# Predictive and probabilistic modelling using machine learning for building indoor climate control

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| Article Info | ABSTRACT |
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| | For the last few decades, thermal comfort has been considered an aspect of sustainable building evaluation methods and tools. However, estimating the indoor air temperature of buildings is a complicated task due to the nonlinear behaviour of heating, ventilation and air conditioning systems combined with complex dynamics characterized by the time-varying environment with disturbances. This issue can be alleviated by modelling the building dynamics using Gaussian processes since it also measures the uncertainty bounds. The main focus of this paper is designing a predictive and probabilistic room temperature model of buildings using Gaussian processes and incorporating it into model predictive control to minimize energy consumption and provide thermal comfort satisfaction. We exploited the Gaussian processes' full probabilistic capabilities as the mean prediction for the room temperature model and used the model uncertainty in the objective function not to lose the desired performance and to design a robust control scheme. We illustrated the potentials of the proposed method in a numerical example with simulation results. |

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## 1. INTRODUCTION

There is an urge for more intelligent living comfort due to recent technologies that lead us towards the smart city that can provide better living through enhancing the integration of the quality and economic conditions by implementing information systems at different infrastructures of a city. Intelligent energy is one of the primary critical concepts of a smart city, where building residents consume significant energy for gaining thermal comfort [1]. Because of the disintegrate character of the building dynamics in terms of optimization and control, achieving an energy-efficient building climate control scheme that integrates fully automated heating, ventilation, and air conditioning (HVAC) services is still an open question. In building climate control problems, HVAC systems keep room temperature within a comfortable range. For decades, thermal comfort has been considered an aspect of sustainable building evaluation methods and tools [2], [3]. However, estimating the indoor air temperature of buildings is a complicated task due to the HVAC system's nonlinear and complex dynamic characterized by the time-varying environment with disturbances. Developing the building model is the most primary and time-consuming task when the modelling technique relies on physics-based and grey-box methods [4] based on energy and mass balance partial differential equations. On the other hand, the recent developments in machine learning (ML) techniques and the increasing data accessibility, thanks to smart

and accurate sensor measurements, in buildings have empowered the study of data-driven building models due to their simplicity, modern stage of automation, and less engineering effort. In these circumstances, several research works have been investigated considering the time-varying user comfort preference [5], the continuous electricity supply to the consumer in the presence of random operation in building energy consumption based on artificial neural networks in a smart grid [6]. Optimized energy and comfort management scheme for intelligent and sustainable buildings is provided in [7]. A comprehensive review focusing on thermal comfort predictive models and their applicability for energy control purposes is analyzed in [8].

Building climate control must be balance three conflicting demands such as energy efficiency, cost and thermal comfort. Model predictive control (MPC) is an optimal control method to design control law by minimizing a performance index while handling these demands. MPC has been implemented successfully in several directions of building control and operation strategies [9], [10]. Better thermal comfort and more energy savings compared to other control techniques can be achieved by combining MPC and ML such as neural networks [11], random forest [12], support vector machines [13]. However, designing accurate building energy/temperature models is the cornerstone to developing MPC for whole building operation and control due to the presence of external disturbances. This issue can be alleviated by modelling the building dynamics using Gaussian processes (GPs) since it also measures the uncertainty bounds. Unfortunately, most GP based control laws do not take advantage of this information [14], [15]. The main focus of this paper is designing a predictive and probabilistic room temperature model of buildings using GPs. We exploit the GPs full probabilistic capabilities as the mean prediction for the room temperature model and use the model uncertainty in the MPC objective function not to lose the desired performance and to design a robust controller.

We organize the remainder of this paper as follows: We start with introducing the preliminary background of MPC formulation, data preparation, and a methodology for constructing a predictive and probabilistic building model using GPs in section 2. A theoretical analysis of designing an intelligent control combining GP with MPC scheme for building climate control problems is presented in section 3. Afterwards, the potentials of the proposed control scheme are demonstrated in simulation with some numerical results in section 4. Finally, the conclusions of our work are drawn, and further research challenges are discussed in section 5.

## 2. THE COMPREHENSIVE THEORETICAL BASIS

This section starts with the preliminary theory for MPC problem formulation and followed by a data preparation process for building dynamics. Since dynamics of a building is complex and computationally lighter prediction model is desired, we should take care of features that provides as much information as needed. Then for given data, the methodology for designing predictive and probabilistic models using GP is explained. The main reason for selecting GP as a prediction model, because GP can handle uncertainties affected to the building dynamics.

### 2.1. MPC scheme

Consider the following classic MPC optimization problem with input and output constraints:

$$\min_{\mathbf{U},E} \sum_{\tau=0}^{N_p-1} \ell_\tau(\mathbf{y}_{\tau+1+t|t}, \mathbf{x}_{\tau+1+t|t}, \mathbf{u}_{\tau+t|t}, \epsilon_{\tau+1+t|t}) \tag{1}$$

$$\text{s.t. } \mathbf{x}_{\tau+1+t|t} = f(\mathbf{x}_{\tau+t|t}, \mathbf{u}_{\tau+t|t}, \mathbf{d}_{\tau+t|t}) \qquad \tau \in \mathbb{I}_0^{N_p-1}$$

$$\mathbf{u}_{\tau+t|t}^{\min} \leq u_{\tau+t|t} \leq \mathbf{u}_{\tau+t|t}^{\max} \qquad \tau \in \mathbb{I}_0^{N_p-1}$$

$$\mathbf{y}_{\tau+t|t} = C\mathbf{x}_{\tau+t|t} + \mathbf{v}_{\tau+t|t} \qquad \tau \in \mathbb{I}_1^{N_p}$$

$$-\epsilon_{\tau+t|t} + \mathbf{y}_{\tau+t|t}^{\min} \leq \mathbf{y}_{\tau+t|t} \leq \mathbf{y}_{\tau+t|t}^{\max} + \epsilon_{\tau+t|t} \qquad \tau \in \mathbb{I}_1^{N_p}$$

$$\epsilon_{\tau+t|t} \geq 0 \qquad \tau \in \mathbb{I}_1^{N_p}$$

where $t$ is the current time instant, $N_p$ the prediction horizon, $\mathbb{I}_a^b$ denotes set of integer numbers in the interval $[a, b]$, $\mathbf{U} = [\mathbf{u}_0, \ldots, \mathbf{u}_{N_p-1}]$ is the sequence of manipulated variables $\mathbf{u}_{\tau+t|t} \in \mathbb{R}^{n_u}$ to optimize, $\mathbf{x}_{\tau+t|t} \in \mathbb{R}^{n_x}$ is the state vector at $\tau$-steps ahead, $\mathbf{y}_{\tau+t|t} \in \mathbb{R}^{n_y}$ is the output vector, $\mathbf{d}_{\tau+t|t} \in \mathbb{R}^{n_d}$ is a disturbances impacting on the prediction model described by $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \to \mathbb{R}^{n_x}$, $\ell_\tau : \mathbb{R}^{n_y} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_y} \to \mathbb{R}_{\geq 0}$ are

convex stage cost functions, and $E = (\epsilon_1, \ldots, \epsilon_N)$ includes auxiliary variables $\epsilon_k \in \mathbb{R}^{n_y}_{\geq 0}$ used to soften output constraints, which is called slack variable vector.

MPC is a control technique that selects optimal control action based on the future state predictions of the system model. Optimal control actions are calculated by solving an optimization problem so that an objective function is minimized and constraints are satisfied in every step of a controlled system. Then the only first sample of the commanded inputs is applied to the system as its optimal input. This process is repeated all over again to calculate the control signal in every step [16]. The development of the model to predict the outputs/states in the MPC objective function is the most primary and time-consuming task of MPC design. However, the recent developments in ML techniques and the increasing data accessibility, thanks to smart and accurate sensor measurements, in buildings have empowered the study of data-driven models [12], [14] as we discuss below.

## 2.2. Data preparation

To provide thermal comfort in buildings, HVAC systems are usually manipulated to keep room temperature within a comfortable range. However, designing a proper controller to minimize cost in the building system while preserving thermal comfort is challenging task due to the HVAC system's complex dynamic characteristics, uncertain and time-varying environment, and disturbances. For this reason, the data acquisition system e.g. supervisory control and data acquisition (SCADA), has to be set up in such a way that the gathered data should comprise information both from inside (power consumption, water flow and water temperature, human occupation, and $CO_2$ level) and outside (air temperature, air humidity, solar radiation, outside wall temperature, and wind speed) the building. One option to correlate these features is to employ nonlinear autoregressive model with exogenous input (NARX) model architecture [15] that incorporates historical information up to a certain lag order. Then a training dataset $\mathbf{D}$ of $N$ samples consisting of input-output pairs $\mathbf{D} = \{\mathbf{X_{l_y,l_u,l_d}}, \mathbf{Y}\}$ is collected as $\mathbf{X}_{l_y,l_u,l_d} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N]_{l_y,l_u,l_d}$ and $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N]$ with:

$$\mathbf{x}_i = [\mathbf{y}_i(l_y) \ \ \mathbf{u}_i(l_u) \ \ \mathbf{d}_i(l_d)] = \begin{cases} \mathbf{y}_i(l_y) = [y^j_{i-1}, \ldots, y^j_{i-l_y}], j = 1, 2, \ldots N_r. \\ \mathbf{u}_i(l_u) = [u^k_{i-1}, \ldots, u^k_{i-l_u}], k = 1, 2, \ldots N_u. \\ \mathbf{d}_i(l_d) = [d^k_i, \ldots, d^h_{i-l_d}], h = 1, 2, \ldots N_d. \end{cases} \tag{2}$$

where $\mathbf{y} \in \mathbb{R}^{n_y}$ is power/temperature measurement vector, $\mathbf{u} \in \mathbb{R}^{n_u}$ is heating/cooling set-point vector, $\mathbf{d} \in \mathbb{R}^{n_y}$ is external disturbance vector affecting to the building dynamics, $N_r \in \mathbb{R}$ is the total number of rooms, $N_u \in \mathbb{R}$ is the total number of control inputs, $N_d \in \mathbb{R}$ is the total number of disturbance parameters, and $l_y$, $l_u, l_d \in \mathbb{R}$ are corresponding minimal auto-regressive lags to be determined by feature selection algorithms as we discuss next.

The feature selection process is one of the most critical steps in prediction problems since it finds the smallest subset that significantly affects the prediction error and maximizes the likelihood measure of the approximated model. The accuracy of the prediction model dramatically depends on the quality of data and the relevancy of features [17]. A review paper [18] summarizes feature selection applications in building energy management, including filter method [19], wrapper method [20], and embedded method [21]. However, these methods are very general and quite conservative in terms of learning speed. We adopt the algorithm proposed in [22] to select the minimum lag orders to get the most informative features by maximizing the relevancy of the features on the buildings' load consumption and thermal comfort settings.

## 2.3. Learning building model with Gaussian processes

A Gaussian process is an assembly of stochastic variables that any finite collection of these variables follows a multivariate normal distribution over functions with a continuous domain. The Bayesian inference of continuous variables leads to Gaussian process regression where the prior GP model is updated with training dataset to obtain a posterior GP distribution [23]. Due to the possibility to include prior knowledge making the method more attractive as compared to other regression algorithms, GP models have been employed in different research fields [24]-[26]. This section provides the necessary background about GP and framework to build a probabilistic and predictive model for regression problems mainly, adopted from [27], [28].

### 2.3.1. Probabilistic model

Let a triple $(\Omega, \Psi_a, \mathcal{P})$ describe a probability space consisting of sample space $\Omega$, corresponding $a$-algebra $\Psi_a$ and the probability measure $\mathcal{P}$. Then a stochastic process can be expressed by a measurable

function $\Phi_{GP}(\mathbf{x})$ in $\mathcal{X} \subseteq \Omega$, which is fully described by mean function $\mu : \mathcal{X} \to \mathbb{R}$ and covariance function $\kappa : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$:

$$\Phi_{GP}(\mathbf{x}) \sim \mathcal{GP}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}')) \tag{3}$$

$$\mu(\mathbf{x}) = E[\Phi_{GP}(\mathbf{x})] \tag{4}$$

$$\kappa(\mathbf{x}, \mathbf{x}') = E[(\Phi_{GP}(\mathbf{x}) - \mu(\mathbf{x}))(\Phi_{GP}(\mathbf{x}') - \mu(\mathbf{x}'))]. \tag{5}$$

with pair $(\mathbf{x}, \mathbf{x}') \in \mathcal{X}$. The mean function of the $\mathcal{GP}$ distribution illustrates the point where the samples are more likely located, while the variance of the $\mathcal{GP}$ distribution comes from measuring the correlation of any two samples $(\mathbf{x}, \mathbf{x}')$ that is calculated by the covariance function. We refer to [28] for a variety of mean and covariance functions.

Despite the absence of the existence of the probability density function of the GPs, their finite collection follows multivariate Gaussian distribution. This property allows us to write samples as a joint multivariate Gaussian distribution with a mean $\mu$ and variance $\sigma^2$:

$$\mathbf{y}_i = \Phi_{GP}(\mathbf{x}_i) \sim \mathcal{GP}(\mu(\mathbf{x}_i), \sigma^2(\mathbf{x}_i)), \quad i = 1, 2, \ldots, N. \tag{6}$$

### 2.3.2. Predictive model

The GP can be utilized as a prior probability distribution in Bayesian inference [28], allowing function regression to perform. A new given test sample $\mathbf{x}_\star \in \mathcal{X}$ is combined with existing training samples based on the Bayesian framework to obtain a posterior distribution for $\mathbf{y}_\star \in \mathcal{Y}$. For the batch of random variables $[\mathbf{y}_1, \ldots, \mathbf{y}_N, \mathbf{y}^\star] \in \mathcal{Y}$, we write:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \\ \mathbf{y}_\star \end{bmatrix} \sim \mathcal{GP} \left( \begin{bmatrix} \mu(\mathbf{x}_1) \\ \mu(\mathbf{x}_2) \\ \vdots \\ \mu(\mathbf{x}_N) \\ \mu_\star(\mathbf{x}_\star) \end{bmatrix}, \begin{bmatrix} \mathcal{K} & \mathcal{K}_\star \\ \mathcal{K}_\star^T & \kappa_\star \end{bmatrix} \right) \tag{7}$$

with the covariance matrices:

$$\mathcal{K} = \kappa(\mathbf{X}, \mathbf{X}) = \begin{bmatrix} \kappa(\mathbf{x}_1, \mathbf{x}_1) & \kappa(\mathbf{x}_1, \mathbf{x}_2) & \ldots & \kappa(\mathbf{x}_1, \mathbf{x}_N) \\ \kappa(\mathbf{x}_2, \mathbf{x}_1) & \kappa(\mathbf{x}_2, \mathbf{x}_2) & \ldots & \kappa(\mathbf{x}_2, \mathbf{x}_N) \\ \vdots & \vdots & \ldots & \vdots \\ \kappa(\mathbf{x}_N, \mathbf{x}_1) & \kappa(\mathbf{x}_N, \mathbf{x}_2) & \ldots & \kappa(\mathbf{x}_N, \mathbf{x}_N) \end{bmatrix} \in \mathbb{R}^{N \times N} \tag{8}$$

$\mathcal{K}_\star = [\kappa(\mathbf{x}_1, \mathbf{x}_\star), \ldots, \kappa(\mathbf{x}_N, \mathbf{x}_\star)]^T \in \mathbb{R}^{N \times 1}$ is the vector of similarity measure between the training samples and the samples and $\kappa_\star = \kappa(\mathbf{x}_\star, \mathbf{x}_\star) \in \mathbb{R}$ is the self-covariance of the test sample. From (7), we can deduce that the Gaussian prediction $\mathbf{y}_\star$ for the new input $\mathbf{x}_\star$ with the mean $\mu_\star(x_\star)$ and the variance $\sigma_\star^2(x_\star)$ is given as follows:

$$\mathbf{y}_\star = \Phi_{GP}(\mathbf{x}_\star) \sim \mathcal{GP}(\mu_\star(\mathbf{x}_\star), \sigma_\star^2(\mathbf{x}_\star)) \tag{9}$$

$$\mu_\star(\mathbf{x}_\star) = \mu(\mathbf{x}_\star) + \mathcal{K}_\star^T \mathcal{K}^{-1}(\mathbf{Y} - \mu(\mathbf{X})) \tag{10}$$

$$\sigma_\star^2(\mathbf{x}_\star) = \kappa_\star - \mathcal{K}_\star^T \mathcal{K}^{-1} \mathcal{K}_\star \tag{11}$$

## 3. RESEARCH METHOD

This section deals with the optimal control problem for building indoor climate using MPC methodology applied to stochastic dynamic process. For this purpose, a GP model is used to learn the building model and integrated into the MPC scheme to design a robust control using variance information of the GP model. Building climate control must balance three conflicting demands: energy efficiency, cost, and thermal comfort. MPC is an optimal control method to design control law by minimizing a performance index while handling these demands. However, designing accurate building energy/temperature models is the cornerstone to developing MPC for whole building operation and control due to the presence of external disturbances. This issue

can be alleviated by including the variance term into the MPC optimization objective enabling the design of a robust controller thanks to the availability of uncertainty prediction in GP modelling.

One of the significant constraints that should be encountered for the building climate control problem is human thermal comfort. There exist two types of methods to introduce this constraint to the problem: predicted average vote of room users [29] and thermal bounds [30]. Treating MPC with the former type as a constraint or objective function increases the computational burden of the optimization problem. For this reason, we consider the latter as a thermal constraint with linear upper and lower bounds in our proposed control problem. We are interested in the use of GPs for predicting the room(s) air temperature $\mathbf{y}$ as a function of the previous temperature measurements, forecasted weather disturbances $\mathbf{d}$ (solar radiation, outside air temperature and internal heat gains) and manipulated variables $\mathbf{u}$. The control task is to keep the room temperature within a predefined comfort range by commanding a set of different actuators $\mathbf{u}$ such as heating, cooling, ventilation and air conditioning. The goal is to select the optimal control inputs automatically using GP based MPC while satisfying the comfort requirements and minimizing energy costs coming from manipulated set-points. To this end, consider the following GP model-based MPC optimization problem:

$$\min_{\mathbf{U},E} \sum_{\tau=0}^{N_p-1} \|\mathbf{y}_{\tau+t+1|t}\|_{Q_{\mathbf{y}}}^2 + \|\sigma_{\tau+t+1|t}^2\|_{Q_\sigma}^2 + \|\mathbf{u}_{\tau+t|t}\|_{Q_u}^2 + \|\epsilon_{\tau+t+1|t}\|_{Q_\epsilon}^2 \tag{12}$$

$$\begin{aligned}
\text{s.t.} \quad & \mathbf{x}_{\tau+t|t} = \begin{bmatrix} \mathbf{y}_{\tau+t|t} & \cdots & \mathbf{y}_{\tau-l_y+t|t} & \mathbf{u}_{\tau+t|t} & \cdots & \mathbf{u}_{\tau-l_u+t|t} & \mathbf{d}_{\tau+t+1|t} & \cdots & \mathbf{d}_{\tau-l_d+1+t|t} \end{bmatrix} && \tau \in \mathbb{I}_0^{N_p-1} \\
& \mathbf{y}_{\tau+t+1|t} = \mu_t(\mathbf{x}_{\tau+t|t}) + \mathcal{K}_{\tau+t|t}^T \mathcal{K}_t^{-1}(\mathbf{Y}_t - \mu_t(\mathbf{X}_t)) && \tau \in \mathbb{I}_0^{N_p-1} \\
& \sigma_{\tau+t+1|t}^2 = \kappa_{\tau+t|t} - \mathcal{K}_{\tau+t|t}^T \mathcal{K}_t^{-1} \mathcal{K}_{\tau+t|t} && \tau \in \mathbb{I}_0^{N_p-1} \\
& \mathbf{u}_{\tau+t|t}^{\min} \leq \mathbf{u}_{\tau+t|t} \leq \mathbf{u}_{\tau+t|t}^{\max} && \tau \in \mathbb{I}_0^{N_p-1} \\
& -\epsilon_{\tau+t|t} + \mathbf{y}_{\tau+t|t}^{\min} \leq \mathbf{y}_{\tau+t|t} \leq \mathbf{y}_{\tau+t|t}^{\max} + \epsilon_{\tau+t|t} && \tau \in \mathbb{I}_1^{N_p} \\
& \epsilon_{\tau+t|t} \geq 0 && \tau \in \mathbb{I}_1^{N_p}
\end{aligned}$$

where $\|s\|_Q^2 = s^T Q s$ is a weighted quadratic norm and $Q_y, Q_\sigma, Q_u, Q_\epsilon$ are corresponding positive definite matrices. The summary of GP based MPC scheme is given in the algorithm 1.

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**Algorithm 1:** GP based MPC at a time step $t$

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**Input:** Trainig data: $\mathbf{D}_t = \{\mathbf{X}_t, \mathbf{Y}_t\}$, autoregressive lags: $l_y, l_u, l_d$, GP model components: mean and covariance functions.

**Output:** $\mathbf{u}_t$

1 calculate the matrices $\mathcal{K}_t^{-1}$ and $\mu_t(\mathbf{X}_t)$
2 solve MPC problem (12) online for $\mathbf{u}_t, \ldots, \mathbf{u}_{t+N_h-1}$
3 apply only $\mathbf{u}_t$ to the building

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## 4. NUMERICAL RESULTS AND DISCUSSION

In this section, we illustrate the potentials and advantages of the proposed method on a simulation example using a simplified version of the building given in [31]. We consider the following discrete nonlinear system:

$$\begin{cases} \mathbf{x}_{t+1} = A\mathbf{x}_t + B\mathbf{u}_t + E\mathbf{d}_t \\ \mathbf{y}_t = C\mathbf{x}_t + \mathbf{v}_t \end{cases} \tag{13}$$

with $A = \begin{bmatrix} 0.8511 & 0.0541 & 0.0707 \\ 0.1293 & 0.8635 & 0.0055 \\ 0.0989 & 0.0032 & 0.7541 \end{bmatrix}$, $B = \begin{bmatrix} 0.070 \\ 0.006 \\ 0.004 \end{bmatrix}$, $E = \begin{bmatrix} 0.02221 & 0.00018 & 0.0035 \\ 0.00153 & 0.00007 & 0.0003 \\ 0.10318 & 0.00001 & 0.0002 \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T$.

the primary purpose of the control task is to achieve temperature $\mathbf{y}$ comfort while minimizing energy consumption by manipulating the control signal $\mathbf{u}$. In order to solve both classic MPC (1) and GP based MPC (12) problems, we use the values of variables frequently used throughout this paper and summarized in Table 2 for this particular problem. We solve nonlinear optimization problems associated with both MPCs using the IPOPT algorithm in the CasADi framework [32] and execute all simulations in MATLAB 2018b on a machine equipped with an Intel Core i5-5200U (2.7GHz) processor.

Table 1. Meaning and values of the variables used in control optimization problems

| Variable | Units | Description | Control setup |
|---|---|---|---|
| $\mathbf{x}$ | $[^0C]$ | Indoor wall/room/outside wall temperatures | States |
| $\mathbf{u}$ | $[W/m^2]$ | Heating set-point | Control input |
| $\mathbf{d}$ | $[^0C], [W/m^2]$ | Outside temperature, solar radiation, internal heat gain | State disturbances |
| $\mathbf{u}^{min} = 0$ | $[W/m^2]$ | Minimum heating capacity | Input constraint |
| $\mathbf{u}^{min} = 30$ | $[W/m^2]$ | Maximum heating capacity | Input constraint |
| $\mathbf{y}$ | $[^0C]$ | Room temperature | Output |
| $\mathbf{y}^{min} = 21$ | $[^0C]$ | Lower comfort boundary | Output constraint |
| $\mathbf{y}^{max} = 23.5$ | $[^0C]$ | Upper comfort boundary | Output constraint |
| $\mathbf{v} \sim \mathcal{N}(0, 0.02)$ | $[^0C]$ | Measurement Gaussian noise | Output disturbance |
| $\epsilon$ | $[^0C]$ | Comfort band violation | Slack |

To learn the GP model in (9), we generate the data of $M = 2000$ samples as follows: $(i)$ the control signal $\mathbf{u}$ is frozen for three consecutive time steps with uniform distribution in the magnitude between $\mathbf{u}^{min}$ and $\mathbf{u}^{max}$ as specified in Table 1, $(ii)$ obtained signals are applied to the building model described by (13), and the corresponding measurements are collected. We use $M_{train} = 0.6M$ samples for learning the hyprerparameters of the GP model, while the rest $M_{test} = 0.4M$ samples are used to measure the accuracy performance of the specified model. We validate the GP model by measuring the prediction accuracy using the widely used normalized root mean square error (NRMSE) and mean standard log loss (MSLL) provided in [28]. The smaller the former metric is, the better the accuracy is, while this holds vice versa for the latter metric. GP models with zero mean are common in practice, so we set $\mu = 0$ and look for a proper covariance function candidate by considering several combinations of corresponding autoregressive lags. We choose the composite covariance function that is the sum of squared exponential and rational quadratic covariance functions with $l_y$=2, $l_u$=2, and $l_d$=0 as it performs better accuracy compared to other candidates, see Table 2.

Table 2. GP modeling accuracy results (NRMSE/MSLL) on the training data for different autoregressive lags and covariance functions (se - squared exponential, rq - rational quadratic)

| Covariance function | Autoregressive lags | | | | |
|---|---|---|---|---|---|
| | $l_y$=3, $l_u$=2, $l_d$=1 | $l_y$=2, $l_u$=2, $l_d$=0 | $l_y$=2, $l_u$=1, $l_d$=1 | $l_y$=1, $l_u$=1, $l_d$=1 | $l_y$=1, $l_u$=1, $l_d$=0 |
| se | 0.061/-1.770 | 0.002/-4.124 | 0.017/-2.946 | 0.045/-1.122 | 0.108/-1.360 |
| se+rq | 0.045/-1.910 | 0.001/-4.829 | 0.024/-3.846 | 0.035/-1.208 | 0.096/-1.642 |
| rq | 0.061/-1.770 | 0.115/-1.284 | 0.097/-2.556 | 0.067/-1.520 | 0.137/-1.595 |

Figure 1(a) illustrates trajectories used during the training and corresponding uncertainty regions predicted by the GP model where the mean values are indistinguishable from the true ones, while Figure 1(b) shows control signals applied to the system. Moreover, the robustness of the chosen GP model is tested with different Gaussian noises $\mathbf{v} =$ and the corresponding trajectory forecasts are demonstrated in Figure 2(a) and Figure 2(b), respectively. One can notice that the uncertainty region enlarges as the noise variance increases. For the sake of better visualization, we cut the first 200 samples off in all Figures. The classic MPC with $N_p = 10$ and the LTC-MPC controllers are tested in simulation by running a temperature from a feasible initial state $\mathbf{y}_0 = 22 \ [^0C]$ and simulation results are obtained for 150 hours. Figure 3(a) shows that the GP-MPC scheme is able to keep the temperature within the thermal comfort margins, whereas a good closed-loop performance is recovered as depicted in Figure 3(b) by using the variance prediction preview information to compute the objective function.



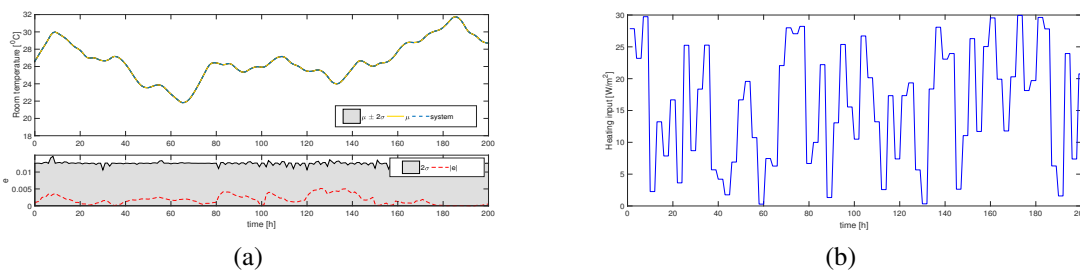(a)                                                        (b)

Figure 1. The prediction accuracy of the GP model for the training data: (a) top plot draws the true (blue), the predicted mean $\mu$ (yellow) and 95 % confidence intervals $\mu + 2\sigma$ (gray) values, while bottom plot shows the absolute error $e$ between true and predicted values and (b) control signal

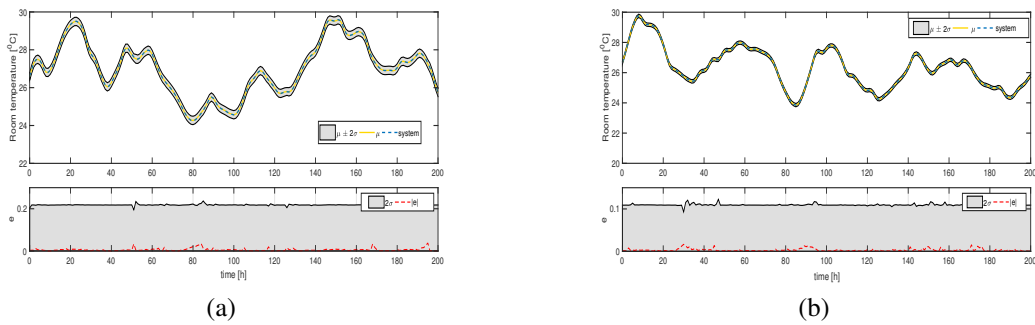(a)                                                    (b)

Figure 2. Effects of introducing different Gaussian noises to the system output: (a) $\mathbf{v} \sim \mathcal{N}(0, 0.02)$ and (b) $\mathbf{v} \sim \mathcal{N}(0, 0.01)$



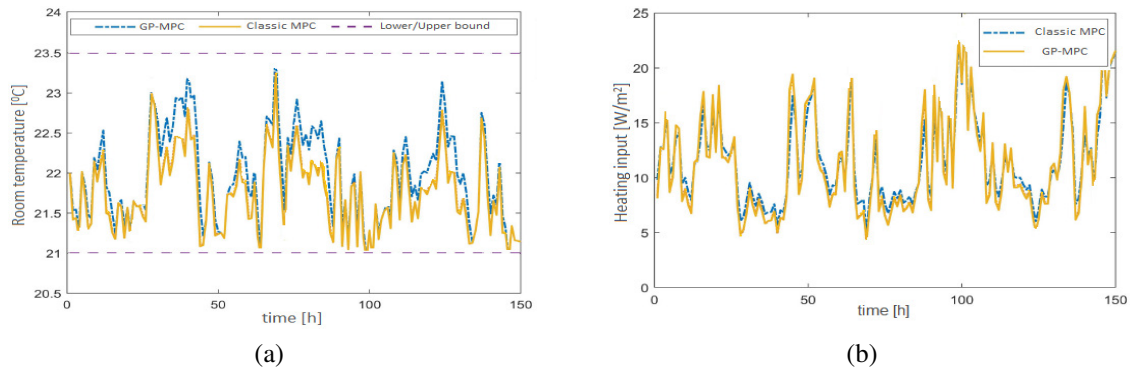(a)                                                    (b)

Figure 3. The closed-loop performances of classic MPC and GP based MPC laws: (a) room temperature and (b) heating input

## 5. CONCLUSION

This paper discussed the use of Gaussian processes for predictive and probabilistic modelling of a building's complex dynamics for thermal comfort. We learned a GP model that predicts a room air temperature as output for a given input vector which is the combination of the previous temperature measurements, forecasted weather disturbances such as solar radiation, outside air temperature and internal heat gains, and manipulated heating set-point. MPC strategy based on GP model was implemented to obtain optimal heating set-points providing user predefined min-max thermal comfort. We exploited the GP model's mean prediction for the room temperature and used the corresponding provided uncertainty bounds in the MPC objective function not to lose the desired performance as compared with classic MPC law in simulation results. Our future work will be devoted to studying robustness analysis of GP based MPC scheme if an uncertain weather forecast is provided and one of the measuring sensors is broken.

## ACKNOWLEDGEMENT

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