

Research of Signal De-noising Technique Based on Wavelet

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Abstract

During the process of signal testing, it is often exposed to interference and influence of all kinds of noise signal, such as data collection and transmission and so noise may be introduced. So in practical applications, before analysis of the data measured, the need for de-noising processing. The signal de-noising is a method for filtering the high frequency noise of the signal and makes the signal more precise. This paper deals with the general theory of wavelet transform, the application of wavelet transform in signal de-noising as well as the analysis of the characteristics of noise-polluted signal. Matlab is used to be carried out the simulation where the different wavelet and different threshold of the same wavelet for signal de-noising are applied. An indicator of wavelet de-noising is presented, it is the indicator of smoothness. Through analysis of the experiment, considering MSE, SNR and smoothness, it can be a good way to evaluate the equality of wavelet threshold de-noising. The results show that the wavelet transform can achieve excellent results in signal de-noising; denoised signal using soft-threshold method is smoother and soft-threshold method is more suitable for more signal detail component; SNR, MSE and smoothness are all important indexes to evaluate the performance of noise reduction; threshold rule, wavelet decomposition level and wavelet function all impact the de-noising performances.

Keywords: wavelet transform, de-noising, threshold, MATLAB

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1. Introduction

Wavelet analysis, as a kind of novel theory, is an important outcome in the history of mathematics development. From the point of mathematics, Wavelet analysis is a kind of mathematical microscope [1-2]; from the view of application, Wavelet analysis is a tool of time-frequency analysis, overcoming the traditional fourier analysis's shortcomings which is complete localization in frequency domain but nonlocalization in time domain, especially suiting to the analysis of non-steady signal. Wavelet transform is of localization in both time and frequency domains, and the frequency distribution of certain time can be calculated, also the mixed signal which is composed of different frequencies can be decomposed into different frequency bands with different frequency ranges [3]. Wavelet analysis is associated closely with many other subjects [4]. Nowadays mathematical and engineering fields are paying much interest to the development of new theory and method concerning wavelet with its application.

Signal de-noising is one of the important research topics in the field of signal analysis [5-7]. At present, there are two de-noising methods, the traditional filtering method and the wavelet de-noising method, when in the actual test, different noise and signal with the choice of different de-noising methods. Traditional de-noising method is based on fourier analysis, can only be used in the circumstances that signal and noise is very small band overlap or completely separate from, and separated the signal and noise by the method of filtering. However, in practice, the signal spectrum and noise spectrum are overlapped, the traditional filtering method can not achieve an effective removal of noise, and the purpose of extracting useful signal. Wavelet analysis is a new mathematical theories and methods developed in the mid-1980s, and it known as the "microscope" of mathematical analysis. Wavelet analysis is a time-frequency analysis method of signal, with the characteristics of multi-resolution analysis, can be focused on any of the details of signal to multi-resolution time-frequency analysis. And it is superior to Fourier analysis algorithm.

The theory of wavelet and principle of signal de-noising based on wavelet are introduced in this paper. Smoothness is used as an indicator. The quality of wavelet de-noising is evaluated using the indicators RMSE, SNR and smoothness. The MATLAB simulation results confirm that wavelet threshold method is effective and enjoys some advantages in removing noises.

2. Wavelet Theory and the Principle of De-noising

Wavelet transform is an analytical method which unites the time domain and frequency domain [8-10]. It has the capacity of the multi-resolution analysis which is localized in time or space. And it is a local time-frequency analysis method that the window size is fixed but its shape can be changed, the time and frequency window all can be changed. Here, its scaling analyzing function can vary its width depending on the frequency information to be analyzed. The scaling analyzing function has a large width in the temporal domain for low frequency components and a small width for the high frequency components. It is very normal for detecting transient signal entrainment anomaly and demonstrates the signal's components. Therefore, it is called a mathematical microscope for analyzing signals [11-13]. Just because of this characteristic, wavelet transform has the adaptability of the signal processing.

The continual wavelet transformation is defined as:

$$WT_x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \psi\left(\frac{t-b}{a}\right) x(t) dt \quad (1)$$

In practice, discrete wavelet transform is used frequently, and it is also required that the signal can be reconstructed. Make $a=a_0^m$ and $b=na_0^m b_0$, then the discrete wavelet transformation is:

$$\varphi_{m,n} = a_0^{-m/2} \varphi(a_0^{-m} t - nb_0) \quad (2)$$

Take $a_0 > 1$ and $b_0 \neq 0$, Then the discrete wavelet transform is:

$$WT_x(m, n) = \int_{-\infty}^{+\infty} \varphi_{m,n}(t) x(t) dt \quad (3)$$

Take $a_0=2$ and $b_0=1$, Equation (2) can be written as a function system.

$$\varphi_{j,k}(t) = \left\{ 2^{-j/2} \varphi(2^{-j} t - k) \right\}, \quad j, k \in \mathbb{Z} \quad (4)$$

The above equation is standard orthogonal basis according to the binary expansion and translation on $L_2(\mathbb{R})$.

In the actual application, besides having the flexibility to choose wavelet basis function, the wavelet also has the following significant advantages:

Orthogonality: reduce the signal redundancy.

Short support: not only show the locality of time domain, but also reduce the amount of computation.

Approximation and Regularity: analyze the singularity of the function.

Symmetry: make easily to deal with the border in the data compression, avoid border distortion, and reduce quantization error.

The wavelet transform is a signal processing technique that represents a transient or non-stationary signal in terms of time and scale distribution. Due to its light computational complexity, the wavelet transform is an excellent tool for on-line data compression, analysis, and de-noising.

Suppose there is an observed signal.

$$f(t) = s(t) + n(t) \quad (5)$$

Where $s(t)$ is the original signal, $n(t)$ is Gaussian white noise with mean 0 and variance σ^2 .

The flow chart of signal de-noising is as shown in Figure 1.

- 1) Select the appropriate wavelet and wavelet decomposition level, make wavelet decomposition for the noise signal $f(t)$, get corresponding coefficient of wavelet decomposition;
- 2) Deal with the threshold for coefficients $w_{j,k}$ which come from the wavelet decomposition, and obtain the estimation value $\hat{w}_{j,k}$ of the wavelet coefficients of original signal $s(t)$;
- 3) Make wavelet inverse transform for these estimated value $\hat{w}_{j,k}$, and get the signal after de-noising.



Figure 1. Flow Chart of Signal De-noising

There are hard threshold method and soft threshold method for the wavelet coefficients estimation which put forward by Donoho. The basic idea is to remove the small coefficient and shrink or keep the large coefficient. By hard threshold function, we compare wavelet coefficient absolute value with the threshold value, turn the absolute value which is less than or equal to the threshold into zero, and keep the one which is greater than the threshold; by soft threshold function, we turn the absolute value which is less than the threshold into zero, and turn the one which is greater than the threshold into the D-value of coefficient and the threshold. On this basis, there are many improved threshold algorithm, the key of the de-noising is to find a suitable number of λ as a threshold, then if the wavelet coefficients are lower than λ , set the coefficient 0, and if the wavelet coefficients are higher than λ , maintain or contract.

Hard-threshold method is expressed as follows:

$$\hat{w}_{j,k} = \begin{cases} w_{j,k}, & |w_{j,k}| \geq \lambda \\ 0, & |w_{j,k}| \leq \lambda \end{cases} \quad (6)$$

Soft-threshold method is defined as:

$$\hat{w}_{j,k} = \begin{cases} \text{sign}(w_{j,k})(|w_{j,k}| - \lambda), & |w_{j,k}| \geq \lambda \\ 0, & |w_{j,k}| \leq \lambda \end{cases} \quad (7)$$

Where $\lambda = \sigma\sqrt{2\log(N)}$ In the case of white noise, its standard deviation can be estimated from the median of its detail coefficients (d_j), with $j = 1 \cdots L$, and is computed as follows:

$$\sigma = \frac{MAD(|d_j|)}{0.6745} \quad (8)$$

Where MAD is the median absolute deviation of the corresponding sequence.

Commonly used threshold selection rules are: rigrsure, sqtwolog, heursure and minimaxi rules. Stein unbiased risk threshold (rigrsure) is an adaptive threshold selection principle based on non-partial likelihood estimator. For a given threshold, its likelihood estimating value is first computed. Then minimize non-likelihood of λ , the threshold could be

determined. The definition of the universal threshold is shown as $\lambda = \sigma\sqrt{2\log L}$, Where L is the sample number, σ is standard deviation of the noisy signal. Heuristic Threshold combines the two thresholds concerned above. The choice of the variable threshold has the best predictor. When the signal to noise ratio is very small, the fixed threshold is better than *rigrsure* rule. Minimal Great Variance Threshold (Minimaxi Rule) is shown as:

$$\lambda = \begin{cases} \sigma(0.3936 + 0.1829 \log_2 n), n > 32 \\ 0, n < 32 \end{cases}$$

In order to compare the effect of reducing noise in the use of different wavelet bases and different combination of threshold rules, three performance indexes are used to evaluate the noise reduction effect, which are signal to noise ratio (SNR), mean square error (MSE) and smoothness .

Signal to noise ratio refers to the signal power to noise power ratio. It is often used as the de-noising effect evaluation index. SNR is measured in decibels. The larger SNR is, the better the de-noising effect. The definition of SNR is shown in Equation 9.

$$SNR = 10 \lg \left(\frac{\sum_{n=1}^N s^2(n)}{\sum_{n=1}^N [f_d(n) - s(n)]^2} \right) \quad (9)$$

Where, N is the number of sample points, $s(n)$ is original signal without noise, $f_d(n)$ is de-noised signal.

Mean square error measures the degree of similarity of the de-noised signal and original signal without noise. Error is smaller, which illustrates the de-noised signal more faithful to the original signal, which is means, error is better noise reduction effect. The definition of MSE is shown in Equation 10.

$$MSE = \sqrt{\frac{1}{N} \sum_n (f_d(n) - s(n))^2} \quad (10)$$

MSE and SNR do not fully reflect the de-noising effect. Here is another evaluation indicator, smoothness index. The definition is shown in Equation 11.

$$r = \frac{\sum_{n=1}^N [f_d(n+1) - f_d(n)]^2}{\sum_{n=1}^N [f(n+1) - f(n)]^2} \quad (11)$$

Where, $f(n)$ is the original noised signal. The indicator can reflect the degree of smoothing of the de-noised signal,

3. Simulation results and analysis

Take heavy sine signal with Gaussian white noise as an example, where the SNR is 12.5166dB, MSE is 0.9947, the smoothness is 1 (as shown in Figure 2).

Firstly, FFT method was applied on heavy sine signal de-noising and simulated on MATLAB. The basic idea of FFT de-noising method is suppressing the high frequency portion of the signal and retaining the low frequency signal. FFT de-noising process can be divided into the following steps. (1) Make FFT operation of signal; (2) according to the frequency spectrum of signal, noise portion is suppressed; (3) Make inverse fourier transform for the transformed spectrum and obtain the de-noising signal. Figure 3 is the frequency spectrum of heavy sine signal with noise under FFT. It can be seen that the signal energy is concentrated in the low frequency and the signal energy quickly decays to zero after the frequency of 5Hz and there is almost no energy at 20Hz. As shown in Figure 4, the signal is filtered by low-pass filters with different widths respectively. FFT noise reduction does not well preserve the peak and mutation portion of useful signal. It can't distinguish

between the high frequency portion of the signal and high frequency interference induced by noise effectively. If the low-pass filter is too narrow, there are still a lot of noise in the signal after filtering; if the low-pass filter is too wide, a part of the useful signal would be filtered out as the noise.

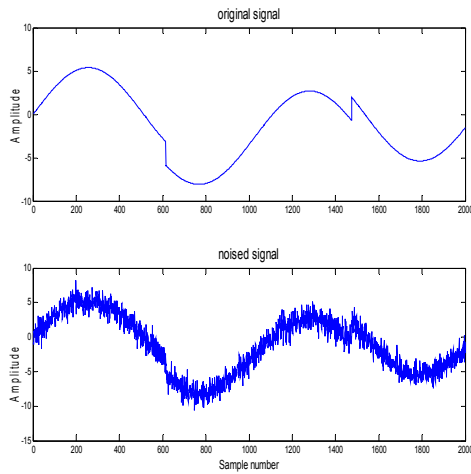


Figure 2. Original and Noisy Heavy Sine Signal

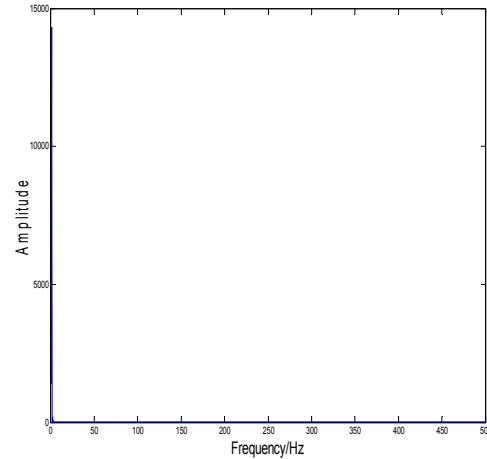


Figure 3. Frequency Spectrum of Heavy Sine Signal with Noise under FFT

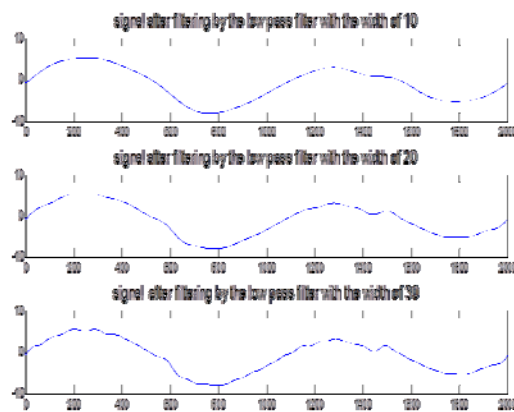


Figure 4. signal de-noising by FFT method

Comparing with FFT fourier transform de-noising, wavelet de-noising is more suitable for non-stationary signal, and it can remove the noise in the signal effectively, while retaining the original signal for more details. The wavelet transform has concentrating ability. Generally, during the wavelet transformation, the wavelet coefficient of useful signals is large, and the energy is concentrated; the wavelet coefficient of decomposed noise is small. According to the signal and noise in wavelet decomposition coefficients, noise can be suppressed through the different decomposition level of wavelet coefficient threshold processing. The coefficient which is larger than the threshold is saved, and the coefficient which is smaller than the threshold is turn to zero.

The noisy heavy sine signal was decomposed with on 5 scales using db4 (daubechies wavelets) wavelet. The daubechies wavelets are compactly supported wavelets with extremal phase and highest number of vanishing moments for a given support width. Wavelet decomposition diagram is shown in Figure 5. The approximate signals are as shown in Figure 6, the detail signals are shown as in Figure 7. It can be seen that the detail signals have more close correlation with the noise.

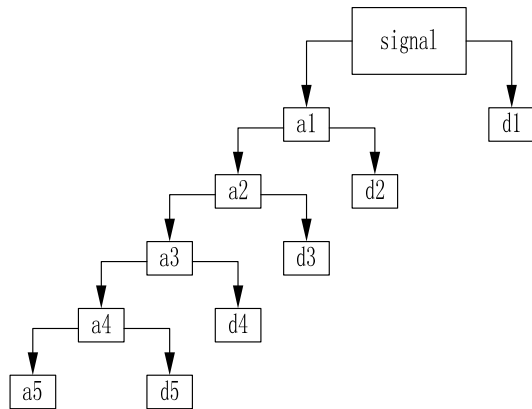


Figure 5. Wavelet Decomposition

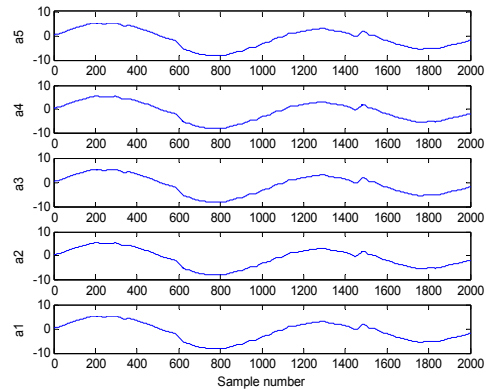


Figure 6. Approximate Signals of each Scale after Wavelet Decomposition

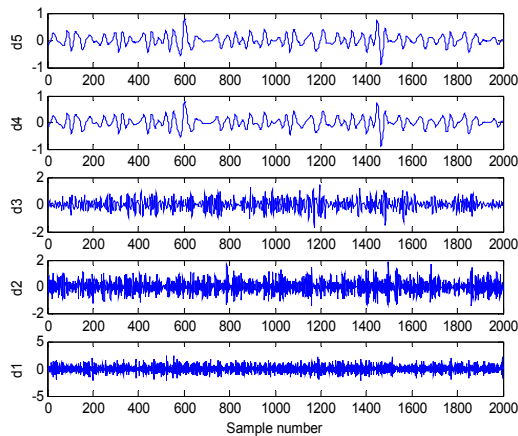


Figure 7. Detail Signals of each Scale after Wavelet Decomposition

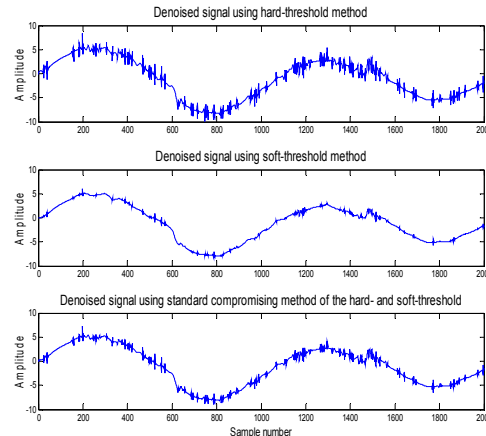


Figure 8. De-noised Signal using Soft-threshold, Soft-threshold and Compromising of the Hard- and Soft- threshold Methods

The hard- and soft-threshold methods are widely used in applications, they have some advantages and disadvantages. The db4 was chosen here in heavy sine signal de-noising. The decomposition scale was five. The hard-, soft-threshold and standard compromising method of the hard- and soft-threshold methods were used respectively. It can be seen from Figure 8 that the de-noised signal using hard-threshold method is not smooth. The de-noised signal using soft-threshold method is smooth, but may lose some signal characteristics. Table 1 is the de-noising performances of the three methods. Table 1 shows that the standard compromising method of the hard- and soft-threshold overcomes the shortcomings of other two kinds of methods to a certain extent.

Table 1. De-noising Performances of the Three Threshold Methods

Methods	SNR	MSE	Smoothness
hard-threshold	17.1509	0.5834	0.3092
soft-threshold	19.7671	0.4317	0.0178
Compromising of Soft and Hard threshold	19.7175	0.4342	0.1125

Commonly, threshold can be determined by rigrsure, heursure, sqtwolog and minimaxi rules respectively. Db4 wavelet was used with soft-threshold method. the noisy signal was

decomposed on 5 scales. The de-noised results using four threshold rules is shown in Figure 9. The summary of the results is shown in Table 2. De-noising performances vary with the threshold rules, there is no significant difference between SNR and MSE, but the difference of smoothness is relatively apparent.

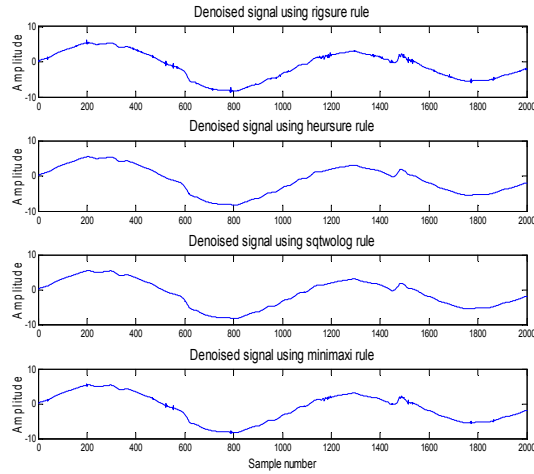


Table 2. De-noising Performances of the Four Threshold Rules

Rules	SNR	MSE	Smoothness
rigrsure	24.3310	0.2553	0.0100
heursure	24.7757	0.2425	4.8270e-04
sqtwolog	24.6553	0.2459	4.7410e-04
minimaxi	24.7530	0.2432	0.0037

Figure 9. De-noised signal using soft-threshold, soft-threshold and compromising of the hard- and soft- threshold methods

Wavelet decomposition level is one of the important factors which effect the de-noising performances. Here wavelet bases functions of db4 were used with soft-threshold method and rigrsure rule. The influence of wavelet decomposition level on noise reduction performance was studied. It can be seen from the Figure 10, with the increasing of wavelet decomposition level, signal de-noising performance is improved obviously.

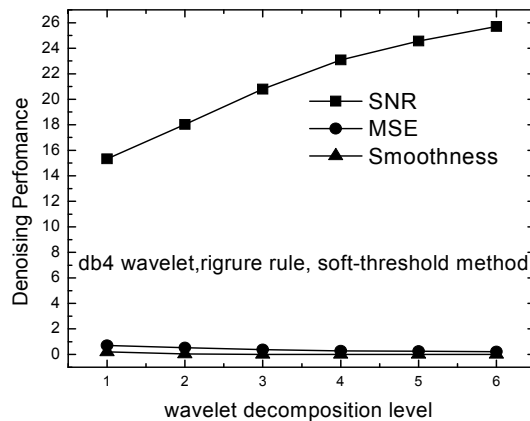


Figure 10. Influence of Wavelet Decomposition Level on De-noising Performance

The selection of wavelet is also very important for signal de-noising. Wavelets of dbH were used with classic hard-threshold and soft-threshold methods and rigrsure rule. The decomposition level was 6. Here, the de-noised result using different daubechies wavelets is shown in Table 3. The de-noising effect is evaluated by three evaluation functions. The de-noising effect using soft-threshold is better than that using hard-threshold. Different wavelets can generate different de-noising effect. In the wavelets of dbH, db4, db5, and db8 are superior to the other wavelets for the de-noising of heavy sine signal with Gaussian white noise.

Table 3. Evaluation Results by using Different Daubechies Wavelets

Wavelet	Method	SNR	MSE	Smoothness
db1	hard-threshold	20.5932	0.3881	0.0418
db2	hard-threshold	17.8403	0.5329	0.1626
db3	hard-threshold	20.5096	0.3919	0.1050
db4	hard-threshold	20.2284	0.4048	0.0936
db5	hard-threshold	20.3036	0.4013	0.0232
db6	hard-threshold	20.2395	0.4043	0.0220
db7	hard-threshold	20.1787	0.4071	0.0456
db8	hard-threshold	21.6280	0.3445	0.0311
db9	hard-threshold	18.9472	0.4691	0.0565
db10	hard-threshold	19.4107	0.4447	0.0916
db1	soft-threshold	23.0930	0.2911	0.0107
db2	soft-threshold	24.2863	0.2537	0.0096
db3	soft-threshold	25.0027	0.2336	0.0051
db4	soft-threshold	25.6989	0.2156	0.0038
db5	soft-threshold	25.1663	0.2293	0.0021
db6	soft-threshold	24.5422	0.2463	0.0030
db7	soft-threshold	25.0242	0.2330	0.0027
db8	soft-threshold	25.5575	0.2192	0.0017
db9	soft-threshold	23.7345	0.2703	0.0045
db10	soft-threshold	24.6691	0.2428	0.0054

4. Conclusion

Wavelet method using for de-noising is an important aspect of wavelet analysis applied to the actual. This article described several commonly used principles of wavelet de-noising method, and achieved wavelet de-noising method based on threshold in the matlab. The results are as follows:

1) Comparing with FFT fourier transform de-noising, wavelet de-noising can retain more details of the original signal and is more effective.

2) Hard-threshold method is more rough than other ways, but smoother signal can be obtained from the soft-threshold method, and many more details of the signal components can be reserved, so soft-threshold method which is more suitable for more signal detail component.

3) In addition to SNR and MSE, smoothness is an important index to evaluate the performance of noise reduction. In some cases, it is very difficult to evaluate the effect of de-noising correctly using only SNR and MSE. The adopting of smoothness will make the evaluation more comprehensive.

4) Threshold rule, wavelet decomposition level and wavelet function are all important factors which impact the de-noising performances.

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