# The Method of Signal Decomposition to Reduce the PAPR 

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#### Abstract

A new method of signal decomposition based on the baseband to reduce the PAPR which Decompose the OFDM signal into two signals, The new method and its degree of PAPR improvement are introduced in detail. Two thresholds are introduced, compare threshold Tc and decompose threshold Td .Simulation the results in different thresholds .The results show that the method can reduce the PAPR greatly.


Keywords: OFDM, PAPR, ALT Alternative Time Decomposition, compare threshold, decompose threshold

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## 1. Introduction

OFDM (Orthogonal Frequency Division Multiplexing) is a kind of multicarrier modulation technology, It distribute the data flow to orthogonal spectrum transmission channel, It can use spectrum resource effectively, at the same time it can resistant multipath fading effectively. It is the main technology of LTE. More than one carrier added together to output the time domain signal. But if too much user data in the same phase, the ratio of the OFDM instantaneous power and the average power is high, which lead to the high PAPR value. High signal peak can lead to clamping in D/A converter, and make the power amplifier saturation which cause signal crosstalk and spectral distortion. It produce interference in the sender, worsen the performance of the system. At present there are 3 classes method to reduce PAPR [1]: predistortion technique, coding technique/ probability technique.

## 2. OFDM System and Definition of PAPR

OFDM modulation process finished in baseband. Binary bit steam changed into complex data after a modulation processing such as QPSK, 16QAM, 64QAM, subcarrier frequency data is $X_{k}$, after IFFT it transformed to the time domain data ${ }_{n}$, calculation formula is [2-8]:

$$
\begin{equation*}
x_{n}=\frac{1}{N} \sum_{k=0}^{N-1} X_{k} e^{\frac{j 2 \pi k n}{N}} \quad k, n \subset\{0,1,2, \ldots N-1\} \tag{2-1}
\end{equation*}
$$

PAPR is the ratio of OFDM symbol peak power with the symbol average power, The time domain signal is ${ }_{n}$ in the baseband signal.

$$
\begin{equation*}
\operatorname{PAPR}(d B)=10 \log _{10} \max _{0 \leq n \leq N-1}\left\{\left|x_{n}\right|^{2}\right\} \quad 0 \leq n \leq N-1 \tag{2-2}
\end{equation*}
$$

For the worst case of PAPR, When N points of time domain signal is 1,after IFFT the frequency signal will be the impact signal, the peak power of signal is N , the average power is 1 the maximum PAPR of baseband signal is $10 \log _{10} N$ [9-16].

OFDM symbol is complex numerical, according to the central limit theorem, as long as the subcarrier number N is large enough, the real part $X_{r}$ and the imaginary part ${ }^{X_{j}}$ of $x_{n}=x_{r}+j x_{i}$ are subordinated to Gaussian distribution. The mean value is 0 ,and the variance is 0.5 [8].

$$
\begin{equation*}
p(x)=\frac{1}{\sqrt{2 \pi} \delta} e^{-\frac{\left(x-m_{x}\right)}{2 \delta^{2}}} \tag{2-3}
\end{equation*}
$$

According to the probability theory, the amplitude value of OFDM symbol is $r=\sqrt{x_{r}^{2}+x_{i}^{2}}, r$ is the Rayleigh distribution, the probability density function of the $r$ [18]is

$$
\begin{equation*}
P(r)=2 r e^{-r^{2}} \tag{2-4}
\end{equation*}
$$

The power of OFDM symbol is $P=|r|^{2}$,the power distribution is the center $\chi^{2}$, the center distribution has two degrees of freedom, the mean value is 0 , the variance is 1 , the probability density function of the power is[18]

$$
\begin{equation*}
P_{\text {power }}(z)=e^{-z} \tag{2-5}
\end{equation*}
$$

Assume the sampling signal are not related to each other, Defined the cumulative distribution function (CDF) to statistic the characteristics of PAPR, It is the probability distribution of the PAPR value less than a threshold value, the formula is

$$
\begin{equation*}
P(P A P R \leq z)=\left(1-e^{-z}\right)^{N} \tag{2-6}
\end{equation*}
$$

In the project we use probability of the PAPR value more than a threshold. It is complementary cumulative distribution probability (CCDF), the formula is

$$
\begin{equation*}
P(P A P R>z)=1-P(P A P R \leq z)=1-\left(1-e^{-z}\right)^{N} \tag{2-7}
\end{equation*}
$$

## 3. Signal Decomposition Method to Reduce PAPR

ALT Alternative Time Decomposition algorithm is proposed in this paper, In the baseband the OFDM symbol are decomposed into two sub-signals, after the decomposition the PAPR can be reduced greatly, send the two sub-signals out, In the receiver the two sub-signals are added together.

Setting two thresholds in this algorithm: compare threshold $T_{c}$ and decompose threshold $T_{d}$. OFDM baseband time domain signal is $x_{n}=\left|x_{n}\right| e^{j \theta}$, Peak signal is defined as Peak, decomposed into $X_{n 1}$ and $X_{n 2}$, the number of original signal's subcarriers is N , The amplitude of first $\mathrm{N} / 2$ subcarriers compared with $T_{c}{ }^{*}|P e a k|$, If it larger than this threshold, $x_{n 1}$ and $X_{n 2}$ are equal to the half of original signal, but if smaller than the threshold,
$x_{n 1}=\mid$ Peak $^{*} T_{d} \mid * e^{j \theta} \quad x_{n 2}=x_{n}-x_{n 1}$. The left N/2 subcarriers compared with $T_{c}^{*} \mid$ Peak $\mid$. If it larger than this threshold, ${ }_{n 1}$ and ${ }^{X_{n 2}}$ are equal to the half of original signal, but if smaller than the threshold, $x_{n 2}=\left|\operatorname{Peak}^{*} T_{d}\right| * e^{j \theta}, x_{n 1}=x_{n}-x_{n 2}$.

The function is $\mathrm{n}=0 . . \mathrm{N} / 2-1$

$$
\left\{\begin{array}{l}
x_{n 1}=x_{n 2}=\frac{x_{n}}{2},\left|x_{n}\right| \geq T_{c}^{*} \mid \text { Peak } \mid  \tag{3-1}\\
x_{n 1}=\mid \text { Peak }^{*} T_{d}\left|* e^{j \theta}, x_{n 2}=x_{n}-x_{n 1},\left|x_{n}\right|<T_{c}^{*}\right| \text { Peak } \mid
\end{array}\right.
$$

$\mathrm{n}=\mathrm{N} / 2 \ldots \mathrm{~N}-1$

$$
\left\{\begin{array}{l}
x_{n 1}=x_{n 2}=\frac{x_{n}}{2},\left|x_{n}\right| \geq T_{c}^{*} \mid \text { Peak } \mid  \tag{3-2}\\
x_{n 2}=\mid \text { Peak } / 2\left|* e^{j \theta}, x_{n 1}=x_{n}-x_{n 2},\left|x_{n}\right|<T_{c}^{*}\right| \text { Peak } \mid
\end{array}\right.
$$

In this algorithm threshold will affect the power differences of two decomposed subsignal. We will analysis the $T_{c}$ threshold's influence to signal decomposition.
(1) $\mathrm{Tc}=1 / 2, T d=1 / 2$

$$
\left\{\begin{array}{l}
\frac{\mid \text { Peak } \mid}{2}<|x|<\mid \text { Peak }\left|, \mathrm{x}_{1}=x_{2}=\frac{x}{2}, \frac{\mid \text { Peak } \mid}{4}<\left|x_{1}\right|=\left|x_{2}\right|<\frac{\mid \text { Peak } \mid}{2}\right.  \tag{3-3}\\
0<|x|<\frac{\mid \text { Peak } \mid}{2}, \mathrm{x}_{1}=\frac{\mid \text { Peak } \mid}{2}, x_{2}=x-\mathrm{x}_{1}, 0<\left|x_{2}\right|<\frac{\mid \text { Peak } \mid}{2}
\end{array}\right.
$$

(2) $\mathrm{Tc}=1 / 3, \mathrm{Td}=1 / 2$

$$
\left\{\begin{array}{l}
\frac{\mid \text { Peak } \mid}{3}<x|<| \text { Peak }\left|, \mathrm{x}_{1}=x_{2}=\frac{x}{2}, \frac{\mid \text { Peak } \mid}{6}<\left|x_{1}\right|=\left|x_{2}\right|<\frac{\mid \text { Peak } \mid}{2}\right.  \tag{3-4}\\
0<|x|<\frac{\mid \text { Peak } \mid}{3}, \mathrm{x}_{1}=\frac{\mid \text { Peak } \mid}{2}, x_{2}=x-\mathrm{x}_{1}, \frac{\mid \text { Peak } \mid}{6}<\left|x_{1}\right|=\left|x_{2}\right|<\frac{\mid \text { Peak } \mid}{2}
\end{array}\right.
$$

(3) $\mathrm{Tc}=1 / 4, \mathrm{Td}=1 / 2$

$$
\left\{\begin{array}{l}
\frac{\mid \text { Peak } \mid}{4}<x|<| \text { Peak }\left|, \mathrm{x}_{1}=x_{2}=\frac{x}{2}, \frac{\mid \text { Peak } \mid}{8}<\left|x_{1}\right|=\left|x_{2}\right|<\frac{\mid \text { Peak } \mid}{2}\right.  \tag{3-5}\\
0<|x|<\frac{\mid \text { Peak } \mid}{4}, \mathrm{x}_{1}=\frac{\mid \text { Peak } \mid}{2}, x_{2}=x-\mathrm{x}_{1}, \frac{\mid \text { Peak } \mid}{4}<\left|x_{1}\right|=\left|x_{2}\right|<\frac{\mid \text { Peak } \mid}{2}
\end{array}\right.
$$

(4) $\mathrm{Tc}=1 / 6, \mathrm{Td}=1 / 2$

$$
\left\{\begin{array}{l}
\frac{\mid \text { Peak } \mid}{6}\left\langle x \nmid \text { Peeak }, \mathrm{x}_{1}=x_{2}=\frac{x}{2}, \frac{\mid \text { Ped } \mid}{12}<\right| x_{1}\left|=\left|x_{2}\right|<\frac{\mid \text { Ped } \mid}{2}\right.  \tag{3-6}\\
0<\chi x\left|<\frac{\mid \text { Pec } k \mid}{6}, \mathrm{x}_{1}=\frac{\mid \text { Pec } k \mid}{2}, x_{2}=x-x_{1}, \frac{\mid \text { Peck } \mid}{3}<x_{1}\right|=\left|x_{2}\right|<\frac{\mid \text { Peck } \mid}{2}
\end{array}\right.
$$

From the formula when the Tc is $1 / 3$, no matter what range the amplitude of x signal in, amplitude of ${ }^{X_{n 1}}$ and ${ }^{X_{n 2}}$ are in the same range. When the Tc is $1 / 2$, the amplitude of $x$ in different range, There is a obvious difference between two sub-signals, which need different power in the sender.

In order to ensure the similar power , we will choose Tc=1/3.

## 4. Simulation and Analysis of Results

In the simulation, we use OPSK and 16QAM, the total subcarriers are $\mathrm{N}=2048$, user subcarriers are 1024, Total 10000 OFDM symbols Two stage sampling. The first stage sampling is 1 , the second stage sampling is 8 , after the decomposition, filtering the frequency. compare threshold is $1 / 3$, decompose threshold is $1 / 2$.

In the signal processing, time domain and frequency domain in Figure 1:


L = 1 Decomposition Time Sighał 2 Decomposition Frequency Signal 2













Figure 1. Signal decomposition method time and frequency domain sampling 1:8

In the figure, sampling is 1 after ALT, there is no high peak in time domain signal,there are not large the signal amplitude difference, in the spectrum, there are spectrum leakage. In the protection interval of frequency after filtering, there is a slight change in time domain. In order to approximate to analog signal do eight times oversampling.

The CCDF curve of OFDM signal after signal decomposition method is shown in Figure 2 and Figure 3.


Figure 2. CCDF of PAPR of QPSK under signal decomposition method


Figure 3. CCDF of PAPR of 16QAM under signal decomposition method

From the simulation results, under QPSK modulation, after the first stage sampling the PAPR can be reduced nearly 9dB, after filtering, time domain signal changed, PAPR will increase greatly. after the 8 sampling the PAPR of new signal and original signal is nearly 3 dB .

From the simulation results, under 16QAM modulation, after the second sampling, PAPR total reduced by 3 dB , the average power of the original signal is nearly the same as the decomposed signals.

In difference sampling method, the PAPR result will be influenced. We will show the influence in Figure 4, In the simulation, we use OPSK, the total subcarriers are N=2048, user subcarriers are1024,Total 10000 OFDM symbols Two stage sampling. The first stage sampling is 2 , the second stage sampling is 4 , after the decomposition, filtering the frequency. compare threshold is $1 / 3$, decompose threshold is $1 / 2$.


Figure 4. Signal decomposition method time and frequency domain sampling 2:4

After the first stage sampling 2, decompose the signal with the ALT, the frequency leakage is different from the last frequency Figure 1.

The CCDF curve of OFDM signal after signal decomposition method is shown in Figure 5. After the first stage ,the PAPR are reduced nearly 8 dB , after the second stage the PAPR are reduced nearly 2.5 dB .


Figure 5. CCDF of PAPR of QPSK under signal decomposition method

## 5. Conclusion

A new signal decomposition method is proposed in this paper. the computational complexity is small in baseband, can reduce PAPR greatly. The influence of Compare threshold to signal power is deduced. This is a method decompose the original signal to two sub-signals in low PAPR then transmited and synthesized in the receiver. The different sampling stage value will influence the PAPR value.

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