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A Curve-fitting Calibration Method applied for Ultrasonic Flow-meter

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Abstract

As the influence of fluid distribution in the internal pipe, the measurement characteristics of theory and practice exist significant differences in Ultrasonic Flow-meter (USF). Through analysis of fluid state, the method of curve-fitting is applied for the calibration of USF. Experimental results show that the USF can achieve level-1 accuracy with just a correction of 5 flow points, and this method performs a low computational complexity and strong practicality.

Keywords: *Curve-fitting, Calibration, Ultrasonic Flow-meter, Reynolds number*

1. Introduction

USF uses the propagation characteristics of ultrasonic wave in fluid to get the information about fluid velocity. Due to the ultrasonic signal is sensitive to external factors, and the law of different velocity in closed conduits is complicated. The measurement characteristics of theory and practice exist significant differences in USF, which would seriously affect the accuracy of USF [1-2]. At present, based on the ISO standard of <With the USF of time-transit method to measure the fluid flow in closed pipe>and the Chinese standard <JJG1030-2007USF>, many researchers have studied the problem caused by the heavy workload of calibration, but these methods still can't adapt to the mass production [3-4].

In this paper, the curve-fitting method is adopted for the calibration of USF after the analysis of fluid state. This method not only can acquire a better accuracy of USF, but also can reduce the workload of calibration and promote enterprise production.

2. The Reason of Calibration in USF

2.1. The Principle of USF

In study of USF, the time difference method possesses a majority, and whose principle of measure is shown in Figure 1. The ultrasonic transducer1 (*U*1) and ultrasonic transducer2 (*U*2) alternately transmit or receive ultrasonic pulses (with an incident angle θ). Because the ultrasonic velocity in the downstream and upstream is different, the time for the ultrasonic signal to reach the two ultrasonic transducers is different, with a time difference ΔT. The linear average velocity v_L is obtained by capturing this time difference

$$V_L \approx \frac{C^2}{2D\tan\theta}\Delta T \quad (1)$$

$$\overline{V_L} = \frac{1}{L}\int_L V(r)dl = \frac{1}{R}\int_0^R V_{\max}(\frac{R-r}{R})^{\frac{1}{n}}dr = \frac{n}{n+1}\bullet V_{\max} \quad (2)$$

Where *C* is the velocity of ultrasonic wave in fluid, *D* is the diameter of this pipe.

Because velocity from Eq.(1) is linear average velocity, the volume flow should be computed using profile average velocity. The flow of fluid in the pipe presents a non-uniform distribution, so, the measured linear velocity differentiates the profile velocity. Some scholars have raised method of measure by multi-channel so that the measured data can be better close

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to the real values. But it still needs to be amended by software compensation, and will bring on the complexity of design and increase the cost. To fully understand the characteristics of the media measured is the premise of accurate measurement. Hence, the analysis of fluid characteristics is an essential link in improving the accuracy of USF [5-6].

2.2. The Analysis of Flow Fluid Characteristics

Under the ideal conditions of that the pipe wall is smooth and the fluid viscosity is zero, the flow rate of fluid in pipe is in uniform distribution, as shown in Figure 2(a). The adjacent particle of fluid has shear stress because of the existence of viscous effect in the actual fluid. When the fluid flows, the velocity is zero at the wall, which increases while away from the wall to the axis, and is shown in Figure 2(b). When the fluid is in the state of full development of flow, the velocity distribution of fluid changes along with the increase of velocity, meanwhile, the Reynolds numbers increase. Velocity will go through three states— laminar, transient and turbulent, which will directly lead to the differences between profile-linear velocity [7-8]

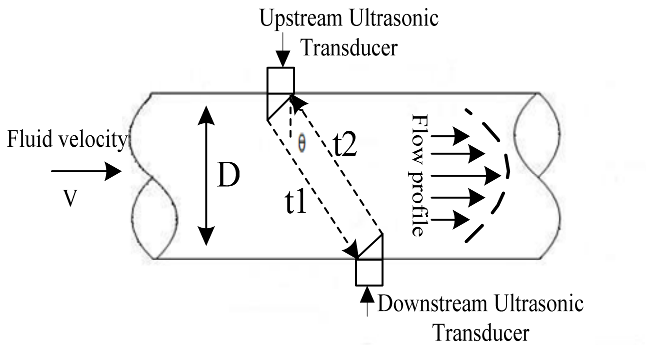
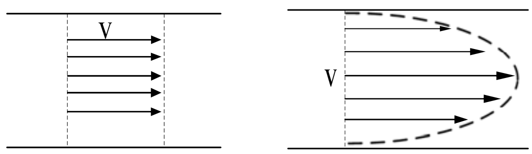


Figure 1. Flow measurement principle of the USF



(a) The ideal state (b) The actual state

Figure 2. The Distribution of Fluid Velocity

Reynolds number is a unit to describe the fluid flow by the theory of fluid mechanics. $R_e = \frac{V_s \bullet D}{\upsilon}$, where v_S is the profile velocity, D is the diameter of the pipe, υ is the kinematic viscosity. From the Reynolds number experiments, it can be concluded that [9-12]:

- When Re<2300—laminar, the particle has a linear motion smoothly along the pipe axis in parallel. Relationship of velocity between the particle and axis is showed in the following: $V_r = V_{max}[1-(\frac{r}{R})^2]$, the correction coefficient is a constant at this situation.
- When Re>3750—turbulent, the fluid doesn't remain stratified flow any more, but in all directions. Relationship of velocity between the particle and axis is showed as follow: $V_r = V_{max}(1-\frac{r}{R})^{\frac{1}{n}}$, n is a function related to Reynolds number, r is the distance from axis of the pipe, R is the diameter of the pipe. The linear average velocity and the profile average velocity can be calculated respectively as follow:

$$\overline{V_s(t)} = \frac{1}{s}\int_s V(r)ds = \frac{1}{s}\int_s V_{max}(\frac{R-r}{R})^{\frac{1}{n}} \bullet 2\pi r dr = \frac{2n^2}{(2n+1)(n+1)} \bullet V_{max} \quad (3)$$

$$K = \frac{\overline{V_s(t)}}{V_L(t)} = \frac{2n}{2n+1} \quad (4)$$

So the coefficient of correction can be obtained from the above two equation shown in Eq.(4)

$$K = \frac{\overline{V_s(t)}}{V_L(t)} \approx \frac{1}{1.12 - 0.011 * \log_{10} Re} \quad (5)$$

$$\overline{V_L} = \frac{1}{L}\int_L V(r)dl = \frac{1}{R}\int_0^R V_{max}(1-\frac{r^2}{R^2})dr = \frac{2}{3}V_{max} \quad (6)$$

- when 2300<*Re*<3750— transient, the relationship of profile-linear velocity has not concluded up to now. The chief reason is that this state isn't stable and the velocity distribution is extremely complicated. The relationship of profile-linear velocity is difficult to express using definite functions, which can only be obtained from experimental equation. One commonly used equation is as Eq.(5).

For the turbulent and transient flow, the theoretical equation of correction is hard to implement due to its complexity. In this paper, the curve-fitting of least-square is applied to the calibration.

3. The Calibration Method

We all know that the fluid exists three states, laminar, transient and turbulent flow by previous analysis. Because the law of fluid movement varies widely in each fluid state, this paper intends to calibrate the USF in each fluid state respectively.

- Laminar, the distribution of fluid velocity likes a parabolic curve shown in Figure 3. As is deduced from [9], the relationship of profile average velocity and maximum velocity is as follow: $\overline{V}_s = \frac{1}{2}V_{max}$. The linear average velocity can be calculated by integral as follow.

$$K = \frac{\overline{V_S}}{\overline{V_L}} = \frac{1}{2}V_{max} / \frac{2}{3}V_{max} = \frac{3}{4} \quad (7)$$

$$D = \sum_{i=1}^{n}[f(x_{Li}, a_1, \ldots\ldots, a_n) - x_{Si}]^2 \quad (8)$$

So the correction coefficient can be computed as (7)

- Turbulent and transient, whose velocity distribution is so complicated that no conclusion could be drawn about the relationship of profile-linear velocity, and the velocity distribution is shown in Figure 3. To this problem, the polynomial is used to calibrate the error of profile-linear velocity.

If to calibrate n+1 flow points, such as (x_{Li}, x_{Si}), where i=0, 1, 2, ……n, and x_L is linear velocity, x_S is profile velocity. The function f(x_{Li}) is demanded to match the real data of flow points as close as possible. In industrial application, the functional relationship between x_L and x_S is often obtained through a lot of experimental data such as (x_{L1}, x_{S1}) ,……,(x_{Ln}, x_{Sn}). Through processing of experimental data, the functional equation can be described like this: x_S=f(x_L,a_1,a_2,……, an). These parameters (i.e. a_1,a_2,……, an) determined by the least squares method are selected to make the sum of squares of deviation to minimum, as described in Eq.(8):

The number *D* can reflect the degree of fitting between function x_S=f(x_L,a_1,a_2,……, a_n) and experimental data— (x_{Li}, x_{Si}). The value of *D* is smaller, the consistency between the value of function and the data of experiment is better.

These parameters in fitting curve are obtained by the following means: assuming the got data of these m sampling points are shown Table1.

Table 1. sample points

x_L	x_{L1}	x_{L2}	x_{L3}	……	x_{Lm}
$f(x_L)$	x_{S1}	x_{S2}	x_{S3}	……	x_{Sm}

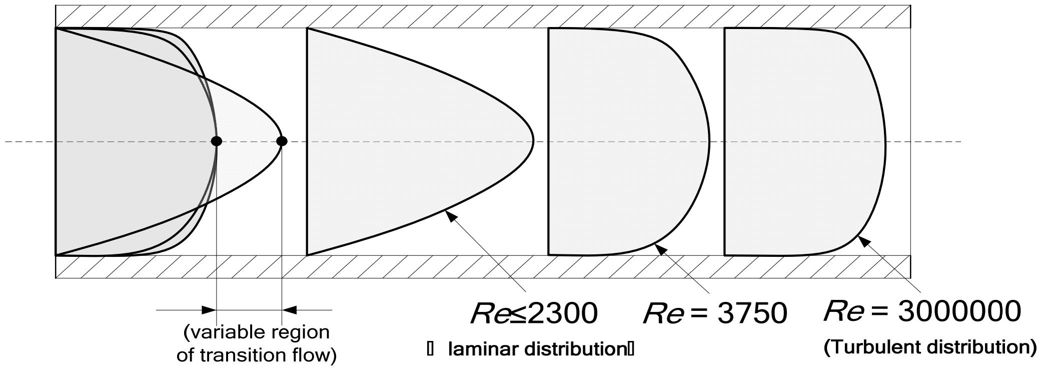


Figure 3. The distribution of velocity in pipe

The polynomial is used to interpolate, if $p_n(x_L)=a_0+a_1x_L+\ldots\ldots+a_nx_L^n$, and enabling the equation $p_n(x_{Li})=f(x_{Li})(i=1,2,\ldots\ldots, m)$, then,

$$\begin{cases} a_0 + a_1 x_{L1} + \ldots\ldots + a_n x_{L1}^n = x_{S1} \\ a_0 + a_1 x_{L2} + \ldots\ldots + a_n x_{L2}^n = x_{S2} \\ \ldots\ldots \\ a_0 + a_1 x_{Lm} + \ldots\ldots + a_n x_{L_m}^n = x_{Sm} \end{cases} \quad (9)$$

Setting an auxiliary function called error function as Eq.(10),
which is a multivariate function related to $a_0,a_1,\ldots\ldots, a_n$. To get the minimum of φ, methods of extreme value of multivariate function are adopted. Partial derivative to each independent variable is set to zero as shown in eq.(10)

$$\begin{cases} \frac{\partial \varphi}{\partial a_0} = 2\sum_{i=1}^{m} [a_0 + a_1 x_{Li} + \ldots\ldots + a_n x_{Li}^n - x_{Si}] = 0 \\ \frac{\partial \varphi}{\partial a_1} = 2\sum_{i=1}^{m} [a_0 + a_1 x_{Li} + \ldots\ldots + a_n x_{Li}^n - x_{Si}] x_{Li} = 0 \\ \ldots\ldots \\ \frac{\partial \varphi}{\partial a_n} = 2\sum_{i=1}^{m} [a_0 + a_1 x_{Li} + \ldots\ldots + a_n x_{L_i}^n - x_{Si}] x_{Li}^n = 0 \end{cases} \quad (10)$$

Eq.(10) is written as linear equation group relating to the unknown character($a_0,a_1,\ldots\ldots, a_n$).

$$\begin{cases} a_0\sum 1 + a_1\sum x_{Li} + \ldots\ldots + a_n\sum x_{Li}^n = \sum x_{Li} \\ a_0\sum x_{Li} + a_1\sum x_{Li}^2 + \ldots\ldots + a_n\sum x_{Li}^{n+1} = \sum x_{Li}x_{Si} \\ \ldots\ldots \\ a_0\sum x_{Li}^n + a_1\sum x_{Li}^{n+1} + \ldots\ldots + a_n\sum x_{Li}^{2n} = \sum x_{Li}^n x_{Si} \end{cases} \quad (11)$$

$$\beta = \begin{bmatrix} \sum 1 & \sum x_{Li} & \sum x_{Li}^2 & \cdots & \sum x_{Li}^n \\ \sum x_{Li} & \sum x_{Li}^2 & \sum x_{Li}^3 & \cdots & \sum x_{Li}^{n+1} \\ \vdots \\ \sum x_{Li}^n & \sum x_{Li}^{n+1} & \sum x_{Li}^{n+2} & \cdots & \sum x_{Li}^{2n} \end{bmatrix} \quad (12)$$

Which can be written as βu=C, where

$$\varphi(a_0,a_1,\cdots,a_n)=\sum_{i=1}^{m}[a_0+a_1x_{Li}+\cdots+a_nx_{Li}^n-x_{Si}]^2=\sum_{i=1}^{m}[p_n(x_{Li})-x_{Si}]^2 \quad (10)$$

$$u=[a_0,a_2,\ldots,a_n]^T,C=[\sum x_{Si},\sum x_{Li}x_{Si},\ldots,\sum x_{Li}^n x_{Si}]^T \quad (11)$$

It is easy to get the solution of equation, from which the curve equation fitted can be acquired.

If taking the workload of calibration and accuracy into account, here we take m=3, n=2, and then we have $p_n(x_L)=a_0+a_1x_L+a_2x_L^2$. There exists the common flow points between laminar flow and transient flow, turbulent flow and transient flow, such as sampling point1 and point3. So, the entire process of calibration only needs 5 sampling points. Based on the experimental platform of DN100 USF, the field sampling data obtained are shown in TABLE 2.

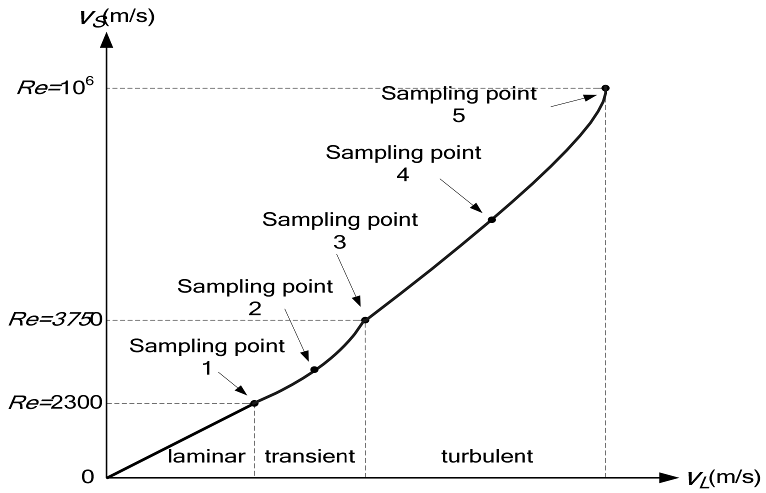


Figure 4. Scheme of calibration

Table 2. Data Sheet of Field Sampling

sampling points	point1	point2	point3	point4	point5
v_L	0.0447	0.0581	0.0727	0.1299	0.1823
V_S	0.0335	0.0512	0.0683	0.1123	0.1745

By sampling flow point1, point2 and point3, the fitting curve obtained is as follow: v_S=-0.0394+1.8703v_L-5.3534v_L^2. By sampling flow point3, point4 and point5, the fitting curve obtained is as follow: v_S=0.0116-0.2851v_L+34.7174v_L^2.

4. Analysis of Experimental Results

To test the performance of the scheme, the sampling data of three additional point of flow is obtained in each fluid state. The results are showed as Table 3-5. By the curve fitting method for USF, calibration results from the above tables can properly reflect the true situation.

Table 3. Laminar

Flow-points (m/s)	0.02010	0.02470	0.03650	Average accuracy	Repeat-ability
Measured value v_S(m/s)	0.02016	0.02485	0.03668		
accuracy	0.3%	0.64%	0.51%	0.48%	0.17%

Table 4. Transient

Flow-points (m/s)	0.04860	0.06260	0.07140	Average accuracy	Repeat-ability
Measured value v_S(m/s)	0.04855	0.06277	0.07152		
accuracy	-0.09%	0.28%	0.17%	0.12%	0.19%

Table 5.Turbulent

Flow-points (m/s)	0.09520	0.15830	0.17970	Average accuracy	Repeat-ability
Measured value v_S(m/s)	0.09551	0.15823	0.18005		
accuracy	0.33%	-0.04%	0.20%	0.16%	0.18%

Accuracy, repeatability, respectively within the range of ±1%、±0.2%, can meet the requirement of Level-1 set by the National Metrological Bureau.

5. Conclusion

In this paper, the nonlinear model is generated by analysis of fluid state and test of field calibration, and the curve-fitting method is applied to the process of calibration. As the result shows, this method not only can guarantee the accuracy of USF, but also can reduce the workload of calibration and perform low computational complexity. Also, the actual application ability is improved.

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References

[1] Q Liu, RD Wang, Y Zhu and CT Du. *An algorithm to eliminate stochastic jump measurement of USF with time difference method*. Manufacturing Systems and Industry Application. 2010: 414-421.
[2] MW Li, GS Lv, YG Hu. *Research on improving the accuracy of the ultrasonic flow-meter with time difference method*. 2010 International conference on electrical and control engineering. 2010: 256-259.
[3] JJG 1030-2007 <Verification Regulation of Ultrasonic Flowmeters>· Beijing: The Metrology Press of China, 2007.
[4] ISO/T 4060-2:2005 <Measurement of water flow in fully charged closed conduits>. ISO, 2005.
[5] DI Marco, C Mauro, D Matteo. Measurement of high-pressure natural gas flow using ultrasonic flowmeters. *Measurement*. 1997: 20(2): 69-73.
[6] J Berrebi, PE Martinsson, M Willatzen, J Delsing. Ultrasonic flow metering errors due to pulsating flow. *Flow Measurement and Instrument*. 2004: 15(1): 179-185.
[7] YH Liu, GS Du. The calculation of the profile-linear average velocity in the transition region for ultrasonic heat meter based on the method of LES. *Journal of Hydrodynamics*. 2011: 23(1): 89-94.
[8] Ding Xi-bo, Liang Jia-yi, Liu Jian, Guo Chao. Measurement Algorithm of Two-Dimensional Wind Vector using Ultrasonic Transducers. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2013; 11(1): 517-524.
[9] C Liu, JJ Huang. Several improvement of flow measurement method. *Journal of Shenyang University of technology.* 2002; 24(2): 113-115.
[10] M Bao. Study on main factors affecting the accuracy of gas Ultrasonic Flowmeter. Zhejiang University. 2004.
[11] L Yao. The electronic sensor of water meter and signal processing technology. Beijing: The quality and inspection press of China; 2011.
[12] H Yang, C Wang. Simulation Research on Ultrasonic Guided Waves Detection of Metal Rod Buffer System Bonding Quality. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2013; 11(3): 1251-1257.