

# The Blanket Fractal Dimension based on the Directed Pattern Plate

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## Abstract

Blanket fractal dimension is a kind of fractal dimension, which is usually used in image processing, and doesn't have a direction. This paper will propose to introduce the direction of pixel into the calculation of the Blanket fractal dimension, and use it to detect the edge of image. This algorithm will calculate the new blanket fractal dimension of the image at first, and then detect the edge with the method of the edge segmentation algorithm based on the traditional Blanket fractal dimension. After this, it will eliminate a part of pixels based on the IFS and Collage Theorem. Experiments will show that this algorithm is able to overcome the issue of double-border in the edge segmentation algorithm base on traditional Blanket fractal dimension, and extract the precision edges with the different directions.

**Keywords:** templet with directions, precision edge segmentation, blanket fractal dimension, fractal

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## 1. Introduction

Mandelbort used self-similarity to describe complicated and inexact figures, and pointed out the fractal geometry. To contrast with the Euclidean geometry, fractal geometry was able to describe the natural object with complicated changes in a better way. The fractal dimension was able to use to measure and analyze the texture and the outline with self-similarity [1]. Pentland figured out the blanket fractal dimension based on Mandelbort's idea. He got the information from the surface of object, and then determined the texture as smooth or harsh [2].

In these years, as the speed of computing improved, the fractal theory developed widely. Many edge extraction algorithms with the fractal theory came out. Lili Jiang used the differences between man-made object and natural background on fractal dimension to detect the edge of man-made object [3]. Mingqin Liu used blanket fractal dimension to calculate out the dimension of pixel, and then detected the edge of irregular object under the background of man-made object.S.S [4, 5]. Chen and B.B Chaudhuri pointed out the edge extraction algorithms with the fractal theory [6, 7]. Otherwise, there were edge extraction algorithms based on the Brown Random Field Model, the scale fractal, and multi-fractal [8-12].

## 2. The Classical Blanket Fractal dimension

### 2.1. The Formulation of the Blanket Fractal Dimension

The blanket fractal dimension assumes the image as its height has the direct proportion with the hill of grey level. The grey level is covered by the surface with the distance of  $\varepsilon$ . The distance of up and down is the volume of the image. Changing the amount of  $\varepsilon$  is able to calculate out different volume. The difference of the volumes of the adjacent  $\varepsilon$  is marked as  $A(\varepsilon)$ . Associated upper surface is  $u(\varepsilon)$ , the lower surface is  $b(\varepsilon)$ , the formulation is below:

$$u_{\varepsilon}(i, j) = \max\{u_{\varepsilon-1}(i, j) + 1, \max_{|(m,n)-(i,j)| \leq 1} u_{\varepsilon-1}(m, n)\} \quad (1)$$

$$b_{\varepsilon}(i, j) = \min\{b_{\varepsilon-1}(i, j) - 1, \min_{|(m,n)-(i,j)| \leq 1} b_{\varepsilon-1}(m, n)\} \quad (2)$$

In the formulation  $u_0(i, j) = b_0(i, j) = I(i, j)$ ,  $I(i, j)$  is the grey level of pixel  $(i, j)$ .

The formulation of the volume of image is below:

$$V_\varepsilon = \sum_{i,j} [u_\varepsilon(i, j) - b_\varepsilon(i, j)] \tag{3}$$

The formulation of the image's surface area:

$$A(\varepsilon) = \frac{V_\varepsilon - V_{\varepsilon-1}}{2} \tag{4}$$

Peleg used Mandelbort's idea to figure out the image's surface area of the blanket fractal dimension:

$$A(\varepsilon) = F\varepsilon^{2-D} \tag{5}$$

Above F is a constant value which is related to the image, not related to  $\varepsilon$ . D is the image's blanket fractal dimension. Getting the logarithm from (5):

$$\log(A(\varepsilon)) = \log(F) + (2 - D)\log(\varepsilon) \tag{6}$$

Counter point  $(\log(\varepsilon-1), \log A(\varepsilon-1))$ ,  $(\log(\varepsilon), \log A(\varepsilon))$ ,  $(\log(\varepsilon+1), \log A(\varepsilon+1))$  processes the line bridge and get linear slope  $h = 2 - D$ . After that, it is able to get the image's blanket fractal dimension  $D = 2 - h$ .

**2.2. Analyzing the Blanket Fractal Dimension**

The blanket fractal dimension uses the image's grey level to calculate the volume, and finally transfer the image's volume to the image's blanket fractal dimension. Therefore, analyzing the procedure of calculating the image's volume is beneficial to the analyzing the procedure of calculating the directed blanket fractal dimension.

As the  $u_\varepsilon$  above, the upper surface calculates from the iteration  $\varepsilon$ . During this procedure, letting the seeking point  $u(i, j)$  plus 1 and choosing the biggest point among the five points of its 4-connected amount as the amount of the iteration  $(i, j)$ . When  $\varepsilon = 1$ , as the Figure below, the maximum is from the convolution of the pattern plate and the pixel.

	$u(i-1, j) + 0$	
$u(i, j-1) + 0$	$u(i, j) + 1$	$u(i, j+1) + 0$
	$u(i+1, j) + 0$	

					0					
				0	1	0				
			0	1	2	1	0			
		0	1	2	3	2	1	0		
	0	1	2	3	4	3	2	1	0	
0	1	2	3	4	5	4	3	2	1	0
	0	1	2	3	4	3	2	1	0	
		0	1	2	3	2	1	0		
			0	1	0					
				0						

Figure 1. The Pattern Plate for Calculating the Upper Surface when  $\varepsilon = 1$

Figure 2. The Pattern Plate for Calculating the Upper Surface when  $\varepsilon = 1$

When  $\varepsilon = 5$ , as the figure, the maximum is from the convolution of the pattern plate and the pixel.

As the analyzing above, during the procedure of calculating the image's volume by blanket fractal dimension, because of using the pixel of 4-connected region, its volume has the same weight for each direction. This kind of blanket fractal dimension is a fractal dimension without direction.

### 2.3. The Result of Classical Blanket Fractal Edge Segmentation Algorithm and Analyzing

Using typical lena grey level image as sample,  $\varepsilon = 3$ . The Figure is a result of using classical blanket fractal dimension to process the edge segmentation algorithm.



Figure 3. The Result of using Classical Blanket Fractal Dimension to Process the Edge Segmentation Algorithm

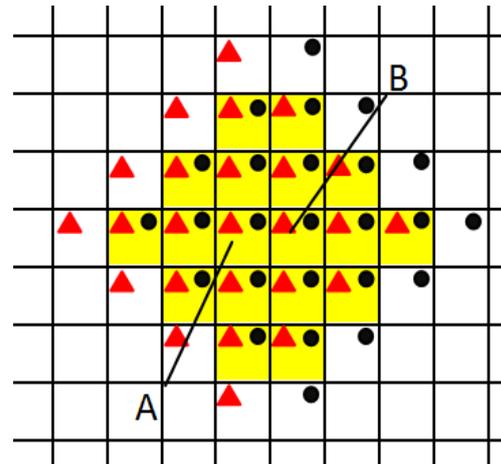


Figure 4. When  $\varepsilon = 3$ , the Regions of Adjacent Pixel under the Effect are Covered

From analyzing the Figure 3, it is clear to see that in the result of using classical blanket fractal dimension to process the edge segmentation algorithm, the phenomenon of double edge is more remarkable. It is because the weights of pixel under the effect from each direction are same (as the Figure 2), and the major regions of adjacent pixel under the effect are same, (as Figure 4). When  $\varepsilon = 3$ , the region of adjacent two pixel A, and B under the effect has 25 points. The region of A under the effect is the red triangle. The region of B under the effect is the black circle. The yellow background is the region for both of A and B under the effect and it has 18 points.

As the analyzing above, during the procedure of classical blanket fractal dimension, the major region of the adjacent pixel under the effect for both two points are covered. If the region of the registration of adjacent pixel includes edge, the results of adjacent pixel A and B are closed. It is easily happened the phenomenon of double edge in such situation.

## 3. Directed Blanket Fractal Dimension

### 3.1. The Formula of Directed Blanket Fractal Dimension

Putting the direction of pixel into the procedure of calculating the upper and lower is able to direct the direction to the blanket fractal dimension. From the directed blanket fractal dimension, examining whether the edge is in the adjacent region of pixel in one direction is able to examine the edge in different directions.

The Figure 5 below shows the 12 situations of expanding the calculation of upper and lower surface.

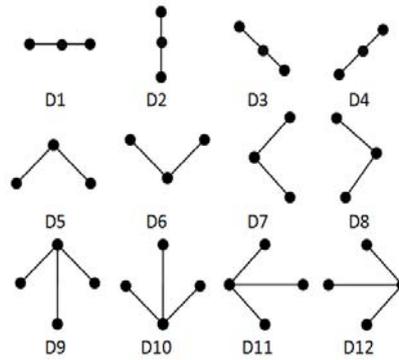


Figure 5. The 12 Situations of Expanding the Calculation of Upper and Lower Surface

Using the D7 as the example, the calculation of upper surface is:

$$u_{\epsilon}(i, j) = \max\{u_{\epsilon-1}(i, j) + 1, u_{\epsilon-1}(i-1, j+1), u_{\epsilon-1}(i+1, j+1)\} \tag{7}$$

Putting the direction into the calculation of upper and lower surface, as the formulation (1), the result will be directed blanket fractal dimension.

### 3.2. Analyzing the Directed Blanket Fractal Dimension

**Definition:** In the procedure of calculating the blanket fractal dimension, the convolution template which uses the iteration or the pre-calculated pixel with different weights is directed pattern plate.

Using the same way in part 1 to analyze the affected region in directed blanket fractal dimension pixel, using the number 7 in Figure 5, when  $\epsilon = 5$ , getting the maximum from the pattern plate and pixel convolution, as Figure 6.

									0
								1	
							2	0	
						3	1		
					4	2	0		
				5	3	1			
				4	2	0			
				3	1				
				2	0				
					1				
									0

Figure 6. When  $\epsilon = 5$ , getting the Maximum from the Pattern Plate and Pixel Convolution

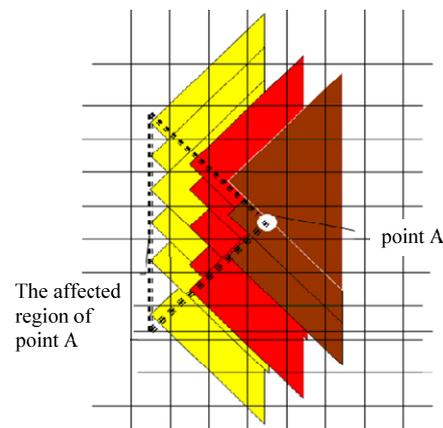


Figure 7. The Affected Region of Blanket Fractal Dimension Pixel

As the analyzing of affected region of pixel in Figure 6, the Figure 7 shows the affected region of blanket fractal dimension pixel A when  $\epsilon = 3$ . In Figure 7, yellow, red, and brown represent the pixel in different x-coordinates under the effect from A. It is clear to see that in the whole figure, the affected region of A is the region inner the dashed triangle.

#### 4. The Classical Blanket Fractal edge Segmentation Algorithm with Directed Pattern Plate

The iteration function family is an important theory of fractal geometry because it explains the generation type and controls the imagine generation. The collage theory explains the level of self-similarity of a focus point in fractal geometry and the corresponding fixed point.

This article uses affine transformation as iteration function family, uses the standard deviation of the pixel and its adjacent fractal dimension after affine transformation as the evaluation for the pixel similarity.

##### 4.1. Based on the Rotation of Pixel

In the digital image, there are only 8 situations based on the rotation of pixel: identity transform, central vertical line double over, central horizontal line double over, main diagonal double over, back-diagonal double over, clockwise rotation  $90^\circ$ , clockwise rotation  $180^\circ$ , and clockwise rotation  $270^\circ$ . The Figure 8  $L_1 - L_8$  represents the 8 situations. Among then, the  $L_0$  is the original image.

1	2	3	1	2	3	3	2	1
4	5	6	4	5	6	6	5	4
7	8	9	7	8	9	9	8	7
$L_0$			$L_1$			$L_2$		
7	8	9	1	4	7	9	6	3
4	5	6	2	5	8	8	5	2
1	2	3	3	6	9	7	4	1
$L_3$			$L_4$			$L_5$		
7	4	1	9	8	7	3	6	9
8	5	2	6	5	4	2	5	8
9	6	3	3	2	1	1	4	7
$L_6$			$L_7$			$L_8$		

Figure 8. 8 Situations based on the Rotation of Pixel

##### 4.2. The Classical Blanket Fractal Edge Segmentation Algorithm with Directed Pattern Plate

First of all, using directed blanket fractal dimension and the idea of classical blanket fractal edge segmentation algorithm to calculate out each pixel of blanket fractal dimension for examining the imagine edge, and then bases on the different pixels in the edge, the pixel fractal in adjacent region with same direction expand in different ways. Using iteration function family and collage theory as basic theory, the rotation after the pixel parallel moving in the affected region as specific function, and the divided difference of the fractal pixel in associated 8 connected regions after the rotation parallel moving as the level of similarity, for eliminating a few similar pixel. The pixel which is eliminated is determined as the most similar to the recent pixel, and it the double edge from the same edge. The procedure in detail below:

Step 1: picking up the 12 situation in Figure 3, or making the suitable pattern plate basing demander, calculating each directed pixel blanket fractal dimension in the image.

Step 2: analyzing the affected region of pixel in chosen pattern plate.

Step 3: basing on demander, determining threshold, the pixel which has the fractal higher than this threshold in procedure 1 is remarked as the point of edge.

Step 4: Rotating the pixel which is remarked as the edge point and its 8 connected region as the 8 different ways of rotating in figure 6. After that, parallel moving them into its affected region.

Calculating the nine points, itself and 8 connected points, and finding out the difference between these 9 points and its moving place. Determining the threshold. If the difference is lower than the threshold, the pixel should be redefined as non-edge point.

### 4.3. The Results and Analyzing

Using the classical lena grey level image as the sample,  $\varepsilon = 3$  in this case. The Figure 9 and 10 are the results of the blanket fractal edge segmentation algorithm from D7 and D10 in the Figure 5. Among them, the red one is the eliminated point according to the step 4.



Figure 9. The Results of the Blanket Fractal Edge Segmentation Algorithm from D7



Figure 10. The Results of the Blanket Fractal Edge Segmentation Algorithm from D10

From contrasting the results of Figure 3 and 9, it is clear to see that the phenomenon of double edge in Figure 9 is more remarkable than Figure 7 because Figure 9 has processed the eliminating. The covered part of affected region of the blanket fractal dimension and adjacent pixel is very small, as Figure 11 shows. In the Figure 7, the affected region of A is marked as red triangle, and B's is marked as black circle. The Figure 11 proves that the affected region in the 4 adjacent connected points have no covered part after processing the directed blanket fractal dimension. This is able to make the phenomenon of double edge weaker.

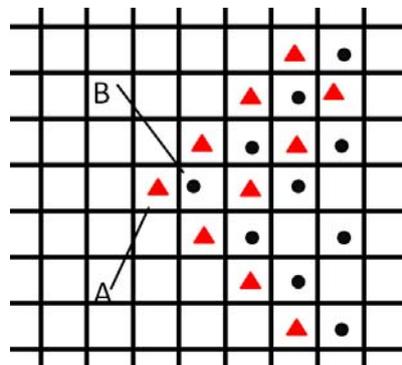


Figure 11. The Affected Region of Pixel in Figure 7 as  $\varepsilon = 3$

From contrasting the Figure 9 and 10, it is clear to see the difference of the result of examining the edge in different directions. For example, in the right side of the image, the Figure 9 eliminates a lot of edges in minus  $45^\circ$  and remains the positive  $45^\circ$ , but in the Figure 10, most positive  $45^\circ$  is eliminated, and the negative parts are remained.

The Figure 5 shows all of the 12 pattern plates are able to examine the edge in different directions. Since this calculation is based on the theory of fractal geometry, different with only contrasting the grey levels in classical edge dimension, it is able to expand the pattern plates in Figure 6 to more shapes, and get more edges with complicated directions.

## 5. Conclusion

This article points out that directing the direction of pixel to the calculation of fractal dimension is able to get the fractal dimension with different adjacent constructions of pixel. Using such directed fractal dimension to process the extraction of edge, using the difference of the fractal dimension of the different edge points in the same adjacent region, and using fractal geometry's the iteration function family and collage theory to eliminate the points in the pixel's associated affected region. From above, this calculation is able to achieve these:

- (1) Reducing the phenomenon of double edge and finding out the edges in different directions.
- (2) The edge which is found out is able to process the exact locating.
- (3) It is able to provide more information of edge to the deeper using during the image processing.

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