

# Study on Application of Quantum BP Neural Network to Curve Fitting

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## Abstract

Based on the Quantum mechanics in a Quantum phase-shifted door and two negates controlled both general door, using them as the basic unit composed of activation function neural Network, this paper puts forward a Quantum state with input and output of the Quantum neuron Quantum state model and three layers of Quantum BP neural Network model of Network learning algorithm and performance to carry on the thorough study. Experimental results show that compared with BP neural Network, QBP appears quickly learn and convergent effect, with better fitting ability than the BP neural Network.

**Keywords:** quantum gate, Quantum Neural Network, Quantum Back-propagation algorithm, curve fitting

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## 1. Introduction

The curve fitting is to describe the function relationship of coordinates with approximate continuous curve or discrete point groups on the surface, and to analyze the relationship between two variables by curve fitting equation. The curve fitting and quick fitting accuracy of data analysis and processing plays an important role. With the development of science and technology and the neural network theory, the in-depth study of artificial neural network topology special structure and learning algorithm, make it has huge parallel distributed information processing capability, which can quickly and accurately the curve fitting.

However, with the popularization and application of the actual problems emerge unceasingly, the limitation of neural network also appeared gradually. For example, the natural learning speed is too low to accord with the brain capacity and real-time response characteristics of homework; the ability to learn and training, Memory disastrous; Hardware implementation difficulties, these limitations and led the artificial neural network and intelligent processing capacity in the gap remains, restricted the development of artificial neural network theory, but also promote the artificial neural network combined with other theories of cross-disciplinary research. Quantum computing gradually appear with the strong ability to calculate, many scholars began to consider the Quantum theory combined with Neural network in essence improve Neural network computing performance. This is the Quantum Neural network [1-2] (Quantum Neural Networks, referred to as QNN).

Quantum neural network, the combination of quantum computing and artificial neural network, may have become an important means of future information processing. This is Kouda in [3], based on the basis of the research of Quantum mechanics in a Quantum phase door and two negates controlled composed of Quantum neuron model[4], establish a Quantum BP neural Network model (Quantum Back Propagation Network, QBP), studies its Network of training algorithm and performance. And will QBP model is applied to the curve fitting and BP network contrast experiment results show: The results show that the convergence rate of QBP networks is the faster than that of BP network. QBP network has better fit ability.

## 2. Quantum BP Neural Network

### A. Quantum Neuron Model

Quantum gate [5] is the physical foundation of quantum calculation. In a sense, quantum gate may represent the quantum calculation and contain the characteristics of

quantum calculation such as quantum parallelism. So by quantum gate of arbitrary quantum gate, this paper USES the network by a phase shift is composed of two doors and negate controlled, as the basic unit of the calculation of the activation function neural network to constitute the new quantum neuron model. For the sake of easy application, the quantum state and universal quantum gate group are expressed by in the plural form.

Quantum bits with classic difference: a quantum state is at  $|0\rangle$  and  $|1\rangle$  coherent superposition state.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

Here is a plural said probability amplitude and normalized, meet demands. Using a complex function to describe the state of quantum state, plural functions of representation for:

$$f(\theta) = e^{i\theta} = \cos\theta + i\sin\theta \tag{2}$$

One said that  $i = \sqrt{-1}$  imaginary number unit.  $\theta$  is expressed the phase of quantum state is expressed the phase.  $|0\rangle$  With the probability amplitude of the plural functions,  $|1\rangle$  said the real probability amplitude of the imaginary part with it. Is a quantum state can be described as

$$|\Psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \tag{3}$$

According to the formula (2), based on a definition of phase door and two negater constitute the quantum neuron control model shown in Figure 1. Its input use multiple quantum states superposing form, through the three quantum branch of the quantum state input and phase-shift separately, to get more quantum states superposing output.

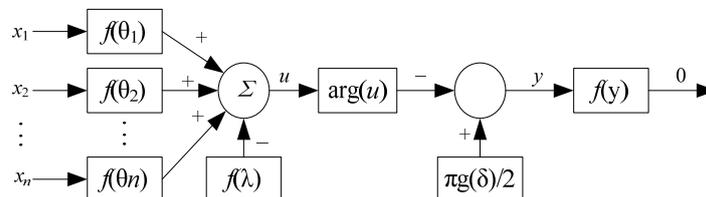


Figure 1. Quantum neuron model

In Figure 1,  $x_i (i=1,2,L n)$  Said the input to a quantum state of neurons,  $\theta_i (i=1,2,L n)$  is as the phase transfer coefficient weights,  $\lambda$  for the threshold for phase control factor,  $\delta$  the coefficient,  $O$  for output state,  $arg(u)$  to extract phase is  $u$  plural, namely  $arg(u) = \text{actag}(\text{Im}(u) / \text{Re}(u))$ , among them,  $\text{Im}(u)$  for the plural  $u$  the imaginary part of the complex  $u$ ,  $\text{Re}(u)$  for  $u$  real, Function  $f$  is defined as formula (2),  $g(x)$  is the sigmoid function.

Here is  $I_i$  as  $i$  the input to a set of neurons in the phase of the  $x_i$  quantum state of the quantum neuron output using formula said is as follows:

$$u = \sum_{i=1}^n f(\theta_i)f(I_i) - f(\lambda) \tag{4}$$

$$y = \frac{\pi}{2} g(\delta) - arg(u) \tag{5}$$

$$O = f(y) \tag{6}$$

In the quantum neuron model, there are two types of parameter form, one kind is corresponding  $\theta_i$  to the phase of phase door and parameters of the threshold value  $\lambda$ , Another turn control parameters  $\delta$  correspond to net gater control. With traditional neurons, quantum neuron weights  $f(\theta_i)$  and input factor  $x_i = f(I_i)$  in the outcome, it is through the phase shift of normal phase shift stoke neurons.

**B. Quantum BP Network Structure**

According to the above the quantum neuron model, using BP neural network of artificial network structure [4] [6] [10] [11], and in consideration of the network runtime, we establish a three-layer quantum BP neural network, as shown in Figure 2. It can be seen from the graph network structure and the traditional BP network, only a hidden layer, namely no connection between neurons, layer and layer between neurons connect each realized. Difference lies in its input and output of multiple quantum neuron structure.

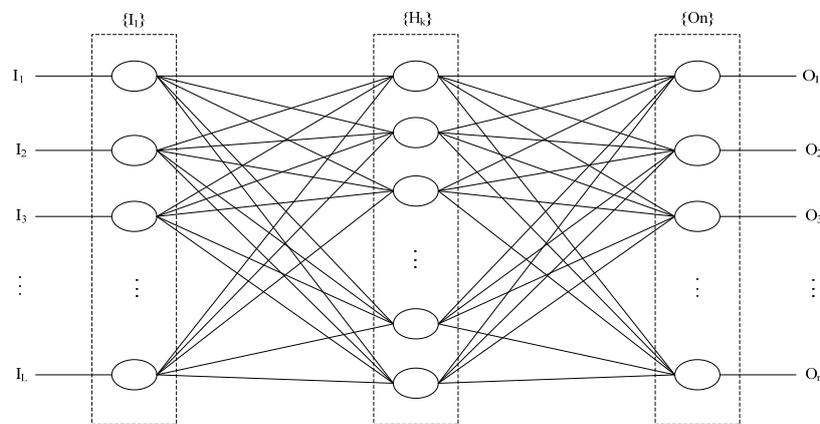


Figure 2. Three layers of quantum BP neural network

In the QBP diagram, the input layer has L neurons, the output layer has K neurons and the hidden layer is of N neurons. When the input data input to the input by input, each neuron will make between the input value between  $[0,1]$  convert between  $[0, \pi/2]$  quantum state between the phase of the value, the phase modulation by the quantum state of each neuron input output. Expression is:

$$y_{il} = \frac{\pi}{2} I_l \tag{7}$$

$$IO_l = f(y_{il}) \tag{8}$$

Among them, the input layers  $I_l (l=0,1,L L)$  says input the  $l$  neuron input,  $IO_l$  for t the  $l$  neuron input and output of its output of neurons were sent to the quantum state of hidden layers.

The output of each neuron hidden layer, its expression

$$u_{ik} = \sum_l^L f(\theta_{lk,l}) * IO_l - f(\lambda_{ik}) \tag{9}$$

$$y_{2k} = \frac{\pi}{2} g(\delta_{1k}) - \arg(u_{1k}), k = 0, 1, L, K \quad (10)$$

$$H_k = f(y_{2k}) \quad (11)$$

Among them, the input is  $\theta_{1k}$  layer of  $l$  neuron to  $k$  neuron hidden layer of phase rotation coefficient,  $\lambda_{1k}$  hidden for the threshold value of  $k$  neuron,  $\delta_{1k}$  hidden layer for the coefficient of the first phase of  $k$  neuron control factor,  $H_k$  hidden for the output of  $k$  neuron. The output of each neuron output layer, its expression:

$$u_{2n} = \sum_k^k f(\theta_{2n,k}) * H_k - f(\lambda_{2n}) \quad (12)$$

$$y_{3n} = \frac{\pi}{2} g(\delta_{2n}) - \arg(u_{2n}) \quad (13)$$

$$OP_n = f(y_{3n}) \quad (14)$$

Among them,  $\theta_{2n,k}$  first for hidden  $k$  neuron to output  $n$  neuron layer of phase rotation coefficient,  $\lambda_{2n}$  for the output layer, the  $n$  neuron threshold for output layer, coefficient of A neuron first phase control factor,  $OP_n$  hidden layer for the  $n$  neuron output.

In quantum neuron, quantum equivalent to activate  $|1\rangle$  neuron, quantum equivalent of  $|0\rangle$  neuron, so any suppression of neurons in the quantum state is defined as the activation and inhibition of superposition state, the output value of neurons in the final for activation probability of state. So the three layers of BP neural network, N neuron output layer of the final output for:

$$O_n = |Im(OP_n)|^2 \quad (15)$$

### C. Training Algorithm

In order to train three layers of quantum BP neural network, with plural BP algorithm[7], define the reverse transmit of the net as Quantum BP algorithm (QBP algorithm) [8,9], approximate mean square error of the steepest descent algorithm in network, to adjust the phase rotation coefficient  $\theta$ , threshold coefficient  $\lambda$  and phase control factor  $\delta$ , makes training error is less than expected training goal.

Definition of mean square error function E, its expressions for:

$$E = \frac{1}{2} \sum_n^N (t_n - O_n)^2 \quad (16)$$

among them,  $t_n$  the output layer is  $n$  neuron expected output,  $O_n$  the output layer is  $n$  neuron actual output. Input/output by the network, the network can see formula is the final output error phase rotation coefficient  $\theta$ , layer threshold coefficient  $\lambda$  and phase control factor  $\delta$  of function, so they can change  $E$  adjustment error. The of BP network propagation process of training are as follows:

Firstly, see whether the relative error is less than the target error. If the target is not less error, error back propagation. According to the steepest descent method, reverse readjusted layer parameters.

(1) parameters adjustment in the output layer of each neuron

After adjusting the hidden first  $n$  neuron phase control factor  $\delta_{2n}$  into:

$$\delta_{2n}^{new} = \delta_{2n}^{old} + \eta \frac{\pi}{2} (t_n - O_n) \text{Im}(f'(y_{3n})) g'(\delta_{2n}) \quad (17)$$

among them,  $\eta$  are learning coefficient, in training reflects learning rate. It's selection of neural network training speed and parameters of precision greatly. Error is not implied layer of explicit function, so the chain in calculus calculate partial derivative rules.

The output layer of phase rotation coefficient  $\theta_{2n,k}$ , threshold adjustment  $\lambda_{2n}$  coefficient for:

$$\theta_{2n,k}^{new} = \theta_{2n,k}^{old} - \eta d_n \arg'(u_{2n}) m_n \quad (18)$$

$$\lambda_{2n}^{new} = \lambda_{2n}^{old} - \eta d_n \arg'(u_{2n}) s_n \quad (19)$$

among them,

$$m_n = \frac{\cos(\theta_{2n,k} + y_{2n}) \text{Re}(u_{2n}) + \sin(\theta_{2n,k} + y_{2n}) \text{Im}(u_{2n})}{(\text{Re}(u_{2n}))^2} \quad (20)$$

$$s_n = \frac{-\cos(\lambda_{2n}) \text{Re}(u_{2n}) - \sin(\lambda_{2n}) \text{Im}(u_{2n})}{(\text{Re}(u_{2n}))^2} \quad (21)$$

$$d_n = 2(t_n - O_n) \text{Im}(OP_n) \text{Im}(f'(y_{3n})) \quad (22)$$

(2) parameters adjustment in the hidden layers

$$\delta_{1k}^{new} = \delta_{1k}^{old} - \eta \frac{\pi}{2} \sum_n^N d_n \arg'(u_{2n}) m_n g'(\delta_{1k}) \quad (23)$$

$$\theta_{1k,j}^{new} = \theta_{1k,j}^{old} + \eta \sum_n^N d_n \arg'(u_{2n}) m_n \arg'(u_{1k}) m_{1k} \quad (24)$$

$$\lambda_{1k}^{new} = \lambda_{1k}^{old} - \eta \sum_n^N d_n \arg'(u_{2n}) m_n \arg'(u_{1k}) s_{2k} \quad (25)$$

among them:

$$m_{1k} = \frac{\cos(\theta_{1k,j} + y_{1l}) \text{Re}(u_{1k}) + \sin(\theta_{1k,j} + y_{1l}) \text{Im}(u_{1k})}{(\text{Re}(u_{1k}))^2} \quad (26)$$

$$s_{1k} = \frac{-\cos(\lambda_{1k}) \text{Re}(u_{1k}) - \sin(\lambda_{1k}) \text{Im}(u_{1k})}{(\text{Re}(u_{1k}))^2} \quad (27)$$

Then, perform input/output of quantum calculation by the new parameters obtained by this three-layer again BP neural network. For there is no parameter in the input layer, the calculation starts from the hidden layer. Get the final output value network error after, if not less error if target is repeated error back propagation, adjusting parameters. Repeat the above course until the error is less than the target.

### 3. Analysis of the Characteristics of QBP Network of Curve Fitting

To test the above three layers of QBP network performance, it will be used to test the curve fitting, network nonlinear mapping capability.  $y = f(x)$  function is known part of the corresponding relationship as shown in Table 1.

Table 1. Part of the corresponding relationship between function  $y = f(x)$

x	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1
y	-0.832	-0.423	-0.024	-0.344	1.282	3.456	4.02	3.232	2.102	1.504
x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
y	0.248	1.242	2.344	3.262	20.52	1.684	1.022	2.224	3.022	1.984

According to the data in Table 1, the use of three layers of BP network and QBP network, on the same network structure and training algorithm under the function curve fitting, the parameters of the network for: input and output layer number of neurons in hidden layer is 1, number of neurons were taken 6, 7, 8, 9, 10, 12 and 15, for different network structure, learning rate  $\eta$  between 0.1 and 3 in every 0.1 training 20 times, target error for 0.001, the maximum iteration for 50. According to the experimental results, QBP using 1-8-1 structure, when  $\eta = 1.8$  can completely fitting curve, 20 times training for completely fitting average iteration, 20% to 16, convergence rate is 60%, this function curve fitting diagram, as shown in Figure 3, Its corresponding iteration error, as shown in Figure 5(a).

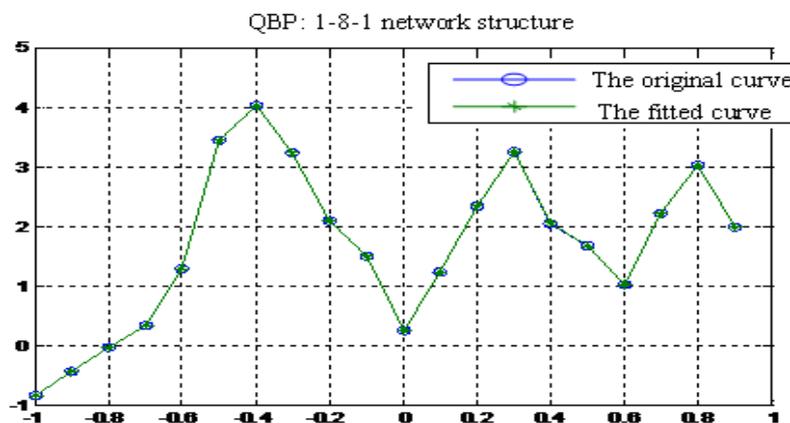


Figure 3. The fitted curve

Corresponding BP is also used 1-8-1 structure, function curve fitting, can realize completely still exist many differences, convergence rate only 10%, Figure 4 is 1-8-1 structure, using BP when  $\eta = 0.2$  get a better fitting, its corresponding iteration error, as shown in Figure 5(b). With the increasing number of neurons in hidden layer, BP, 1-10-1 using structure, when  $\eta = 0.2$  can realize the function curve fitting, after completely 20 times the completely fitting training, average rate of 10% for 25 iteration convergence rate of 40%. When adopting QBP 1-10-1 structure, the fitting average rate can be of 75% through 20 times training function, the average iteration time can be of eight and the convergence rate can be of 100%, while using BP 1-15-1 structure, through 20 times after training fully fitting rate can reach 80%, the average iteration of 10 for Convergence rate 100%. Conclusion it is easy to see QBP by more than BP network, network with strong function fitting ability.

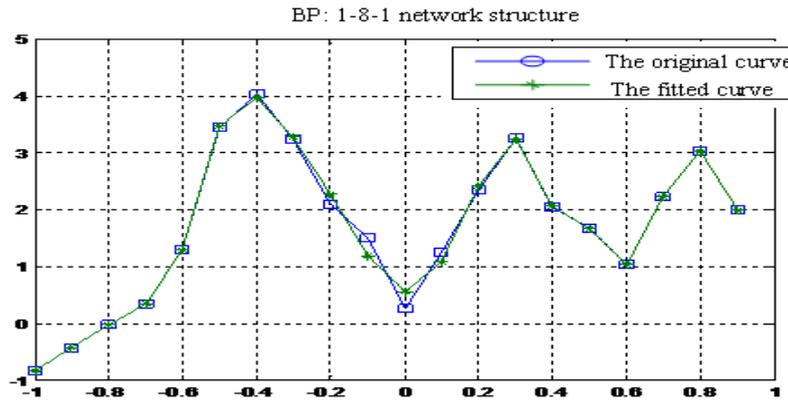


Figure 4. The fitted curve

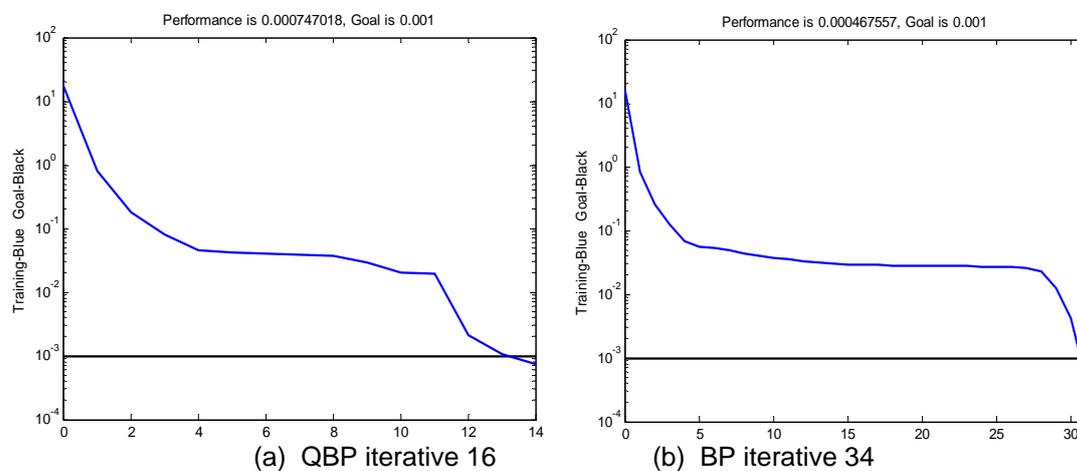


Figure 5. QBP and BP1-8-1 network structure

#### 4. Conclusion

Based on analysis of the quantum neuron model which regards the quantum gate group (i.e. phase-shift gate and controlled not gate) as the basic unit of the calculation, using plural BP learning rule, this article establishes a three-layer QBP network model and applies it in function fitting. For the controlled negater of the quantum neuron structure has the function of microcosmic emendation for output, and of for quantum BP neuron can play the role of output, and network calibration parameter, and phase shift parameters exists in the network which varies from 0-1, this will make the quantum neuron output continue to vary and to the minimum. Experimental results show that compared with BP neural network QBP appears a much higher rate of learning. The QBP network has better fitting ability.

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