

Robustness of Improving Active Maglev Motorized Spindle Equilibrium Position

Zhang Yanhong^{*1,2}, Zhao Dean¹, Zhang Jiansheng², Zheng Zhongqiao^{2,3},
Zou Yiqin^{1,2}, Zhang Yongchun^{1,2}

¹School of Electrical and Information Engineering, Jiangsu University, 301 Xuefu Road, Zhenjiang, Jiangsu Province, China, 212013

²School of Electronic Information & Electric Engineering, Changzhou Institute of Technology, 299 South Tongjiang Road, Changzhou, Jiangsu Province, China, 213002

³School of Mechatronic Engineering and Automation, Shanghai University, 99 Shangda Road, BaoShan District, Shanghai, China, 200444

*Corresponding author, e-mail: zhangyh@czu.cn^{1,2}, dazhao@ujs.edu.cn¹, zhangjs@czu.cn², zhengzq@czu.cn²

Abstract

The equilibrium position of active maglev motorized spindle had great influence on the industrial processing accuracy, in order to improve the performance of active maglev motorized spindle and processing accuracy, a new control method and control device of active maglev was put forward based on H_∞ mixed sensitivity. Firstly, a mathematical model was established among the electromagnetic force on the rotor, control current and position displacement of the rotor center. Secondly, the H_∞ mixed sensitivity control method was used, the selection method of weighting function was discussed and H_∞ robust controller was designed, the experimental results showed that for active maglev motorized spindle, the designed controller had better static and dynamic performance, position precision, so that the robustness of the motorized spindle equilibrium position was further improved, which met the requirement of high precision industrial process.

Keywords: active maglev, motorized spindle, equilibrium position, H_∞ mixed sensitivity, robustness

Copyright © 2013 Universitas Ahmad Dahlan. All rights reserved.

1. Introduction

Active magnetic levitation is a new supporting technology without contact between rotor and stator, it maintains the stable levitation of rotor by controllable attraction or pushing force of electromagnet, and it is characterized by no wear, no lubrication, no oil pollution, no mechanical noise and adjustable stiffness and damping and applied in many fields such as maglev train, magnetic bearing, magnetic guide rail, micro-electromechanical control, lithography and so on as well [1-3]. It is generally known that controller is the critical component of magnetic levitation system, which influences its dynamic characteristics, and it does not only concern its robustness, but also decides its bearing characteristic, stiffness and damping characteristics. The PID controller is generally used as controller of active magnetic levitation which is easily realized, but it can hardly meet the demands of good robustness because it is difficult to adjust three parameters of K_p , K_i and K_d , so there has been considerable interest in the research of active magnetic levitation controller. With development and application of robust control theory and intelligent control theory, the robust controller becomes one of research directions in the fields of active magnetic levitation controller.

H_∞ robust control method is widely applied in various fields because of its good antijamming capability, but there were few reports about its applications in active magnetic levitation control system. Firstly, the structure of active magnetic levitation control system is analyzed and the mathematical model is built. Secondly, the H_∞ mixed sensitivity control method is introduced and the corresponding robust controller is designed based on H_∞ mixed sensitivity. Lastly, the experiment is done and experimental results show that the designed controller had better static and dynamic performance, position precision and robustness.

2. Structure of Active Magnetic Levitation System

A typical structure of active magnetic levitation system is mainly composed of electromagnet, displacement sensor, controller, power amplifier and rotors, which is shown in Figure 1.

The displacement sensor can measure the change x of the rotor position, and transform x into electrical signal U_x , U_x is compared with the expected voltage signal U_r , which is correspond to the voltage signal of stable levitation position of the rotor, the deviation signal U_e is the difference between U_r and U_x , the controller derives the control signal U_c from U_e , the amplifier transforms the control signal into control current. In such a way, the current of the electromagnet is adjusted, the electromagnetic force is changed, and the stable levitation of the rotor is maintained. So active magnetic bearing modulates the equilibrium position of the rotor by controlling the coil current of differential electromagnet.

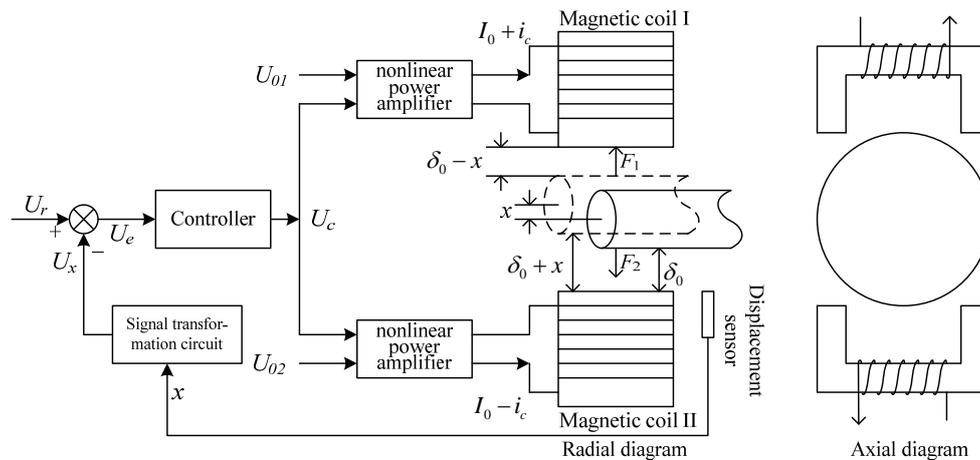


Figure 1. Structure of Active Magnetic Levitation System

3. Model of Active Magnetic Levitation System

The following assumptions as well [4] are made in the active magnetic levitation system shown as Figure 1.

- The leakage flux of the winding is neglected.
- The reluctance of core and rotor is neglected.
- The hysteresis loss and eddy-current loss of magnetic material are neglected.
- The rotor is considered as a single mass.
- The control system adopts differential structure.

Then the electromagnetic force on the rotor is:

$$F_1 = \frac{\mu s_0 N^2}{4} \left(\frac{I_0 + i_c}{\delta_0 + x} \right)^2 \quad (1)$$

$$F_2 = \frac{\mu s_0 N^2}{4} \left(\frac{I_0 - i_c}{\delta_0 - x} \right)^2 \quad (2)$$

Where the meanings of the symbols are as follows:

μ - air permeability of ferromagnetic materials

s_0 - total pole face area

N - number of turns per coil

I_0 - offset component of the current of the magnetic coil

i_c - control current

δ_0 - nominal air gap

x - position displacement of the rotor center in the degree of freedom

The motion equation of the rotor center of the radial bearing by electromagnetic force and else external force is:

$$m\ddot{x} = \Delta F = F_1 - F_2 = \frac{\mu S_0 N^2}{4} \left[\left(\frac{I_0 + i_c}{\delta_0 + x} \right)^2 - \left(\frac{I_0 - i_c}{\delta_0 - x} \right)^2 \right] \quad (3)$$

As $i_c \ll I_0$, $x \ll \delta_0$, neglecting quadratic term and higher order term of i_c and x , the Taylor series expansion of the equation (3) can be obtained at the equilibrium point $x = 0$, $i_c = 0$ as follows:

$$m\ddot{x} = C_1 x - C_2 i_c \quad (4)$$

Where:

$$C_1 = \frac{\mu S_0 N^2 I_0^2}{\delta_0^3} \text{ - displacement stiff coefficient of active magnetic bearing system}$$

$$C_2 = \frac{\mu S_0 N^2 I_0}{\delta_0^2} \text{ - current stiff coefficient of active magnetic bearing system}$$

By Laplace transformation, the following equation can be obtained:

$$ms^2 X(s) = C_1 X(s) - C_2 I(s) \quad (5)$$

The open-loop transfer function of the system can be obtained after rearranging.

$$G(s) = \frac{X(s)}{I(s)} = \frac{-C_2}{ms^2 - C_1} \quad (6)$$

There is a pole $s = \sqrt{\frac{C_1}{m}}$ in transfer function of the equation (6), which is in the right-half plane. So the feedback control must be introduced to stabilize the system.

The mathematical model of five DOF (degrees of freedom) magnetic levitation systems can be obtained from the Equation (3) as follows:

$$\begin{cases} \Delta F_{xa} = \frac{\mu S_0 N^2}{4} \left[\left(\frac{I_{a0} + i_{ac}}{\delta_0 + x} \right)^2 - \left(\frac{I_{a0} - i_{ac}}{\delta_0 - x} \right)^2 \right] \\ \Delta F_{xb} = \frac{\mu S_0 N^2}{4} \left[\left(\frac{I_{b0} + i_{bc}}{\delta_0 + x} \right)^2 - \left(\frac{I_{b0} - i_{bc}}{\delta_0 - x} \right)^2 \right] \\ \Delta F_{xc} = \frac{\mu S_0 N^2}{4} \left[\left(\frac{I_{c0} + i_{cc}}{\delta_0 + x} \right)^2 - \left(\frac{I_{c0} - i_{cc}}{\delta_0 - x} \right)^2 \right] \\ \Delta F_{xd} = \frac{\mu S_0 N^2}{4} \left[\left(\frac{I_{d0} + i_{dc}}{\delta_0 + x} \right)^2 - \left(\frac{I_{d0} - i_{dc}}{\delta_0 - x} \right)^2 \right] \\ \Delta F_{xe} = \frac{\mu S_0 N^2}{4} \left[\left(\frac{I_{e0} + i_{ec}}{\delta_0 + x} \right)^2 - \left(\frac{I_{e0} - i_{ec}}{\delta_0 - x} \right)^2 \right] \end{cases} \quad (7)$$

From above equations, it can be known that the attraction force of the electromagnet is changeable with the control current i_{nc} ($n = a, b, c, d, e$, five dof) and the displacement x , in addition, the change of work point also can influence the change of the attraction force. The robustness of the system is improved by adjusting the coil current based on the change of these parameters.

4. Robust Controller Design of Active Magnetic Levitation System

4.1. H_∞ Mixed Sensitivity Robust Control

The robustness of control system refers to the ability to keep stability of controlled object under parameter or structure perturbations[5-6]. The control objective of active magnetic levitation system is to maintain the stable levitation of rotor without contact in equilibrium position by adjusting the coil current of electromagnet. Based on this objective, applying robust control theory, the control of active magnetic levitation can be set as mixed sensitivity control, whose structure weighting as well [7-8] is shown as Figure 2, where $P(s)$ is augmented object model, $K(s)$ is controller model, $W_1(s)$ is weighting performance function introduced to restrain the influence to control error by noise and interference, $W_2(s)$ is weighting output function of controller introduced to restrain input extremity, $W_3(s)$ is weighting model perturbations function introduced to meet robust stability.

Sensitivity function is $S(s)$, it is an important indicator of measuring tracking error, the formula is as follows.

$$S(s) = \frac{1}{1 + G(s)K(s)} \tag{8}$$

Where,

$G(s)$ is augmented controlled object. The lower the sensitivity is, the smaller the tracking error of the system is, then the better the quality index of system response is.

Complementary sensitivity function is $T(s)$.

$$T(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} = I - S(s) \tag{9}$$

It is the important indicator of measuring the system robustness, where

$$S(s) + T(s) = I$$

$W_1(s)S(s)$ means performance requirement, $W_2(s)R(s)$ and $W_3(s)T(s)$ means robust stability requirement of the system as well [9-10]. If input signal is given equilibrium position of rotor and deviation position of rotor is the output of generalized controlled object, the H_∞ robust control design problem of active magnetic levitation system exactly is finding a regular rational controller $K(s)$, which makes closed-loop control system shown as Figure 2 stable and the norm of closed-loop control system to be less than a given upper bound γ (positive number), that transfer function $Ty_{1u_1}(t)$ from input signal $u_1(t)$ to output signal $y_1(t)$ meets

$$\|T_{y_{1u_1}}(s)\|_\infty \triangleq \left\| \begin{matrix} W_1(s)S(s) \\ W_2(s)R(s) \\ W_3(s)T(s) \end{matrix} \right\|_\infty < \gamma \tag{10}$$

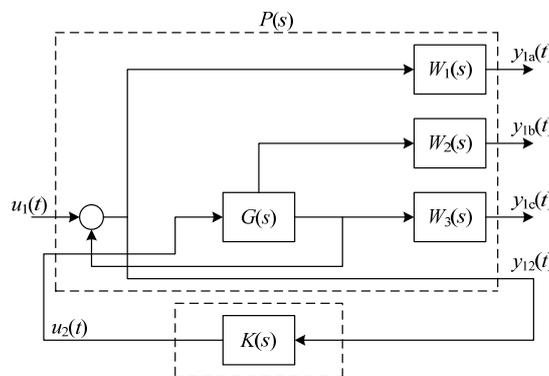


Figure 2. The Closed-Loop Structure with Mixed Sensitivity

4.2. Selection of the Weighting Functions

The robust controller design of active magnetic levitation system mainly is selecting weighting functions $W_1(s)$, $W_2(s)$ and $W_3(s)$ of $S(s)$, $T(s)$ and $R(s)$, which directly decide the performance of the system. The general requirement of selecting weighting functions is that it ensure that introduce of weighting function does not influence the stability of the system, it is required that the weighting functions are stable and the non minimum phase system, and their order can not be too high, otherwise the order of the controller will be increased because the order of H^∞ controller is the sum of controlled object and of weighting function. So we should select the lowest order weighting function whenever possible on the premise of ensuring design requirements as well [11-15].

$W_1(s)$ selected should make the amplitude of active magnetic levitation system as high as possible at low frequency, which brings the system good tracking performance and antijamming capability. At high frequency, the overshoot of $W_1(s)$ is adjusted by selecting appropriate gain k_1 . $W_1(s)$ whose sheared frequency should be less than the system is required with low-pass property. Based on above principles, $W_1(s)$ selected is shown as follows.

$$W_1(s) = k_1 \frac{\alpha_1 s + 1}{\alpha_2 s + 1} \quad (11)$$

$W_3(s)$ selected can reduce high frequency noise with larger amplitude at high frequency and has high-pass property. $W_3(s)$ with added gain k_3 is shown as follows.

$$W_3(s) = k_3 \frac{s}{\beta s + 1} \quad (12)$$

When solving the problem of mixed sensitivity design, DGKF solution based on Riccati equation is used in Matlab toolbox, but augmented object model $P(s)$ of this paper does not meet one of necessary conditions of the solution that the matrix D_{12} is full column rank, so weighting function $W_2(s)$ is introduced. In this paper, weighting function $W_2(s)$ is a constant gain k_2 , that is $W_2(s) = k_2$.

The weighting functions are obtained by trial and error as follows.

$$\begin{cases} W_1(s) = 400 \cdot \frac{0.05s + 1}{100s + 1} \\ W_2(s) = 0.02 \\ W_3(s) = 100 \cdot \frac{s}{0.001s + 1} \end{cases} \quad (13)$$

5. Experiment and Analysis of Active Maglev Electric Spindle

The parameters involved in active magnetic levitation of this paper are shown as follows: $m = 12$ kg, $\delta_0 = 0.5 \times 10^{-3}$ m, $I_0 = 3.0$ A, $\mu_0 = 4\pi \times 10^{-7}$ Vs/Am, $s_0 = 340$ mm², $N = 190$. Substituting these parameters to equation (6), the open-loop transfer function of active magnetic levitation system can be obtained.

$$G(s) = -\frac{1.542 \times 10^{-3}}{s^2 - 9.254}$$

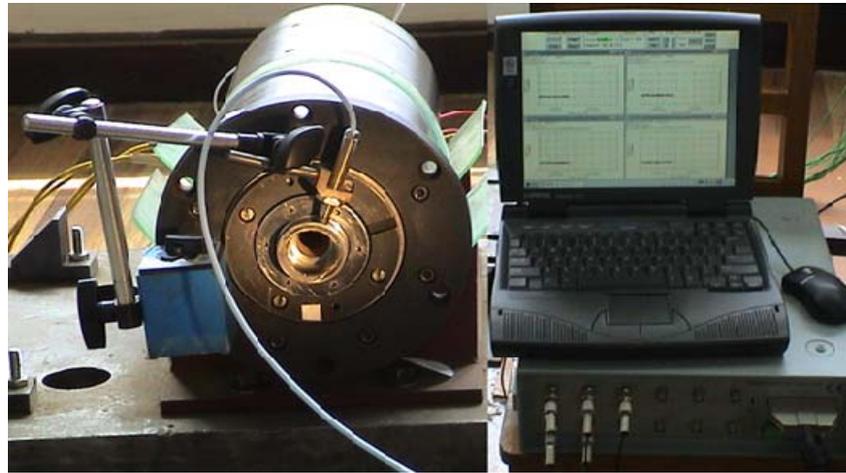
By program and calculation using MATLAB Robust Control Toolbox as well [16], the optimized parameter γ can be obtained.

$$\gamma = 1.3321$$

So the controller is:

$$K(s) = \frac{3.228s^2 + 25.487s + 20.535}{0.021s^2 + 5.096s + 0.002} \quad (10)$$

Experiments are carried on using active maglev electric spindle in laboratory whose experimental test equipments and experimental subject are shown as Figure 3, where (a) is active maglev electric spindle, which samples position signal by non-contacting eddy current sensors and (b) is signal regulator and dedicated testing environment.



(a) Experimental subject

(b) Signal regulator and dedicated testing environment

Figure 3. The Figure of Testing Equilibrium Position of Active Maglev Electric Spindle

We only do comparison test in four radial degrees of freedom because of near-zero axias load. Figure 4 is steady state output waveforms of rotor displacement sensors with H_∞ controller, The sensor output value and corresponding displacement output value of roto are shown in Table 1, Table 2, Table 3 and Table 4, where the sensitivity of the sensor is $20\text{mV}/\mu\text{m}$.

Figure 4 shows that the displacement sensor outputs are relatively stable as rotor reaches the equilibrium position using H_∞ controller. Table1 shows that the displacement output signals are less than $1\mu\text{m}$, and the biggest steady state position error is $0.763\mu\text{m}$. Obviously, the robustness of the equilibrium position is improved evidently after H_∞ controller.

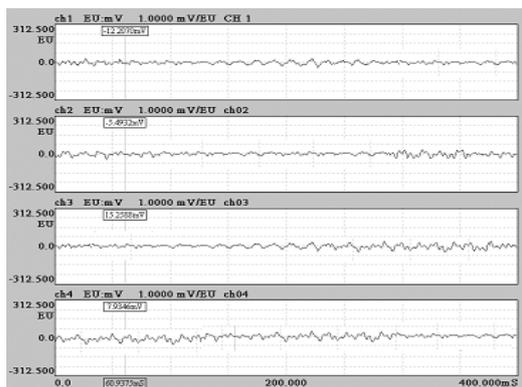


Figure 4. The output waveforms of rotor displacement sensors with H_∞ controller

Table 1. First Group Results

Channel number	Sensor output value (mV)	displacement(μm)
1# channel (front horizontal sensor)	2.44	0.122
2# channel (front vertical sensor)	11.60	0.580
3# channel (back vertical sensor)	10.91	0.518
4# channel (back vertical sensor)	7.80	0.354

Table 2. Second Group Results

Channel number	Sensor output value (mV)	displacement(μm)
1# channel (front horizontal sensor)	9.16	0.458
2# channel (front vertical sensor)	1.83	0.092
3# channel (back vertical sensor)	6.10	0.305
4# channel (back vertical sensor)	10.28	0.506

Table 3. Third Group Results

Channel number	Sensor output value (mV)	displacement(μm)
1# channel (front horizontal sensor)	12.20	0.610
2# channel (front vertical sensor)	5.49	0.275
3# channel (back vertical sensor)	15.26	0.763
4# channel (back vertical sensor)	7.93	0.397

6. Conclusion

According to the needs of industrial process, the new control method and control device of active maglev has been put forward on the basis of H^∞ mixed sensitivity. The work is done as follows.

A kind of active magnetic levitation system is designed in paper, firstly, the mathematical model of active magnetic levitation system is built, then the control method based on H^∞ mixed sensitivity robust is analyzed and the robust controller is designed, lastly, the simulation and experiment based on four dof active maglev system is done. From the output value of sensor and corresponding displacement, on the condition that there is the existence of interference, it is quick to reach steady state. So the result of experiment shows that the controller designed basen on robuste control theory has higher control precision and stronger robust stability than original PID controller.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant No. 51175052), the Natural Science Foundation project for graduate of Changzhou Institute of Technology (Grant No. YN1216).

References

- [1] G Schweitzer, H Bleuler, A Traxler. Basics, Properties and Applications of Active Magnetic Bearings. Beijing: New Times Press.1994: 20-23.
- [2] Wei Jiang, Zhao H B. *Variable Structure Control for Active Magnetic Bearings*. Proceeding of the 5th International Symposium on Magnetic Bearings. Kanazawa. 1996; 39: 215-219.
- [3] J.C. Doyle. Feedback Control Theory. Beijing: TsingHua University Press. 1993: 23-25.
- [4] Namerikawa T, Fujita M, Matsumura F. *Wide Area Stabilization of a Magnetic Bearing Using Exact Linearization*. Proceeding of the 6th International Symposium on Magnetic Bearings. Virginia. 1998; 23:733-742.
- [5] Rong Mei, Mou Chen. Robust Position Control of Electro-mechanical Systems. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2013; 11(3): 19-26.
- [6] LI Kebai, ZHAO Yuhua. Robust Control of Urban Industrial Water Mismatching Uncertain System. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2013; 11(2): 13-19.
- [7] Shen Tielong. H_∞ control theory and application. Beijing: TsingHua University Press. 1996:50-55.
- [8] Wu Min, Gui Weihua. Modern Robust Control. Changsha: Central South University of Technology Press. 1998: 30-35.
- [9] Trumper DL, Olson SM, Subrahmanyam PK. Linearizing Control of Magnetic Suspension Systems. *IEEE Transactions on Control Systems Technology*. 1997; 5: 427-437.
- [10] Matsumura F, Namerikawa T, Hagiwara K, Fujita M. Application of Gain Scheduled H_∞ Robust Controllers to a Magnetic Bearing. *IEEE Transactions on Control Systems Technology*. 1996; 4: 484-493.
- [11] Wu Xudong, Xie Xueshu. Weighting function matrix selection in H_∞ robust control. *Journal of Tsinghua University (Science and Technology)*. 1997; 37(1): 27-30.

-
- [12] Sun Jianliang, Peng Yan, Liu Hongmin. Dynamic Model of Gauge Control System based on Thickness Gauge and Design of H_∞ Robust Controller. *Journal of Mechanical Engineering*. 2009; 45(6): 160-170.
- [13] Wang Youmin, Si Miaoli. H_∞ Control of Disturbance Attenuation Problem for Electro Hydraulic Position Servo System. *Transactions of the Chinese Society for Agricultural Machinery*. 2004; 35(6): 164-166.
- [14] Li Qunming, Zhu Ling, Xu Zhen. Robust controller design of maglev ball system. *Journal of Central South University (Science and Technology)*. 2007; 38(5): 922-927.
- [15] Hu Chunhua, He Ren, Li Nan. Robust Controller Design for Energy Control System of Hybrid Electric Vehicle. *Transactions of the Chinese Society for Agricultural Machinery*. 2011; 42(12): 62-66.
- [16] Xue Dingyu. Design and Analysis of Feedback Control System–MATLAB Language Application. Beijing: TsingHua University Press. 2000:102-110.