

Reliability Estimation based on Step-Stress Accelerated Degradation Testing by Unequal Interval Time Series Analysis

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Abstract

This paper proposes a reliability estimation method based on Step-Stress Accelerated Degradation Testing (SSADT) data analysis using unequal interval time series analysis. A Multi-Regression Time Varying Auto-Regressive (MRTVAR) degradation time series model is proposed. Product SSADT data are treated as unequal interval composite time series and described using MRTVAR time series model and utilized to predict long-term trend of degradation. By using the suggested method, product reliability is obtained. An example is presented as a verification of the modeling technique and estimation method. A reasonable estimation of lifetime and reliability of the product is obtained by employing the present method.

Keywords: Accelerated Degradation Testing, Time Series, Reliability, Unequal Interval

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1. Introduction

For long lifetime and high reliability products, it is difficult to obtain failure time data in a short time period. Hence, Accelerated Degradation Testing (ADT) is presented to deal with the cases that few or no failure time data could be obtained but degradation data of the primary parameter of the product are available. Step-Stress ADT (SSADT) is commonly used for the advantage that it needs only a few test samples to conduct a life test. For reliability and lifetime evaluation in SSADT, previous works use deterministic functions to represent the product performance degradation process. However, it does not represent performance degradation information adequately. It is necessary to add stochastic information description to performance degradation process. Time series analysis can represent stochastic information. During the last two decades, considerable research has been carried out in time series analysis. However, only few papers have studied the degradation data analyze method based on time series method. Moreover, SSADT data analysis based on time series method has not been reported in literature at present.

Product performance degradation data of SSADT is time series. Hence, a time series method is proposed to analyses SSADT data. There are several problems for applying time series method to SSADT data analysis as follows.

1. Product lifetime can be obtained by converting SSADT data at each test stress level into use-stress level. According to section 2 in this paper, this converted data is usually unequally spaced time series. Using traditional time series analysis method, which is only suitable for equally spaced series, might cause big prediction error.
2. Traditional degradation modeling method only considers degradation process of product performance. However, certain control characteristic of accelerated test equipment also causes product degradation.
3. Product degradation includes multiple factors. Certain factors interact with each other. This interaction can be represented by combining each model of them.
4. Product lifetime is predicted based on modeling SSADT time series. However, accurately prediction of time series model depends on large sample size. Traditional direct prediction is not effective enough to time series model.

In this paper, four assumptions of SSADT are put forward:

- 1 Product performance level degrades monotonously.
 - 2 The failure mechanism is unchanged at all stress levels.
 - 3 The remaining life of specimens depends only on the current cumulative fraction failed and current stress.
 - 4 There is no failure unrelated to degradation failure mechanism.
- There are certain notations of SSADT in this paper:

k is number of the stress levels;

S_i is i^{th} stress level, $i = 1, 2, \dots, k$;

Δt is sampling interval;

m_i is number of measurements under S_i ;

m is totally number of measurements, $m = \sum_{i=1}^k m_i$;

τ_i is the time scale of S_i , $\tau_i = \Delta t \cdot m_i$

2. SSADT Data Conversion

For modeling SSADT data at use-stress level, it is necessary to convert SSADT data at each test stress level into use-stress level. A conversion method is presented based on stress-degradation rate relationship and cumulative exposure model.

2.1. Stress-Degradation Rate Relationship

Cumulative degradation measure at S_i is obtained from the preprocessed SSADT data using the equation

$$\Delta y_i = y_{m_i} - y_{0_i} \quad (1)$$

Here, Δy_i is cumulative degradation measure at S_i , y_{0_i} is the first data at S_i .

Based on Eq.1 and τ_i , the degradation rate slp_i at S_i is obtained by the equation

$$slp_i = \frac{\Delta y_i}{\tau_i} \quad (2)$$

In SSADT, there are different degradation rates versus different stress levels. Certain stress-degradation rate relationship can be used to represent relationship between degradation rate and stress. Thus, degradation rate slp_0 at use-stress S_0 can be obtained.

2.2. Cumulative Exposure Model

Time scale τ_i is converted into time scale τ_{0i} at S_0 based on cumulative exposure model. The equation is

$$\tau_{0i} = \frac{\Delta y_i}{slp_0} = \frac{slp_i}{slp_0} \cdot \tau_i \quad (3)$$

Sampling interval Δt is converted by equation

$$\Delta t_i = \frac{\tau_{0i}}{m_i} = \frac{slp_i}{slp_0} \cdot \frac{\tau_i}{m_i} = \frac{slp_i}{slp_0} \cdot \Delta t \quad (4)$$

Here, Δt_i is sampling interval at S_0 .

Δt_i is different each other. Hence, the converted SSADT data at S_0 is unequally spaced data.

3. Time Series Modeling

Product lifetime is obtained by modeling SSADT time series at S_0 , which is unequally spaced and nonstationary time series. A new time series modeling method is put forward to represent the SSADT data at S_0 .

In this paper, $Y(l)$ denotes the level of degradation at l^{th} measurements, $l = 1, 2, \dots, m$. According to the Cramer Decomposition Theorem, any time series $Y(l)$ can be decomposed into deterministic component and stationary random component. $T(l)$ and $S(l)$ denotes trend component and seasonal component, which are deterministic components. $R(l)$ denotes residual stationary random component.

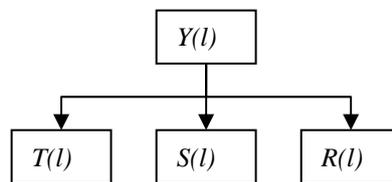


Figure 1. Relationship of components

3.1. Trend Component at S_i Modeling

Based on preprocessed SSADT data, trend component at S_i , denoted by $T'_i(l)$, is different each other and represented respectively by linear regression model

$$T'_i(l) = slp_i \cdot l + y_{0_i} \tag{5}$$

Figure 2 shows an example of preprocessed SSADT data at four stress levels. White line shows $T'_i(l)$. Tab.I in section 5 shows test parameters.

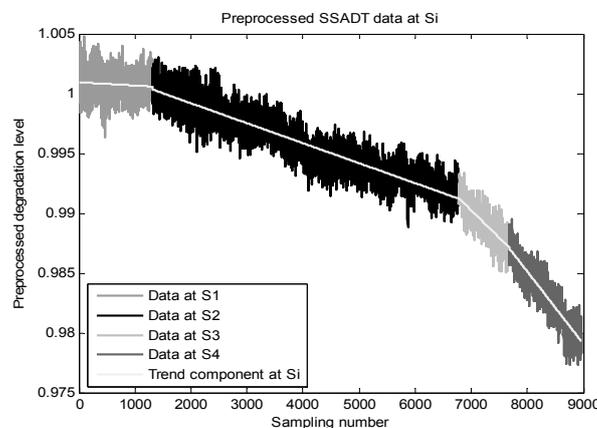


Figure 2. Preprocessed SSADT data at S_i

3.2. Seasonal Component Modeling

Controlled by test equipment, product performance reflects periodical fluctuation of stress level. This paper regards it as seasonal component $S(l)$ and represents it using Hidden Periodicity (HP) regression model

$$S(l) = \sum_{j=1}^q A_j \cos(\omega_j l + \varphi_j) \quad (6)$$

Here, q is number of angular frequency, ω_j is j^{th} angular frequency, A_j is amplitude of ω_j , φ_j is j^{th} phase angle.

Figure 3 shows preprocessed SSADT data of the example above without $T_i'(l)$ at S_i . White line shows $S(l)$.

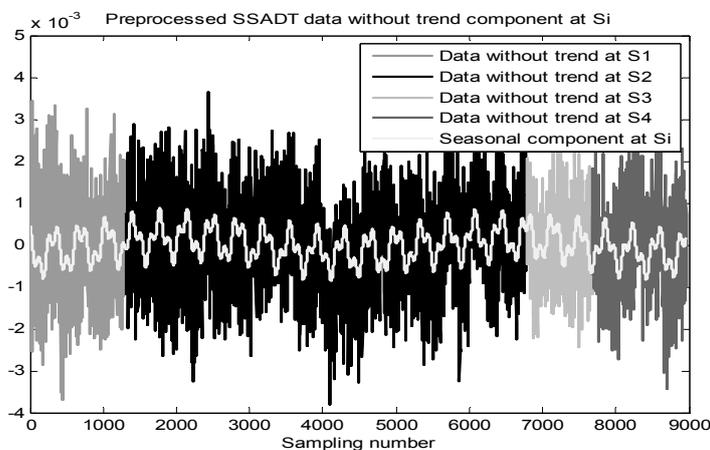


Figure 3. Preprocessed SSADT data without $T_i'(l)$ at S_i

3.3. Residual Component Modeling

$R(l)$ represents residual series at S_0 . This unequally spaced stationary random series is time series with stationary correlation coefficient. t_l denotes the l^{th} sampling time at S_0 , this paper represents $R(l)$ using Time Varying AutoRegressive (TVAR) model

$$R(l) = \sum_{j=1}^p \eta_j(\tau_l) R(l-j) + \varepsilon(\tau_l) \quad (7)$$

Here, $\tau_l = (\tau_{l1}, \tau_{l2}, \dots, \tau_{lp})'$, $\tau_{lj} = t_l - t_{l-j}$, $j = 1, 2, \dots, p$, $\varepsilon(\tau_l)$ obeys $N[0, \sigma_0^2 \phi(\alpha, \tau_l)]$, it is independent white noise.

3.4. Seasonal and Residual Component Combining Modeling

There is interaction between $R(l)$ and $S(l)$. $X(l)$ denotes seasonal-residual component. This paper adds HP regression model in TVAR model and puts forward a Hidden Periodicity Time Varying Auto Regressive (HPTVAR) model

$$X(l) = \sum_{j=1}^p \eta_j(\tau_l) X(l-j) + \sum_{j=1}^q A_j^* \cos(\omega_j l + \phi_j^*) + \varepsilon(\tau_l) \quad (8)$$

Here, A_j^*, φ_j^* are corrected parameters for combining [2].

Figure 4 shows converted preprocessed SSADT data without $T_i'(l)$ at S_0 . White line shows $S(l)$ at S_0 .

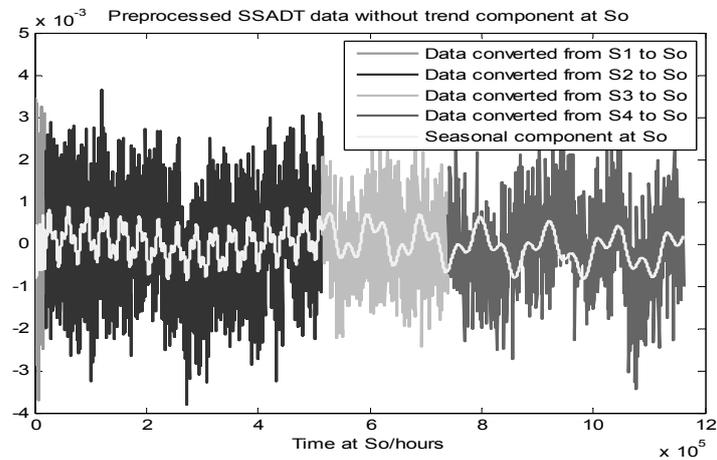


Figure 4. Preprocessed SSADT data without $T_i'(l)$ at S_0

3.5. Trend Component at S_0 Modeling

Trend component at S_0 denoted by $T(l)$ is represented by

$$T(l) = slp_0 \cdot l + y_{0i} \tag{9}$$

3.6. Degradation Time Series Modeling

This paper proposes degradation time series model, Multi-Regression Time Varying Auto Regressive (MRTVAR) model

$$Y(l) = T(l) + X(l) \tag{10}$$

Figure 5 shows converted preprocessed SSADT data at S_0 . White line shows $T(l)$.

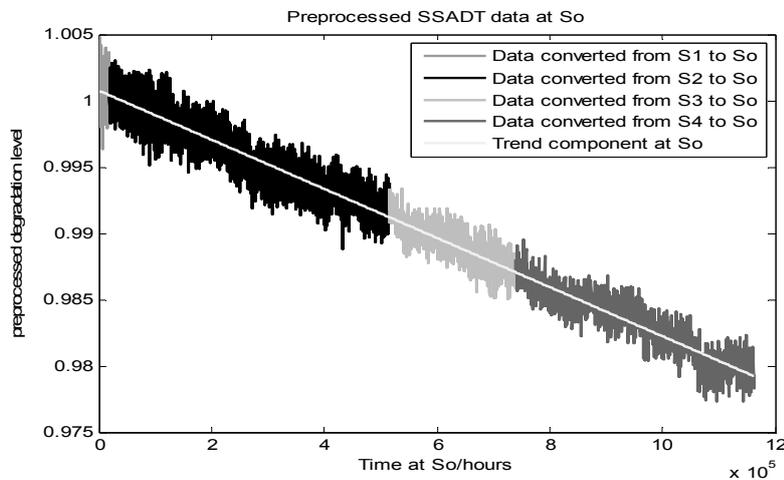


Figure 5. Preprocessed SSADT data at S_0

4. Reliability Estimation

In ADT, failure occurs as product performance level achieves a specified threshold. Product lifetime is time scale from ADT beginning to the first achieving. In this paper, the lifetime is obtained by prediction of degradation model. This predicted lifetime is called pseudo lifetime.

In addition, the accuracy of pseudo lifetime depends on prediction precision of degradation model. The creditability of reliability estimation depends on accuracy of pseudo lifetime.

4.1. Pseudo Lifetime

Pseudo lifetime t_{life} is obtained when prediction data y_{m+u} first achieves the failure threshold at prediction step u_{life} , it is obtained by equation

$$t_{life} = \sum_{i=1}^k \tau_{0i} + u_{life} \cdot \overline{\Delta t} \quad (11)$$

4.2. Prediction Error

Prediction error of time series model is brought by residual component. Its mean square error of grey prediction is obtained by u_{life} and equation

$$\sigma_{m+u}^2 = \sum_{j=1}^u G_{uj}^2 \sigma_{(u-j+1)\varepsilon}^2 \quad (12)$$

Here, G_{uj} is the Green function of TVAR model u -step prediction, $\sigma_{(u-j+1)\varepsilon}^2$ is white noise variance of TVAR model $(u-j+1)$ step prediction $j = 1, 2, \dots, u$.

4.3. Reliability Estimation

Pseudo lifetime distribution and reliability estimation are obtained based on all product pseudo lifetime.

5. Example Verification

A four temperature levels SSADT of ten certain products is conducted as an example to verify the suggest SSADT data analysis method. Sampling interval is one hour. Figure 6 shows the original degradation data of SSADT. Table 1 shows test parameters.

Table 1. Parameters of SSADT

Temperature	Sample size
55°C	1320
70°C	5460
85°C	900
95°C	1320

Firstly, SSADT data of each product is preprocessed by initial value processing for eliminating influence of its initial value difference and normalizing the failure criterion. Figure 7 shows the preprocessed SSADT data.

Secondly, SSADT data at each temperature are converted into it at use-stress level 25°C. Figure 8 shows the Arrhenius temperature stress-degradation rate relationship.

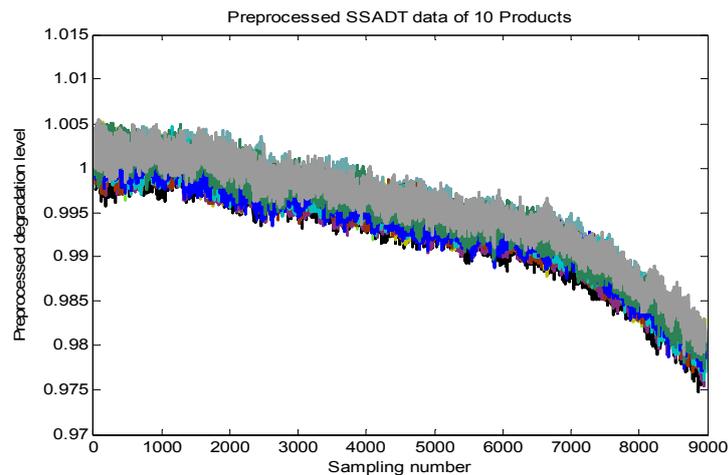


Figure 7. Preprocessed SSADT data of 10 products

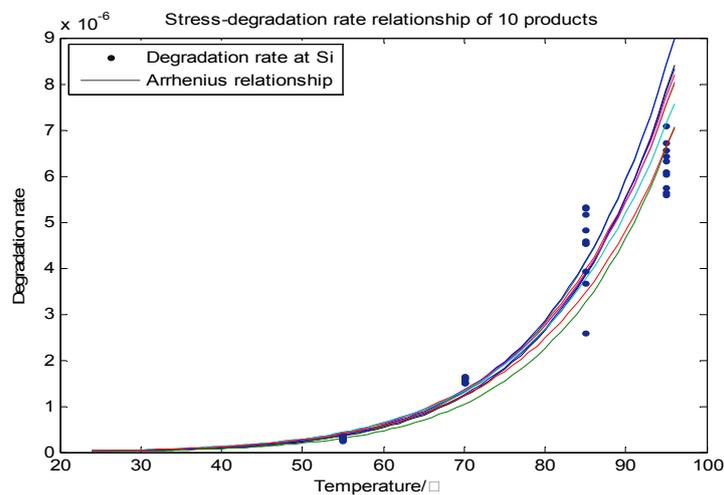


Figure 8. Stress-degradation rate relationship of 10 products

Thirdly, the unequally spaced preprocessed SSADT time series of each product at 25°C is represented by MRTVAR model. Parameters of the TVAR model are estimated by the maximum likelihood method.

Fourthly, set 100 hours as prediction interval and 5000 as prediction step number. SSADT prediction data of each product is obtained. Figure 9 shows the SSADT prediction data. White line shows trend component at 25°C. Grey line shows prediction data.

Fifthly, set 97% of initial value as failure threshold. Table 2 shows pseudo lifetime and prediction error. Set lognormal distribution as pseudo lifetime distribution. Figure 10 shows reliability function. Table 3 shows pseudo lifetime distribution parameters estimated using maximum likelihood method.

This paper also analyzes the same SSADT data of this example based on regression model. Table 2 and Table 3 show the pseudo lifetime prediction and distribution parameters based on regression model, Figure 10 shows the reliability function based on regression model.

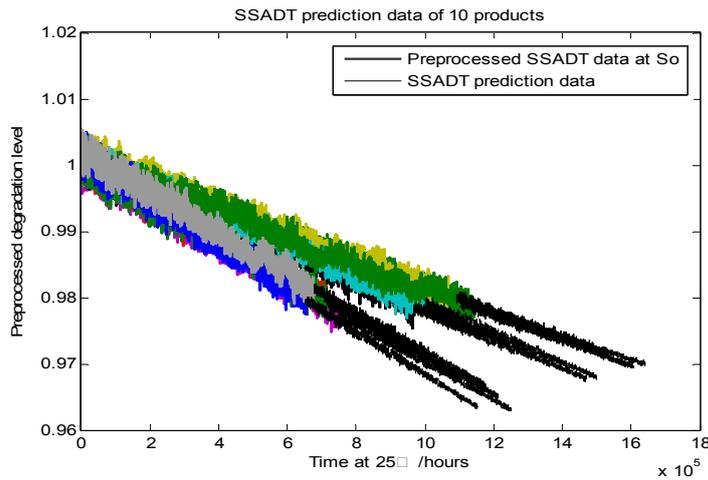


Figure 9. SSADT prediction data of 10 products

Table 2. Pseudo lifetime & prediction error of SSADT

Pseudo Lifetime (hours)		Mean square error	
MRTVAR	Regression	MRTVAR	Regression
1569948	1654080	1569948	1654080
991970	1011170	991970	1011170
1025120	1042520	1025120	1042520
1345578	1366178	1345578	1366178
1564491	1578591	1564491	1578591
1390313	1409213	1390313	1409213
1621545	1639145	1621545	1639145
1024051	1043451	1024051	1043451
928334	950334	928334	950334
1053697	1066597	1053697	1066597

Table 3. Parameters of Pseudo lifetime distribution

Model	MRTVAR	Regression
Log-lifetime mean	14.0231	14.0593
Log-lifetime variance	0.051	0.2692
Median lifetime (hours)	1251504	1276127

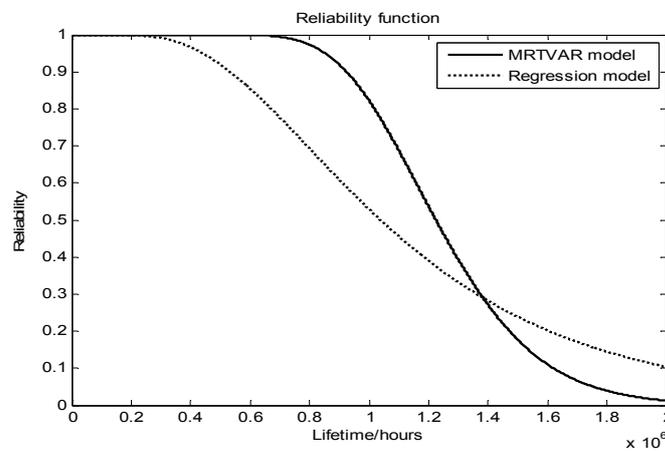


Figure 10. Reliability function

According to Table 2, it is obviously that prediction error at pseudo lifetime based on MRTVAR model is smaller than regression model. Hence, compared with regression model, reliability estimation by SSADT data analysis based on time series method is more creditable.

6. Conclusion

This paper proposes a SSADT data analysis method based on time series analysis. A MRTVAR degradation time series model is proposed based on SSADT data. By using the suggested method, product reliability is obtained. An example is presented as a verification of the modeling technique and estimation method. A reasonable estimation of lifetime and reliability of the product is obtained by employing the present method.

There are also several problems about SSADT data analysis based on time series analyses deserve further research. Indeed, the way that the improvement of prediction precision of time series model enhances creditability of lifetime prediction and reliability estimation in SSADT is not clarified. Thus, it can be the next object of research to improve reliability estimation technology further.

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