theta = -\text{sign}(w)|k_1| \max(d)$. Substitute this value into (26), and we obtain the final form of the derivative of the Lyapunov function, that is:

$$\dot{V}(e_2, w) = -r_2 e_2^2(t) - r_2 w^2(t) + |w(t) k_1|(\text{sign}(k_1)\text{sign}(w) d(t) - \max(d)) \tag{27}$$

Because $\max(d) > d(t)$, then $\text{sign}(k_1)\text{sign}(w) d - \max(d) < 0$ and $\dot{V}(e_2, w) < 0$ apply to every $t \geq 0$.

The control function $v(t)$ is obtained from (25).

$$\begin{aligned} &v(t) = -\frac{1}{k_1} \\ &[(1 - r_2(k_2 + r_2)) e_2(t) + (k_2 + r_2 + r_w) w(t) - \ddot{\phi}(t) + k_2 \dot{\phi}(t) + \text{sign}(w)|k_1| \max(d)] \end{aligned} \tag{28}$$

Using the value of $\{e_1(t), e_2(t)\}$ in (20) and $w(t)$ in (22), the control function $v(t)$ in (28) can be expressed by (29):

$$v(t) = \dot{y}_d(t) + (k_2 + r_2 + r_w) y_d(t) - \frac{M^T T x(t)}{k_1} + \frac{1}{k_1} M \phi(t) - \frac{|k_1|}{k_1} \text{sign}(w) \max(d) \tag{29}$$

with $M = \begin{bmatrix} k_1(k_2 + r_2 + r_w) \\ 1 + (k_2 + r_w)(k_2 + r_2) \end{bmatrix}$.

Note that the control function in (29) requires a value of $\phi(t)$. Consider the $w(t)$ function in (22); $w(t) = k_1 e_1(t) + (k_2 + r_2) e_2(t) + k_1 y_d(t) + k_2 \phi(t) - \dot{\phi}(t)$. The state variables $e_2(t)$ and $w(t)$ have been controlled to zero at each stage. Furthermore, it is expected that $e_1(t) \approx 0$ to obtain $z_1(t) \approx y_d(t)$. Assume that for a value $t \geq T$ that satisfies $e_2(t), w(t) \approx 0$, from (22), we will get $k_1 e_1(t) = \dot{\phi}(t) - k_2 \phi(t) - k_1 y_d(t)$. Next, to obtain $e_1(t) = 0$, the value of $\phi(t)$ is obtained from an ordinary first-order differential equation.

$$\dot{\phi}(t) - k_2 \phi(t) - k_1 y_d(t) = 0 \tag{30}$$

The solution of (30) is a path for internal dynamics, and it depends on the initial value of $\phi(t_0)$ and the path $y_d(t)$. In order to make bounded the internal dynamic, it is necessary to determine the initial value of $\phi(t_0)$ so that the solution of (30) is bounded for every $t \geq 0$.

3.3. Simulations

This section gives some examples to illustrate the implementation of the controls described in the previous section. Example 1. Consider the dynamics of the paper cutting machine [34], which is expressed by (31):

$$\begin{cases} \dot{x}_1(t) &= -0.046x_1(t) - 0.027x_1(t)u(t) + 0.978d \\ \dot{x}_2(t) &= -0.7632x_1(t) + 3.197x_2(t) \end{cases} \tag{31}$$

Suppose $v(t) = -0.046x_1(t) - 0.027x_1(t)u(t)$ then (31) can be written as (32).

$$\begin{cases} \dot{x}_1(t) &= v(t) + 0.978d \\ \dot{x}_2(t) &= -0.7632x_1(t) + 3.197x_2(t) \end{cases} \tag{32}$$

As shown in (32) has formed a normal system with a control function $v(t)$ with an internal dynamic of order one. If $x_1 = 0$, then zero dynamics is $\dot{x}_2(t) = 3.197x_2(t)$, which is unstable and consequently is a non-minimum phase system with relative degrees $\rho = 1$. Simulations were carried out using the data in Table 1. Using Table 1, the control function in (18), which takes the output to the origin, is $v(t) = -1.3103(17.5910x_2 - 11.1970w - 0.7632\,\mathrm{sign}(w))$. Figure 1 shows the simulation results for the stabilisation problem. Figure 1(a) shows that the proposed control function with several parameter values variations successfully stabilises the system output to the origin. The system output initially moves from the initial value away from the origin to a certain maximum point and then towards the origin. The convergence behaviour of the system output is strongly influenced by the value of the backstepping control parameter. The maximum point achieved for a small combination of control parameters is smaller, but the system output takes longer to converge to the origin. As for larger combinations, it takes a relatively short time to converge, but the system will move to a larger maximum point. Figure 1(b) shows the internal dynamics for each backstepping control parameter. In contrast to the system output, the internal dynamics controlled with large parameters move from the initial value to the origin directly. Meanwhile, the internal dynamics, which are controlled with small parameters, allow the internal dynamics to move away from the origin until a certain maximum point before moving towards the origin.

Table 1. Simulation parameters

Parameter	Value	Description
r_2	{3,7,11}	Backstepping parameter
r_w	{5,9,15}	Backstepping parameter
$d(t)$	$rand() \times sin(t)$	Disturbance
k_1	-0.7632	Coefficient variable internal dynamic
k_2	3.197	Coefficient variable external dynamic

Next up for the tracking problem, we use the path $y_d(t) = \cos(t)$. The path for internal dynamics obtained from the equation $\phi(t)$ is obtained from the solution of the following ordinary differential equation.

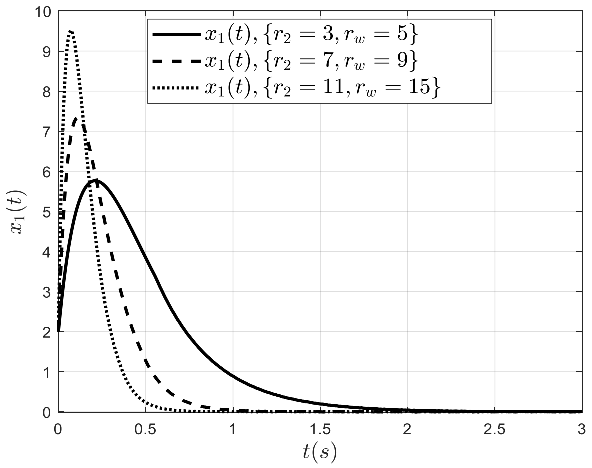
$$\frac{d\phi(t)}{dt} - 3.197\phi(t) + 0.7632\cos(t) = 0 \tag{33}$$

If the initial value is $\phi(t_0) = \phi_0$, then the solution of (33) is (34).

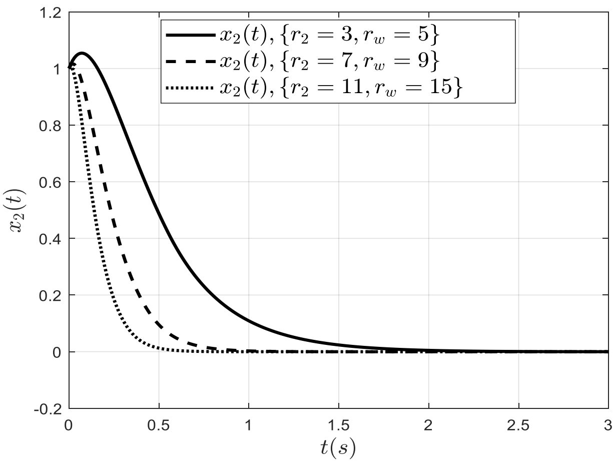
$$\phi(t) = 0.217\cos(t) - 0.068\sin(t) + \exp(3.197)(\phi_0 - 0.217) \tag{34}$$

In order for (34) to be bounded, the term $\exp(3.197t)$ needs to be eliminated. Selecting the initial value of $\phi_0 = 0.217$ will produce a bounded solution of (34) as well as a particular solution, namely $\phi(t) = 0.217\cos(t) - 0.068\sin(t)$. The simulation result for the tracking problem with the path $y_d(t) = \cos(t)$ is shown in Figure 2. Figure 2(a) compares the system's output dynamics with the trajectory. The system output has converged to the trajectory at $t \approx 1$. At $t \in [1,10]$, the system's output and the trajectory remain attached, and there is no dynamic spike even though the system is disturbed. Figure 2(b) shows the internal dynamics. Internal dynamics move from the initial point following a path $\phi(t)$ whose value is bounded at

$[-0.5,0.5]$. This indicates that the backstepping control is successful in bringing the system output to the trajectory while keeping the internal dynamics to a minimum.



(a)



(b)

Figure 1. Comparison of state variable dynamics on paper cutting machines with varying control parameter values with (a) system output and (b) internal dynamic

Example 2 given a bilinear control system with a disturbance function $\dot{x}(t) = Ax(t) + u(t)Bx(t) + N\omega(t)$ with the system output $y(t) = x_2(t)$ and the parameter values are given in [40].

$$A = \begin{bmatrix} 3/16 & 5/12 \\ -50/3 & -8/3 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, N = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{35}$$

The disturbance function is generated using the equation $\omega(t) = \mathrm{rand}(\quad) \times \sin(t)$. The purpose of the control is to drive the system output to the origin $y_d(t) = 0$. For the control parameters, the values of $\{r_1 = 1, r_w = 3\}$ and $\{r_1 = 5, r_w = 7\}$ are used. Comparison of computational results between the proposed method and the robust control method H∞ [13], [40] is shown in Figure 3.

Figure 3 shows that both control methods can bring the output to the origin relatively quickly. Compared with the robust H∞ control, the results depend on the combination of $\{r_1, r_w\}$ values. In the backstepping method, the control performance shows significantly different results for the two variations of the given $\{r_1, r_w\}$ value. For the $\{r_1 = 1, r_w = 3\}$ combination, the backstepping method produces worse performance than the H∞ robust control method. The system output controlled using backstepping takes longer to converge to the origin. Still, the robust H∞ control method produces a higher minimum output value at the beginning of the simulation. By changing the value of the control parameter $\{r_1, r_w\}$ to be larger, the convergence of the backstepping method increases rapidly. The $\{r_1 = 5, r_w = 7\}$ combination can bring the system output with a minimal minimum value to the origin in time $t \approx 0.5$, which is 700% faster than the $\{r_1 = 1, r_w = 3\}$ variation, which takes time $t \approx 3.7$.

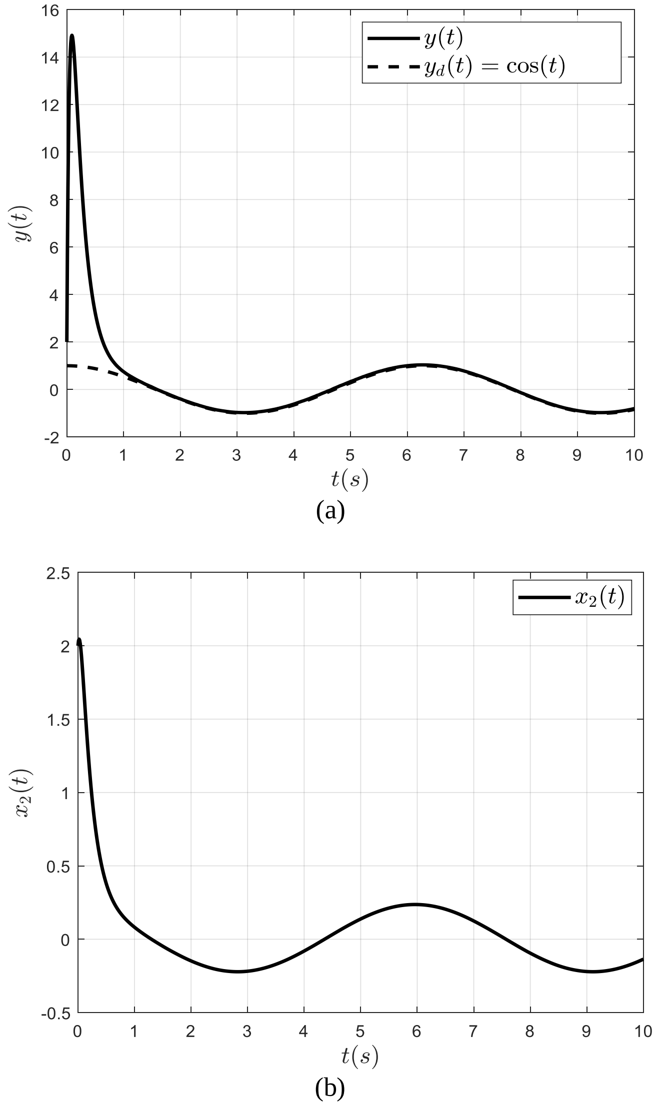


Figure 2. Comparison of state variable behaviour in the tracking problem with (a) system output versus the path and (b) internal dynamic

For tracking problems, select the path $y_d(t) = \cos(t)$. The implementation of the control function in (29) using several variations $\{r_1, r_w\}$ is shown in Figure 4. The calculation of the error value between the system output and the path using IAE for each $\{r_1, r_w\}$ combination is shown in Table 2.

Figure 4 shows, with all given combinations value of $\{r_1, r_w\}$, the system output can follow the path with identical behaviour but at different speeds. At the start of time, the system output moves away from and against the given path. The system output moves back to the given path at a maximum point, shown at time $t \approx 0.7$. The smaller the combination of $\{r_1, r_w\}$ values, the higher the system output value and the longer it takes to converge to the path. These results provide a different IAE value for each combination $\{r_1, r_w\}$ with the characteristic that the lower the combination value, the higher the IAE value for the combination. Table 2 shows the IAE values of each combination shown in Figure 4. The combination $\{r_1 = 3, r_w = 3\}$, the smallest combination, gives the largest IAE value, while the combination $\{r_1 = 12, r_w = 9\}$, the largest combination, gives the smallest IAE value.

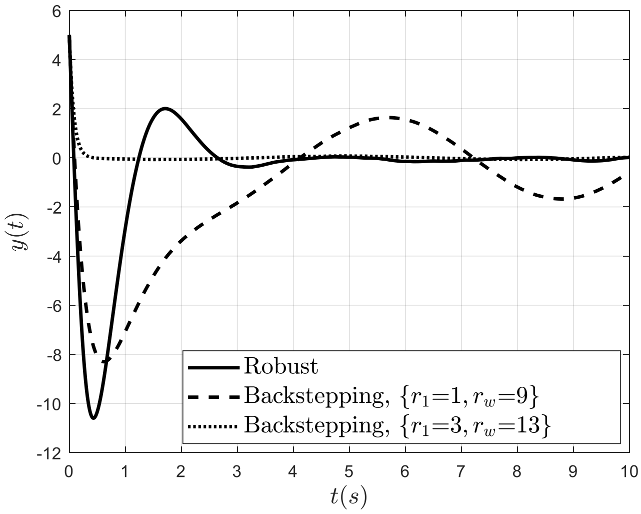


Figure 3. Comparison of the system output stabilised to the origin using the backstepping method with two variations of $\{r_1, r_w\}$ and the robust H∞ control method

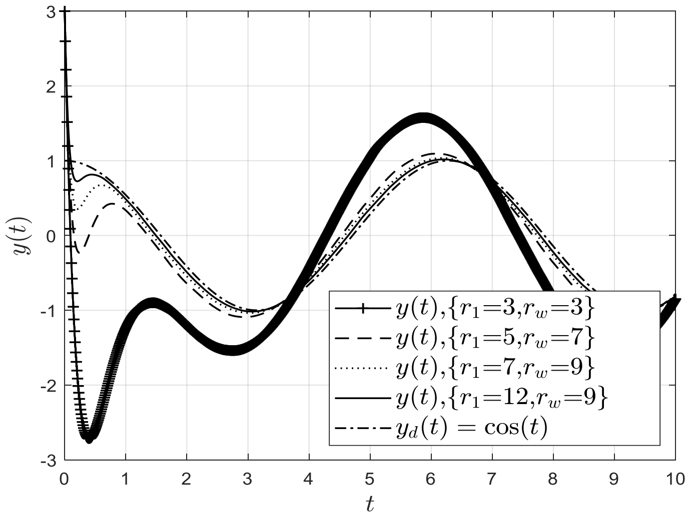


Figure 4. The effect of variations in the value of $\{r_1, r_w\}$ on control performance

Table 2. IAE values based on the combination $\{r_1, r_w\}$ shown in Figure 4

$\{r_1, r_w\}$	IAE	ToC (seconds)
$\{r_1 = 3, r_w = 3\}$	7.63929	0.58891
$\{r_1 = 5, r_w = 7\}$	1.96321	0.54580
$\{r_1 = 7, r_w = 9\}$	1.04368	0.54813
$\{r_1 = 12, r_w = 9\}$	0.60237	0.54495

Table 2 does not adequately explain the change in IAE value due to changes in the value of the combination of $\{r_1, r_w\}$. Next, the IAE calculation is performed using a combination of partitioned $(r_1, r_w) = [1,25] \times [1,25]$ with $\Delta r = 0.5$, and the results are shown in Figure 5. The highest IAE value occurs for the combination of $\{r_1 = 1, r_w = 1\}$, and the lowest IAE value occurs for the combination of $\{r_1 = 25, r_w = 25\}$.

Statistically, the correlation level can be calculated from the combination $\{r_1, r_w\}$ to the IAE value. Using Spearman's non-parametric correlation test, r_1 to IAE is -0.68043 while r_w to IAE is -0.68240 with 1% significance level. This means that $\{r_1, r_w\}$ has enough correlation to lower the IAE value. Furthermore, if multiple regression is performed to obtain the IAE function for the $\{r_1, r_w\}$ combination, the equation $\text{IAE} = -0.10646r_1 - 0.10657r_w + 3.64456$ is obtained. Although the weights of the two coefficients

$\{r_1, r_w\}$are different, the difference is very small. We say that both $\{r_1, r_w\}$ give almost equal weight to changes in IAE values.

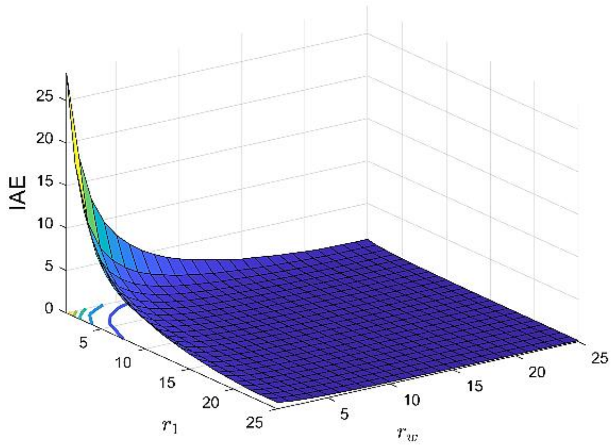


Figure 5. IAE value for all combinations of $(r_1, r_w) = [1,25] \times [1,25]$

4. CONCLUSION

This article discussed the control design for tracking problems using the backstepping method for a non-minimum phase bilinear control system containing disturbance. The disturbance function is assumed to exist only in dynamic externals. The bilinear control system is converted into a normal form using a linearisation transformation based on input and output. The control function is designed using the backstepping method from the normal form. For systems that have internal dynamics, the internal dynamics are stabilised first. For the tracking problem, a path has been defined for the internal dynamics so that the system output follows the path, and the internal dynamics remains bounded. Based on the simulation, the proposed control function successfully brings the output to follow the trajectory for stabilisation problems and tracking problems. The selection of the right parameters will affect the performance of the control and, for a certain value, results in better performance than the Robust method on stabilisation problems. The combination of control parameters is almost symmetrical and inversely proportional to the resulting IAE value.

ACKNOWLEDGEMENTS

This research was financially supported by the 2021 ITB Research Funding. We also thank the reviewers for their suggestions.

REFERENCES

[1] R. R. Mohler, "Bilinear control processes: With applications to engineering, ecology, and medicine," in *Mathematics in Science and Engineering*, vol. 106, 1973, pp. 193–214.

[2] X. Wang and Y. Jiang, "Model reduction of discrete-time bilinear systems by a Laguerre expansion technique," *Applied Mathematical Modelling*, vol. 40, no. 13–14, pp. 6650–6662, Jul. 2016, doi: 10.1016/j.apm.2016.02.015.

[3] Z. Bai and D. Skoogh, "A projection method for model reduction of bilinear dynamical systems," *Linear Algebra and Its Applications*, vol. 415, no. 2–3, pp. 406–425, 2006, doi: 10.1016/j.laa.2005.04.032.

[4] H. R. Shaker, "Optimal filtering scheme for bilinear discrete-time systems: A linear matrix inequality approach," *IFAC Proceedings Volumes*, vol. 45, no. 13, pp. 302–306, 2012, doi: 10.3182/20120620-3-DK-2025.00051.

[5] R. Chabour, G. Sallet, and J. C. Vivalda, "Stabilization of nonlinear systems: A bilinear approach," *Mathematics of Control, Signals, and Systems*, vol. 6, no. 3, pp. 224–246, Sep. 1993, doi: 10.1007/BF01211621.

[6] R. E. Rink and R. R. Mohler, "Completely controllable bilinear systems," *SIAM Journal on Control*, vol. 6, no. 3, pp. 477–486, Aug. 1968, doi: 10.1137/0306030.

[7] R. R. Mohler and R. E. Rink, "Control with a multiplicative mode," *Journal of Fluids Engineering*, Transactions of the ASME, vol. 91, no. 2, pp. 201–205, Jun. 1969, doi: 10.1115/1.3571061.

[8] R. R. Mohler, "Natural bilinear control processes," *IEEE Transactions on Systems Science and Cybernetics*, vol. 6, no. 3, pp. 192–197, 1970, doi: 10.1109/TSSC.1970.300341.

[9] C. Bruni, G. Koch, and G. Dipillo, "Bilinear systems: An appealing class of 'nearly linear' systems in theory and applications," *IEEE Transactions on Automatic Control*, vol. 19, no. 4, pp. 334–348, Aug. 1974, doi: 10.1109/TAC.1974.1100617.

[10] D. Elliott, "Bilinear control systems: matrices in action," *Choice Reviews Online*, vol. 47, no. 03, pp. 47-1477-47–1477, Nov. 2009, doi: 10.5860/CHOICE.47-1477.

[11] S. Bichiou, M. K. Bouafoura, and N. Benhadj Braiek, “Time optimal control laws for bilinear systems,” *Mathematical Problems in Engineering*, vol. 2018, pp. 1–10, 2018, doi: 10.1155/2018/5217427.
[12] D. Gao, Q. Yang, M. Wang, and Y. Yu, “Feedback linearization optimal control approach for bilinear systems in CSTR chemical reactor,” *Intelligent Control and Automation*, vol. 03, no. 03, pp. 274–277, 2012, doi: 10.4236/ica.2012.33031.
[13] A. K. Arum, R. Saragih, and D. Handayani, “Bilinear robust control design for virotherapy model,” in *2019 19th International Conference on Control, Automation and Systems (ICCAS)*, Oct. 2019, vol. 2019-Octob, pp. 82–86, doi: 10.23919/ICCAS47443.2019.8971508.
[14] O. Balatif, M. Rachik, I. Abdelbaki, and Z. Rachik, “Optimal control for multi-input bilinear systems with an application in cancer chemotherapy,” *International Journal of Scientific and Innovative Mathematical Research*, vol. 3, no. 2, pp. 22–31, 2015.
[15] H. Q. Nguyen, A. D. Tran, and T. T. Nguyen, “The bilinear model predictive method-based motion control system of an underactuated ship with an uncertain model in the disturbance,” *Processes*, vol. 7, no. 7, p. 445, Jul. 2019, doi: 10.3390/pr7070445.
[16] Z. Ogonowski, “Two-state bilinear predictive control for hot-water storage tank,” in *Proceedings of the 18th International Conference on Process Control*, 2011, pp. 408–414, [Online]. Available: http://www.kirp.chtf.stuba.sk/pc11.
[17] S. Mechhoud and T. M. Laleg-Kirati, “Observer-based bilinear control of first-order hyperbolic PDEs: Application to the solar collector,” in *Proceedings of the IEEE Conference on Decision and Control, Dec. 2015, vol. 54th IEEE Conference on Decision and Control, CDC 2015*, pp. 4909–4914, doi: 10.1109/CDC.2015.7402986.
[18] A. Tsouli and A. Boutoulout, “Constrained feedback stabilization for bilinear parabolic systems,” *Intelligent Control and Automation*, vol. 06, no. 02, pp. 103–115, 2015, doi: 10.4236/ica.2015.62011.
[19] F. Amato, *Robust control of linear systems subject to uncertain time-varying parameters*, vol. 325. Springer Berlin Heidelberg, 2006.
[20] R. K. Yedavalli, *Robust control of uncertain dynamic systems*. New York, NY: Springer New York, 2014.
[21] M. Corless and G. Leitmann, “Deterministic control of uncertain systems. A Lyapunov theory approach,” in *Deterministic Control of Uncertain Systems, Institution of Engineering and Technology*, 1990, pp. 220–251.
[22] S. Li, J. Yang, W. H. Chen, and X. Chen, *Disturbance observer-based control: Methods and applications*. CRC Press Taylor & Francis Group, 2016.
[23] H. G. Tanner and K. J. Kyriakopoulos, “Backstepping for nonsmooth systems,” *Automatica*, vol. 39, no. 7, pp. 1259–1265, Jul. 2003, doi: 10.1016/S0005-1098(03)00081-5.
[24] C. Ren, J.-H. Ji, H.-Y. Yan, H. Zhang, and J.-Z. Yue, “A backstepping control method for mobile robot path tracking,” in *Proceedings of the 3rd Annual International Conference on Mechanics and Mechanical Engineering (MME 2016)*, 2017, doi: 10.2991/mme-16.2017.94.
[25] Y. Ihedrane, C. El Bekkali, M. El Ghamrasni, S. Mensou, and B. Bossoufi, “Improved wind system using non-linear power control,” *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 14, no. 3, pp. 1148–1158, Jun. 2019, doi: 10.11591/ijeecs.v14.i3.pp1148-1158.
[26] A. Bouchaib, R. Taleb, A. Massoum, and S. Mekhilef, “Geometric control of quadrotor UAVs using integral backstepping,” *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 22, no. 1, p. 53, Apr. 2021, doi: 10.11591/ijeecs.v22.i1.pp53-61.
[27] H. Pang, R. Yao, P. Wang, and Z. Xu, “Adaptive backstepping robust tracking control for stabilizing lateral dynamics of electric vehicles with uncertain parameters and external disturbances,” *Control Engineering Practice*, vol. 110, p. 104781, May 2021, doi: 10.1016/j.conengprac.2021.104781.
[28] A. H. Mary, A. H. Miry, and M. H. Miry, “System uncertainties estimation based adaptive robust backstepping control for DC DC buck converter,” *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 11, no. 1, p. 347, Feb. 2021, doi: 10.11591/ijece.v11i1.pp347-355.
[29] M. Madark, A. Ba-razzouk, E. Abdelmounim, and M. El Malah, “Adaptive backstepping control of induction motor powered by photovoltaic generator,” *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 11, no. 4, p. 2842, Aug. 2021, doi: 10.11591/ijece.v11i4.pp2842-2855.
[30] V. Mañosa, F. Ikhouane, and J. Rodellar, “Control of uncertain non-linear systems via adaptive backstepping,” *Journal of Sound and Vibration*, vol. 280, no. 3–5, pp. 657–680, Feb. 2005, doi: 10.1016/j.jsv.2003.12.033.
[31] S. Alshamali and E. Aljuwaiser, “Design of robust observer-based backstepping control for a satellite control system,” *Mathematical Problems in Engineering*, vol. 2019, pp. 1–9, May 2019, doi: 10.1155/2019/7412194.
[32] J. Wu and Y. Lu, “Adaptive backstepping sliding mode control for boost converter with constant power load,” *IEEE Access*, vol. 7, pp. 50797–50807, 2019, doi: 10.1109/ACCESS.2019.2910936.
[33] H. K. Khalil, “Nonlinear system,” in *Encyclopedia of Neuroscience*, 3rd ed., Berlin, Heidelberg: Springer Berlin Heidelberg, 2008, pp. 2890–2890.
[34] P. Shi, S.-P. Shue, Y. Shi, and R. K. Agarwal, “Controller design for bilinear systems with parametric uncertainties,” *Mathematical Problems in Engineering*, vol. 4, no. 6, pp. 505–528, 1999, doi: 10.1155/S1024123X98000945.
[35] E. K. Kostousova, “On feedback target control for uncertain discrete-time bilinear systems with state constraints through polyhedral technique,” in *AIP Conference Proceedings*, 2017, vol. 1895, p. 110004, doi: 10.1063/1.5007410.
[36] D. D’Alessandro and J. T. Hartwig, “Dynamical decomposition of bilinear control systems subject to symmetries,” *Journal of Dynamical and Control Systems*, vol. 27, no. 1, pp. 1–30, Jan. 2021, doi: 10.1007/s10883-020-09488-0.
[37] K. Mu’tamar, J. Naiborhu, and R. Saragih, “Stabilization of bilinear control system with disturbance using exact linearization and backstepping (In Indonesian*: Stabilisasi sistem kontrol bilinier dengan gangguan menggunakan linierisasi eksak dan backstepping*),” *Jurnal Teknologi*, vol. 14, no. 1, pp. 55–63, Jun. 2021, doi: 10.34151/jurtek.v14i1.3549.
[38] D. Ho and J. Karl Hedrick, “Control of nonlinear non-minimum phase systems with input-output linearization,” in *Proceedings of the American Control Conference*, Jul. 2015, vol. 2015-July, pp. 4016–4023, doi: 10.1109/ACC.2015.7171957.
[39] J. Naiborhu, Firman, and K. Mu’tamar, “Particle swarm optimization in the exact linearization technic for output tracking of non-minimum phase nonlinear systems,” *Applied Mathematical Sciences*, vol. 7, no. 109–112, pp. 5427–5442, 2013, doi: 10.12988/ams.2013.37425.
[40] B. S. Kim and M. T. Lim, “Robust H∞ control method for bilinear systems,” *International Journal of Control, Automation and Systems*, vol. 1, no. 2, pp. 171–177, 2003, [Online]. Available: https://www.koreascience.or.kr/article/JAKO200311921711595.page.

BIOGRAPHIES OF AUTHORS

Khozin Mu'tamar received the B.Sc. degree in Mathematics from the Faculty of Mathematics and Natural Sciences, Universitas Diponegoro, Indonesia in 2010, and M.Sc. degree in Mathematics from the Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung in 2013. He is currently pursuing a PhD degree in Mathematics from the Department of Mathematics, Institut Teknologi Bandung since 2019. He is also a Lecturer with the Mathematics Department at the Universitas Riau since 2014. His current research interests include optimal control in the epidemic models, nonlinear control systems, and numerical optimization. He can be contacted at email: mutamar.khozin@students.itb.ac.id, khozin.mutamar@lecturer.unri.ac.id.

Janson Naiborhu received a B.Sc., M.Sc. and PhD from the Department of Mathematics, Institut Teknologi Bandung, Indonesia in 1989, 1992 and 1995, respectively. He is currently a Professor in the Department of Mathematics, Institut Teknologi Bandung, Indonesia. His current research interests include nonlinear control systems and optimisation. She can be contacted at email: janson@math.itb.ac.id.

Roberd Saragih received B.Sc. and M.Sc. from the Department of Mathematics, Faculty of Mathematics and Natural Sciences in 1986 and 1993, respectively. He received a PhD in Mechanical Engineering from Keio University, Japan in 1998. From 1989, He joined the Department of Mathematics, Institut Teknologi Bandung, Indonesia, where he is currently a Professor of Mathematics. His general area of interests is robust control, system theory and stochastic control. He can be contacted at email: roberd@math.itb.ac.id.

Dewi Handayani received the B.Sc., M.Sc. and Ph.D from Department of Mathematics, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung in 2007, 2011 and 2017 respectively. She is currently a Lecturer and Researcher in Department of Mathematics, Institut Teknologi Bandung, Indonesia. Her current research interests include optimal control for epidemiology model and stochastic control. She can be contacted at email: dewi.handayani@math.itb.ac.id.