The measurement set representation of the body posture based on group theory

Changjian Deng, Xiaona Xie
School of Control Engineering, Chengdu University of Information Technology, Chengdu, China

Article Info

Article history:

Received Jun 22, 2021
Revised Dec 20, 2021
Accepted Jan 11, 2022

Keywords:

Attitude sensor
Body posture
Group theory
Lie group
Representation

ABSTRACT

The foundation of measurement is the representation of measurement set. The paper proposes a body posture measurement set representation method based on concepts of group, fields, and ring. It attempts to explore the intrinsic relationship among numerous different measurement set. The attitude sensor is used to measure the attitude, and the measurement set representation and processing are analyzed based on the lie group theory in the paper. In the paper, the paper maps body posture data into image, and it is easier to identify the error of posture measurement data. Meanwhile, the simulation and test results show that the method can represent body posture data and detect it's defects easily.

Corresponding Author:

Changjian Deng
School of Control Engineering, Chengdu University of Information Technology
No. 24, Block 1, Xuefu Road, Chengdu City, China
Email: chenglidcj@cuit.edu.cn

1. INTRODUCTION

Normally, measurement refers to a group of operations to determine the "quantity value". The purpose of measurement is to make the quantity information contained in the object obvious and comparable. In complex and specific application, the measured quantity may no longer be a scattered string of irrelevant data. These data and their changes may be manifolds or groups. Exploring these relationships is an important method to ensure the accuracy of measurement and control. This paper mainly studies this problem. The development of group theory provides a basis for these studies. Nowadays, group theory has many applications [1]. For example, it is also a powerful tool in signal processing [2], [3], signal representation [4], and mechanics [5]. This paper is mainly concerned with the representation of group theory. The related research of representation includes two catalogues, one is signal representation and detection [6]-[9], and the other is deep learning and object tracking [10]-[12]. The paper focuses on measurement set representation based on group theory. The contributions of paper include: 1) it proposes image representation method; 2) it presents body posture representation and analysis method based on lie group. The paper is organized as follows: in section 2, we present the related work. In section 3, we present the measurement sets and group representation. Section 4 focusing on measurement set map. The experiments and simulation are presented in section 5. The section 6 is conclusion.

2. MEASUREMENT SETS AND GROUP REPRESENTATION

2.1. Fields of measurement

The measurement processing often is undertaken in different domains, for example: the time -frequency electronic magnetic domain; the tensor mechanical domain; the thermotical domain; energy and mass domain; time and space domain; and so on. In nature, most quantity of entity to be measured is represented by signal as signal processing is algebraic in nature. The measurement processing is algebraic in nature:

- The real voltage signal set V in time domain is a group.
- The real voltage signal set V in time domain is a ring.
- The real voltage signal set V in time domain is a field.

Generally, measurable sets normally itself and its map are fields. Domain is structured sets. In group analysis, lagrange theorem and homomorphism principle in group theory are the basis of the analysis of measurement set.

2.2. Continue medium measurement sets

The measurement set may come from different time test, different space test, or different circumstance test. In a rigid body, architecture, material, quantity form their different part and their motivation form a continue measurement sets. Here we discuss two kinds of problem. They may be useful in sensor, motivation analysis, and so on.

2.2.1. The first kind of problem is performance analysis and defect detection of material in sensor technology

For example, the analysis the dielectric coefficient, piezoelectric coefficient, piezo-resistance coefficient, and so on. There are some related software examples of group in engineering application. Puschel and Moura [13] proposed the ISOTROPY software tools, it collections software which applies group theoretical methods to the analysis of phase transitions in crystalline solids.

Method of analysis the measurement related coefficient: as an introduction, here give an example of dielectric coefficient of cubic dielectric capacitance sensor. And it can be proved that this coefficient is scalar quantity. Prove [14]:
assume the tensor of dielectric coefficient is $\epsilon_{\alpha\beta}$ ($\alpha, \beta = 1, 2, 3$), then:

$$D_\alpha = \sum_{\beta=1}^{3} \epsilon_{\alpha\beta} E_\beta$$

select axis, let $E = Ej$

$$D_\alpha = \sum_{\alpha,\beta} \epsilon_{\alpha\beta} E \delta_{\beta_2} = \epsilon_{\beta_2} E = E_{\alpha y} E (\alpha = x, y, z)$$

let axis ‘y’ as basis axis of rotation, do C_4^1 operation:

$$z \to z' = x, x \to x' = -z,$$

then, $D'_x = D_z, D'_z = -D_x$., with the invarity property of C_4 , so, $D'_x = D_x, D'_z = D_z$., then we get:

$$\epsilon_{xy} = \epsilon_{zy}$$

$$\epsilon_{xy} = -\epsilon_{zy}$$

so,$\epsilon_{xy} = \epsilon_{zy} = 0$, and it is same as:

$$\epsilon_{xz} = \epsilon_{yz} = \epsilon_{yx} = \epsilon_{zx} = 0$$

then let E along the diagonal of a cube (C_3 axis), then:

$E_x = E_y = E_z = \frac{1}{\sqrt{3}} E$, then:

$D_x = \frac{1}{\sqrt{3}}\epsilon_{xx}E$

$D_y = \frac{1}{\sqrt{3}}\epsilon_{yy}E$

$D_z = \frac{1}{\sqrt{3}}\epsilon_{zz}E$

do C_3^1 and C_3^2 operation, and consider the invarity of C_3. Then $D_x = D_y = D_z$ and $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = \epsilon$, and get the solution.

2.2.2. The another kind of problem is faults or distortion caused by material defects, the important tool is homotropy group

Nowadays, point defect research in semiconductors has gained remarkable new momentum due to the identification of special point defects that can implement qubits and single photon emitters with unique characteristics. The defect of continuum can be represented using homotropy group. Homotropy groups in order parameter space $T = G/H$ are used to represent and classify defects. The union of defects is obtained by multiplication of homotropy groups.

In the plane ordered space field T, the winding number n is an important parameter. For the images with the same winding number n, they can be changed into the same features by local adjustment. And the kind of defects with same n can be transformed into each other, which is barrier free in topology. They are same homotropy group. Method of analysis the defect of planar spin system, planar spin system [14]:
Its Order parameter: $S = icos\theta + jsin\theta$
Transformation group: G=T(1); $T\phi(\theta) \rightarrow \theta - \phi$
Isotropic group: H=$T_{a\pi}(n), n = 0, \pm1, \pm2, ...,$
Discrete group: $H_0 = e$.
So, $\prod_1(T) = H = Z$, this is additive group of integers.

2.3. Measurement sets representation method

As shown in Table 1, different measurement processing has different method to represent their parameters. Some rules are :
a.) Parameters (measurement sets) can represent the main property of object visually, mapping them into graph-based data though positive and negative direction calculation (mapping). Table 1 give some exist method and its extending. An example is shown in Figure 1.
b.) This graph-based measurement set should be measurable and has some defined mathematical feature (or Algebraic). Table 2 is an example of body posture measurement set and its group representation.

Table 1. Parameters representation

Properties	Graph method
Infra thermo-image	Pixel temperature map to image
Nature sound	Time-frequency mapping
Automotive radar	Geometry parameters mapping
Soil temperature of different place	Temperature map to image

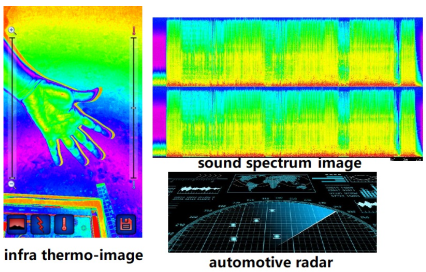

Figure 1. The diagram of measurement set map

Table 2. Parameters measurable representation

Parameters	Algebraic method
Acceleration and Gyroscope signal data set	Perfect linear matrix group
Eulerian angle measurement data	Perfect linear matrix group
Defect data	Homotropy group

When we map these data into image, it is easy to see some defect of sensor or measurement error. The Figure 2 is acc data of three different sensor, the Figure 3 is their gyro data. The measurement environment is static. So, it is very clear that there is something wrong in sensor 2. And this is true case. We use Kalman filter and other methods to deal with the acc data and gyro data, then we get the Eulerian angle data. It is shown in Figure 4.

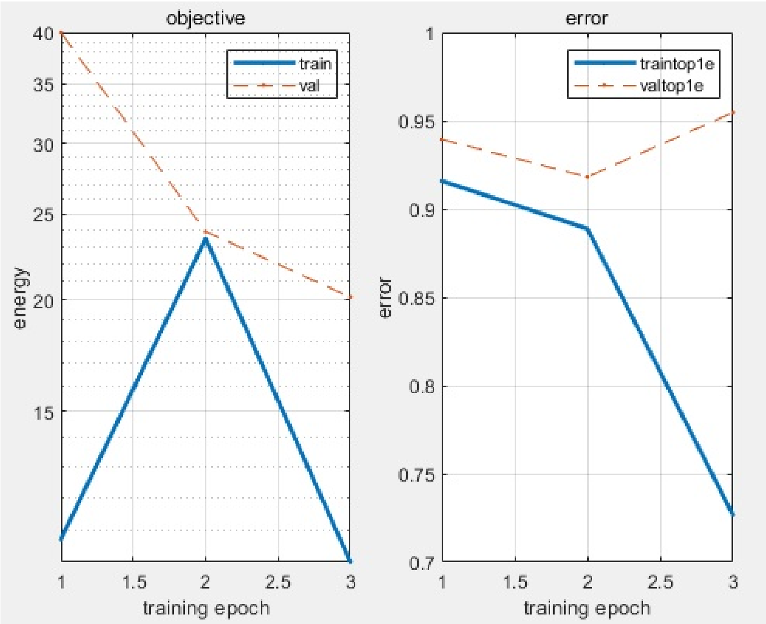

Figure 2. The acc measurement set mapping

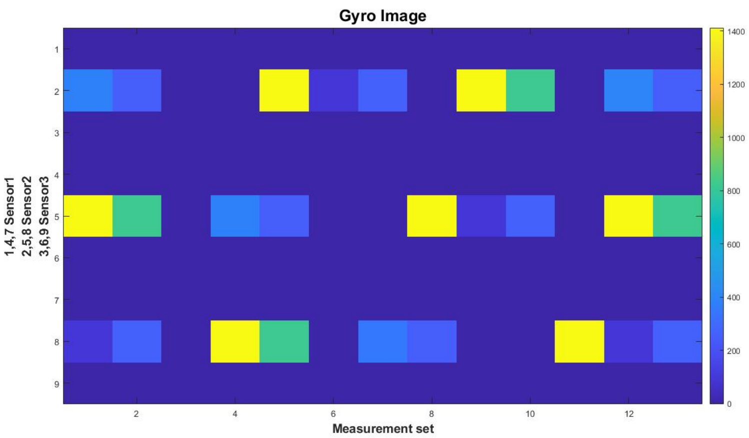

Figure 3. The hyro measurement set mapping

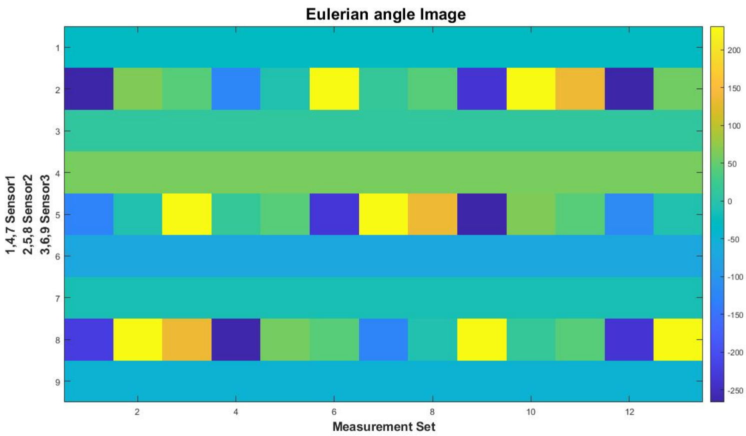

Figure 4. The Eulerian angle data set mapping

In planar operation, the rigid status is defined by the angle of the joint. In n-dimensional real space, it is $SO(2) \times SO(2)... \times SO(2)$. In this Algebraic structure, the group operators are defined based on addition operation. Group addition operation:

$$q_1, q_2 \in \Re^n, q_1 \oplus q_2 = q_1 + q_2$$

zero element:

$$E = 0 = [0, 0, ..., 0]^T$$

inverse operation:

$$q \in \Re^n, \ominus q = -q$$

scalar product:

$$\alpha \in \Re, q \in \Re^n, \alpha \odot q = q \oplus q \oplus ... \oplus q = \alpha \cdot q$$

space rigid body posture often is defined by 3×3 matrix, the group operator is defined by the matrix product, that is SO(3).

Group addition operation:

$$R_1, R_2 \in SO(3), R_1 \oplus R_2 = R_1 R_2$$

zero element:

$$E = I_3 = diag([1, 1, 1])$$

inverse operation:

$$R \in SO(3), \ominus R = R^{-1}$$

scalar product:

$$\alpha \in \Re, R \in SO(3), \alpha \odot q = R \oplus R \oplus ... \oplus R = R \cdot R... \cdot R.$$

3. MEASUREMENT SET MAP

3.1. From ODE (ordinary differential equation) to group

In sensor technology, the polynomials is used to stand for the standalone static characteristics of the sensor system and ordinary differential equation is used to stand for the dynamic feature of the linear time invariant sensor system. More generally, the state space equation is used to stand for the system motion equation. Polynomials are created from names, integers, and other values using the arithmetic operators $+, -, *, and \div$. Ordinary differential equation have different types of ODE problems. Here, we consider it in the algebraic way. We use mapping to represent the characteristic transformation processing (see on Figure 5).

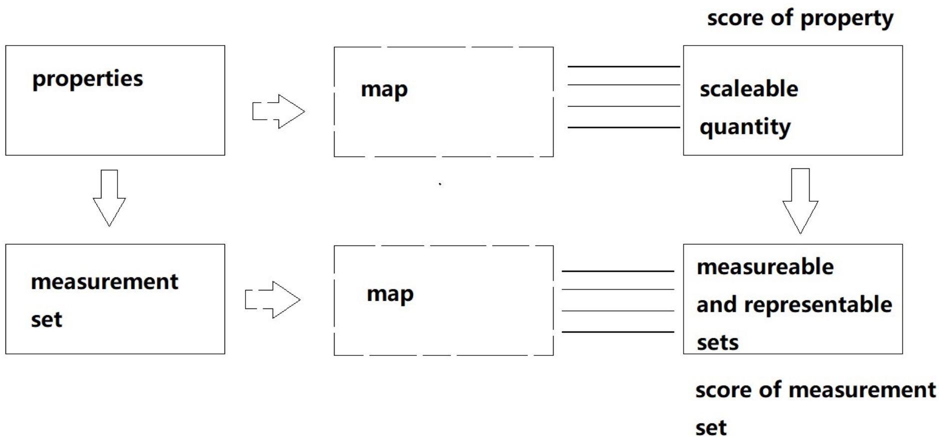

Figure 5. The diagram of measurement set map

The authors denote Algebras by $\mathcal{C}$ the set of complex numbers. A $\mathcal{C}$-algebra $\mathcal{A}$ is a C-vector space that is also a ring [11], [12]. They also proposed that filter spaces is algebras, filter as metrics. For example, if module $\mathcal{M}$ is of is of dimension n with basis $b = (b_0, ..., b_{n-1})$ and $s \in C^n$, then:

$$\phi(s) = s = \sum_{i=0}^{n-1} s_i b_i \tag{1}$$

and its z-transform, fourier transform. Some realization of the abstract space model is shown in Table 3.

Table 3. Realization of abstract model

Concept	Abstract	Realized
Shift opetator	q	$T_1(x) = x$
Space mark	t_n	C_n
K-fold shift operator	$q_k = T_k(q)$	$T_k(x)$
Signal	$\sum s_n t_n$	$\sum s_n C_n(x)$
Filter	$\sum h_k T_k(q)$	$\sum h_k T_k(x)$

Assumption: the static and dynamic feature of (sensor or control) system, are often also defined by error and other parameters partial differential equation. And in physics world, the entity state is reasonable to be represented. So, normally it is represented by field, ring, group, or other kinds of math domain. It is reasonable and convenience that some measurement quantity or quality are represented by group. Any this may be their inner comparable properties of measurement. Lie group analysis (symmetry analysis) finds point transformations which map a given differential equation to itself [15]. Except SO(2), the Aff(2) and SO(3) are also often used.

SO(3) is the group of rotations in 3D space, represented by 3 orthogonal matrices with unit determinant. It has three degrees of freedom: one for each differential rotation axis. The inverse is given by the transpose:

$$R \in SO(3) \subset \mathcal{R}^{3\times3}$$

$$R^{-1} = R^T$$

$$det(R) = 1$$

Aff(2) is the group of affine transformations on the 2D plane. It has six degrees of freedom: two for translation, one for rotation, one for scale, one for stretch and one for shear. Subgroups include Sim(2). After the discussion above, there are some methods to represent the measurement set mapping, they are shown in Figure 5.

The processing or analysis is always undertaking developing while a object is moving. The better method is the timely solution and within the timeout by some distribution or human being action. And also, for the moving properties of objects, the best solution is varying with time [16]. The representation of parameters measurable is shown in Table 4.

Table 4. Parameters measurable representation

General operation methods
Lie Group $\rightarrow$ the normalized measurement set
Measurement set and its map $\rightarrow$ finite fields
Measurable $\rightarrow$ movement or error exist rule
Modern algebra $\leftarrow$ the structure of measurement set

3.2. Joint space mapping

Joint space is calculated by linear interpolation of each joint. For attitude, it is possible to find the rotation axis between the two attitudes, and then interpolate the rotation angle. Linear interpolation of Lie group: $A_\alpha = A_1 \oplus \alpha \odot ((\ominus A_1) \oplus A_2), 0 \leq \alpha \leq 1$. In Joint space:

$$q_\alpha = q_1 \oplus \alpha \odot ((\ominus q_1) \oplus q_2) \tag{2}$$

$$q_\alpha = q_1 + \alpha((-q_1) \oplus q_2) \tag{3}$$

$$q_\alpha = q_1 + \alpha(q_2) - q_1), 0 \leq \alpha \leq 1 \tag{4}$$

for attitude:

$$R_\alpha = R_1 \oplus \alpha \odot ((\ominus R_1) \oplus R_2) \tag{5}$$

$$R_\alpha = R_1 + \alpha((-R_1) \oplus R_2) \tag{6}$$

$$R_\alpha = R_1 \cdot (R_1)^{-1} \cdot R_2)^\alpha, 0 \leq \alpha \leq 1 \tag{7}$$

4. EXPERIMENTS AND SIMULATION

4.1. Action recognition using lie network based on skeleton data set

For skeleton data has following feature [16]: each input is an element on the lie group. The skeletal data can be exhibitted as fully connected convolution-like layers and pooling layers, it has a deep network architecture of the Lie group. The train processing is shown in Figure 6.

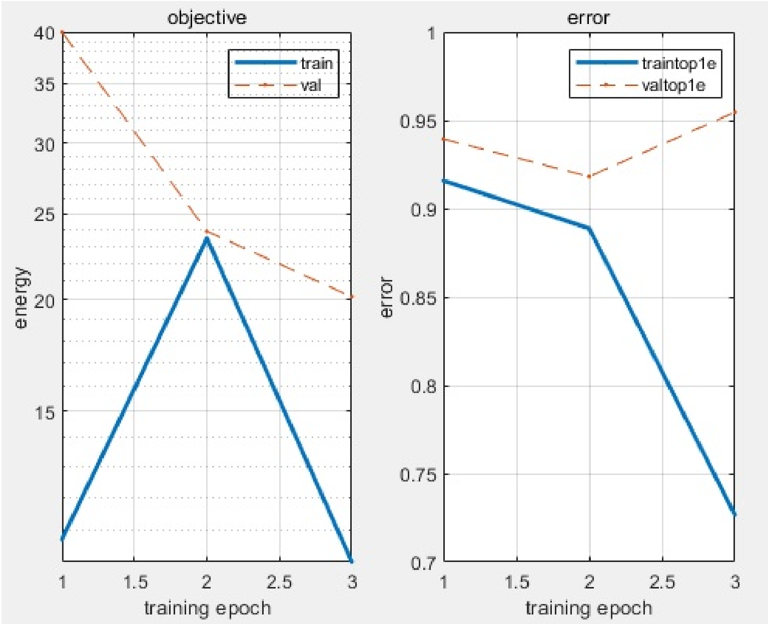

Figure 6. The training processing of lie network

4.2. Body posture recognition by MPU6050

It is the direct angle data obtained by the data fusion algorithm between gyroscope and acceleration sensor. The deploying place MPU6050 sensor is shown in Figure 7. Like skeleton data and lie group feature, different body postures are shown in Figures 8-11. It is obvious that the relationship of Euler angle follows the theory of rigid body mechanics. And it is foundation of recognition of body posture.

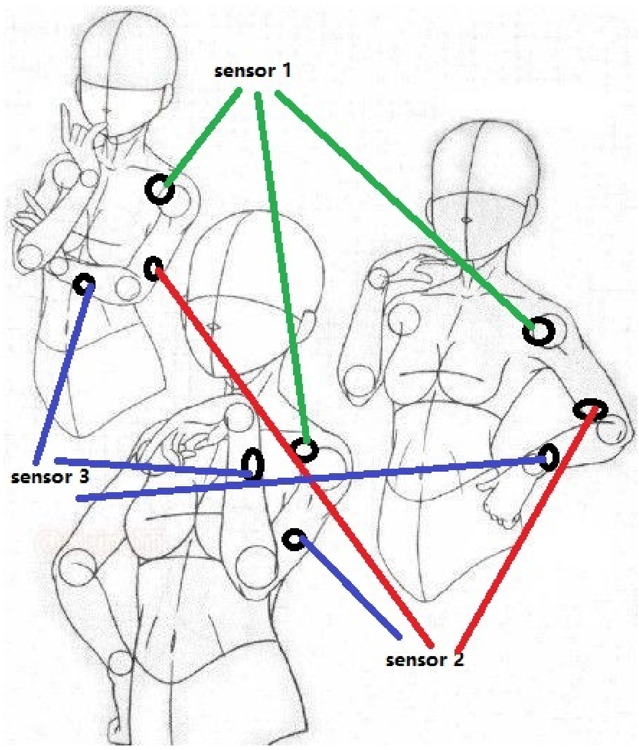

Figure 7. The MPU6050 sensor deploying place on body

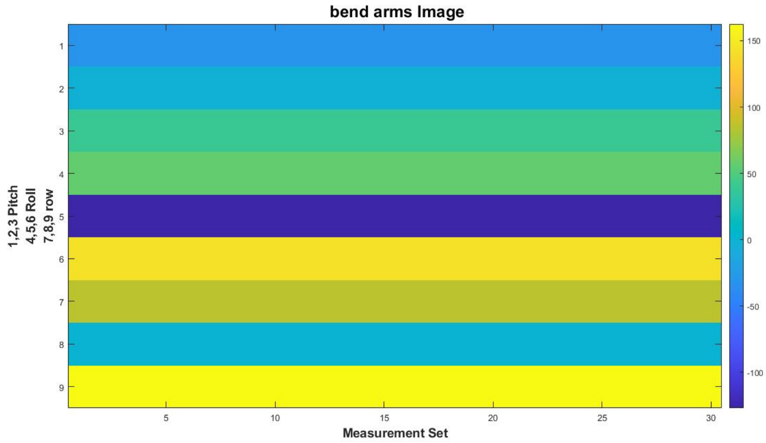

Figure 8. The body posture no. 1

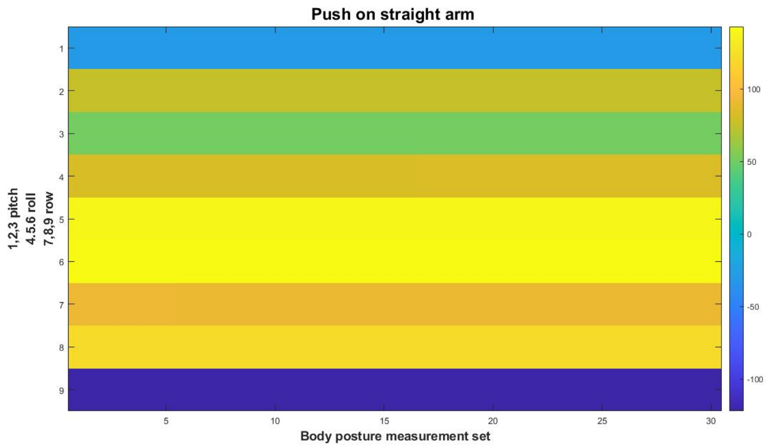

Figure 9. The body posture no. 2

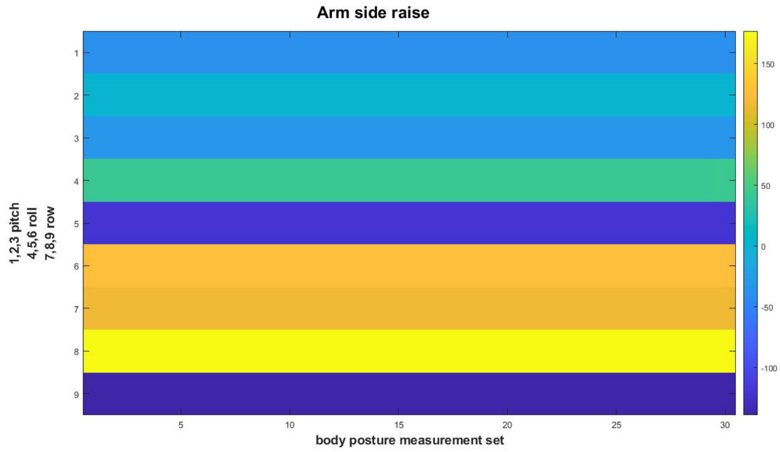

Figure 10. The body posture no. 3

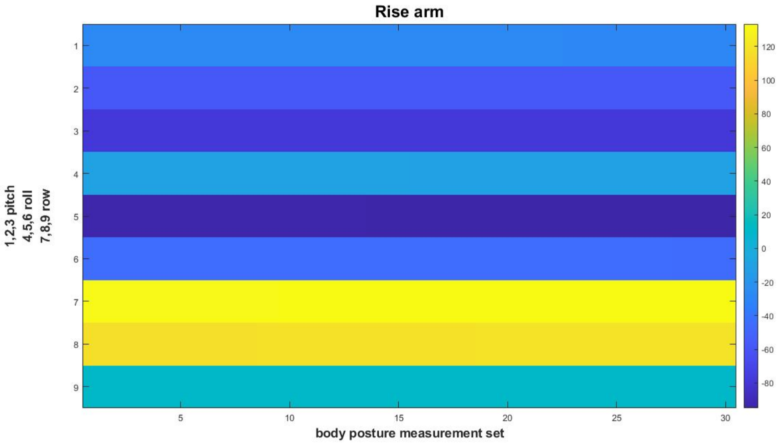

Figure 11. The body posture no. 4

5. FUTURE STUDY

There are some paper [17]-[21] that study the coding-encoding technology, in the future research of the manuscript we should consider the method based on concepts of group. In articles [22]-[25] research the mathematical expression of symmetry in physics using group theory, in the future research of the manuscript we should studies symmetry in physics more deeply.

6. CONCLUSION

The paper proposes a body posture measurement set representation method based on concepts of group, fields, ring. It attempts to explore the intrinsic relationship among numerous different measurement set. Meanwhile, the paper maps body posture data into image. The contributions of paper are: 1) it proposes image representation method; 2) it presents body posture representation and analysis method based on lie group.

ACKNOWLEDGMENT

The paper is supported by 2019RYJ03 Open fund project of Sichuan Key Laboratory of artificial intelligence.

REFERENCES

[1] X. B. Wang, Q. L. Zhang, Y. Zhou, and H. X. Zhou, "Unification of signal transforms on groups: An introduction,"*SCIENTIA SINICA Informationis*, vol. 43, no. 12, pp. 1547-1562, 2013, doi: 10.1360/N112013-00153.

[2] M. Püschel and J. Moura, "Algebraic signal processing theory: foundation and 1-D time," *IEEE Transactions on Signal Processing*, vol. 56, no. 8, pp. 386-391, 2006, doi: 10.1109/TSP.2008.925261.

[3] A. Sandryhaila, J. Kovacevic, and M. Puschel, "Algebraic signal processing theory: 1-D Nearest Neighbor models," *IEEE Transactions on Signal Processing*, vol. 60, no. 5, pp. 2247-2259, 2012, doi: 10.1109/TSP.2012.2186133.

[4] A. Goodall,"Fourier analysis on finite abelian groups: some graphical applications," *Combinatorics, Complexity, and Chance: A Tribute to Dominic Welsh*, 2007, doi: 10.1093/acprof:oso/9780198571278.003.0007.

[5] Y. Gu, "Image singularities of green's functions for anisotropic elastic half-spaces and bimaterials,"*The Quarterly Journal of Mechanics and Applied Mathematics*, vol. 45, no. 1, pp. 119–139, 1992, doi: 10.1093/qjmam/45.1.119.

[6] L. Daniels and T. Bay, "Group theory and the rubik's cube', *Lakehead University*, 2014.

[7] V. Dabbaghian-Abdoly, "An algorithm for constructing representations of finite groups,"*Journal of Symbolic Computation*, vol. 39, no. 6, pp. 671–688, 2005, doi: 10.1016/j.jsc.2005.01.002.

[8] M. C. Colin, H. George, H. Alexander, and F. R. Edmund, "Efficient simple groups," *Communications in Algebra*, vol. 30, no. 10, pp. 4613–4619, 2002, doi: 10.1081/AGB-120023154.

[9] B. Eick and C. R. B. Wright, "Computing subgroups by exhibition in finite solvable groups," *Journal of Symbolic Computation*, vol. 33, no. 2, pp. 129-143, 2000, doi: 10.1006/jsco.2000.0503.

[10] Z. Huang, C. Wan, T. Probst, and L. V. Gool, "Deep learning on lie groups for skeleton-based action recognition," In *Proceedings of the IEEE conference on computer vision and pattern recognition*, 2017, doi: 10.1109/CVPR.2017.137.

[11] N. Boumal, B. Mishra, P. A. Absil, and R. Sepulchre, "Manopt, a Matlab toolbox for optimization on manifolds," *Journal of Machine Learning Research*, vol. 15, no. 1, pp. 1455–1459, 2014.

[12] M. Püschel and J. Moura, "Algebraic signal processing theory: 2-D spatial hexagonal lattice," *IEEE Transactions on Signal Processing*, vol. 16, no. 6, pp. 1506-1521, 2007, doi: 10.1109/TIP.2007.896626.
[13] M. Püschel and J. Moura, "Algebraic signal processing theory: cooley–tukey type algorithms for DCTs and DSTs," *IEEE Transactions on Signal Processing*, vol. 56, no. 4, pp. 1502-1521, 2006, doi: 10.1109/TSP.2007.907919.
[14] H. T. Stokes, D. M. Hatch, and B. J. Campbell, "ISOTROPY Software Suite." iso.byu.edu (accessed Feb. 9, 2021).
[15] R. M. Zhang, "Introduction to applied group theory," Huazhong Univ. Science and Technology Press, 2001.
[16] L. Tagliacozzo, A. Celi, and M. Lewenstein, "Tensor networks for lattice gauge theories with continuous groups," *Physical Review X*, vol. 4, no. 4, 2014, doi: 10.1103/PhysRevX.4.041024.
[17] D. T. Hai, "Leveraging the survivable all-optical WDM network design with network coding assignment," *IEEE Communications Letters*, vol. 21, no. 10, pp. 2190–2193, 2017, doi: 10.1109/LCOMM.2017.2720661.
[18] D. T. Hai, "An optimal design framework for routing and network coding assignment in WDM optical networks with 1+1 protection enhanced by XOR network coding," *IEEE Communications Letters*, vol. 21, no. 10, pp. 2190-2193, 2017, doi: 10.1109/ACCESS.2017.2761809.
[19] D. T. Hai, H. T. Minh, and L. H. Chau, "QoS-aware protection in elastic optical networks with distance-adaptive and reconfigurable modulation format," *Optical Fiber Technology*, vol. 61, 2020, doi: 10.1016/j.yofte.2020.102364.
[20] D. T. Hai, M. Morvan, and P. Gravey, "Combining heuristic and exact approaches for solving the routing and spectrum assignment," *IET OptoElectronics*, vol. 12, no. 2, pp. 65-72, 2018, doi: 10.1049/iet-opt.2017.0013.
[21] D. T. Hai, "Re-designing dedicated protection in transparent WDM optical networks with XOR network coding," In *2018 Advances in Wireless and Optical Communications (RTUWO)*. Nov. 2018, pp. 118-123, doi: 10.1109/RTUWO.2018.8587873.
[22] J. P. Antoine, "Quantum mechanics and its evolving formulations," *Entropy*, vol. 23, no. 1, p. 124 2021, doi: 10.3390/e23010124.
[23] M. Kibler, "Galois fields and galois rings made easy," ISTE Press-Elsevier: London, United Kingdom, 2017. [Online]. Avaliable: https://www.sciencedirect.com/book/9781785482359/galois-fields-and-galois-rings-made-easy.
[24] H. Shen, "Stochastic quantization of an abelian gauge theory," *Communications in Mathematical Physics,* vol. 384, no. 4, pp. 1445-1512, 2021, doi: 10.1007/s00220-021-04114-x.
[25] J. Dai, "Stability in the higher derivative Abelian gauge field theory," *Nuclear Physics B,* vol. 961, pp.1-21, 2020, doi: 10.1016/j.nuclphysb.2020.115267.

BIOGRAPHIES OF AUTHORS

Changjian Deng is associate professor, school of automatic engineering, Chengdu University of Information Technology. He obtained Ph. D degree in University of Electronic Science and Technology of China in 2015. His researches are in fields of intelligent detection technology and device, sensor network and its application. He is a project leader of Sichuan Education Department project; "research on wireless acoustic photography technology for valve leakage monitoring under harsh environment". He can be contacted at email: chenglidcj@cuit.edu.cn.

Xiaona Xie is a associate professor, school of automatic engineering, Chengdu University of Information Technology. Her obtained Ph. D degree in University of Electronic Science and Technology of China in 2014. Her researches are in fields of computer science and technology. She can be contacted at email: xnxieok@163.com.