Indonesian Journal of Electrical Engineering and Computer Science
Vol. 24, No. 3, December 2021, pp. 1399~1404
ISSN: 2502-4752, DOI: 10.11591/ijeecs.v24.i3.pp1399-1404

Centroidal-polygon: a new modified Euler to improve speed of resistor-inductor circuit equation

Nur Shahirah Zulkifli[1], Nooraida Samsudin [2], Suzanna Ridzuan Aw[3], Wan Farah Hanan Wan Osman[4], Shahreen Kasim[5], Tole Sutikno[6]
[1,2,3,4]Department of Information Technology, TATI University College, Chukai, Malaysia
[5]Faculty of Computing Science and Information Technology, Universiti Tun Hussein Onn, Parit Raja, Malaysia
[6]Department of Electrical Engineering, Faculty of Industrial Technology, Universitas Ahmad Dahlan, Yogyakarta, Indonesia

Article Info

Article history:

Received Jul 17, 2021
Revised Oct 6, 2021
Accepted Oct 23, 2021

Keywords:

Centroidal mean
Euler method
First-order circuit
Harmonic-polygon
Resistor-inductor

ABSTRACT

Two types of first-order circuits are resistor-capacitor (RC) and resistor-inductor (RL). This paper focuses on the RL circuit equation. The centroidal-polygon (CP) scheme will be tested using SCILAB 6.0 software. This new scheme (CP scheme) is addressed to improve the speed. For the first order circuit equation, the complexity is focused on the time complexity, which is speed of the time taken to complete the simulation in the electrical part. The CP scheme is compared with the previous studies, polygon (P) and harmonic-polygon (HP). The result shows that the CP scheme is less computational and an alternative to solve the first order circuit equation, and get the result quickly compared with the previous research.

Corresponding Author:

Nur Shahirah Zulkifli
Department of Information Technology, TATI University College
Kemaman, 24000 Chukai, Terengganu, Malaysia
Email: shahirah.zulkifli.md@gmail.com

1. INTRODUCTION

Euler method produces simple numerical solutions and enforces low computational cost to solve the ordinary differential equation (ODE) for a given initial value problem (IVP) [1]–[9]. Although the Euler method gives a simple solution, this method lacks accuracy [5], [9]. The Euler method will contribute to an error at every step size implied in the solution [8]. This is because the contribution of error is proportional to the phase size implied in the solution. Regarding this problem, it makes researchers enhance the Euler method known as the new modified Euler method [2], [8], [10]–[16].

Usually, the improved Euler approach is explicitly applied to mathematical fields only. This new modified Euler named centroidal polygon (CP) [17] has been tested into the linear and non-linear equation. The results proved that the mathematical problem could be solved using CP. Thus, this approach will be tested in the engineering field [18]. In mechanical engineering, the Euler method has also been known for studying the behavior of flow [19], [6], such as fluid dynamic computations. In this research, the CP concept is applied to electrical engineering. Ordinarily, in the electrical field, the calculation of speed will be conducted manually by reading the output from the oscillator using an analytical solution [20]. Thus, this research aims to examine the speed of the circuit equation using the modified Euler method. The arithmetic mean is one of the means used to improve the Euler. Arithmetic mean is the simplest form of mean, contributing to a more straightforward solution of modified Euler. Although simple, by using this mean, the accuracy of Euler improved compared to the original Euler method [7].

Centroidal-polygon scheme is the new scheme that will be created to solve the resistor-inductor (RL) equation. Centroidal mean is the main means to create a new scheme by combining mean and Euler. Centroidal is one of the simple concepts. There use any two positive actual number. The centroid of an object refers to its geometric center. This is a beneficial concept in engineering to find the center of an object and other applications. Centroidal mean already prove by previous research by using Runge-Kutta fourth-order [4], [17], [21]. To ensure the centroidal scheme is applicable in the Euler method. The scheme will be tested in the linear and non-linear equation before applied in the RL circuit equation. Other than that, the centroidal scheme already proves that more accurate compared to the arithmetic mean, harmonic mean, and contra-harmonic mean [22].

In the electrical field, the first-order circuit equation can be solved using ODE [23], [24]. The electrical region is in the circuit equation. Two conditions of the electrical circuit equation apply to the first-order equation of electrical engineering: resistor-capacitor (RC) and resistor-inductance (RL) [25]. The RC circuit consists of a capacitor and a resistor. The RL circuit, meanwhile, consists of the resistor and inductor. This research focuses on the RL circuit equation. RL circuit consists of one resistor and one inductor. It is composed of a first-order RL circuit and is the simplest type of RL circuit equation [26]. RL circuit equation usually reduces a single equivalent inductance and a single equivalent resistance in one complete circuitz [27]. The Euler method's speed measured for the RL circuit equation is determinable by the time taken for each scheme to complete the simulation of comparing the Euler method solution error to the exact solution.

A simple first-order RL circuit equation with a switch, as shown in Figure 1. The RL series circuit is connected across a constant voltage source and a switch. Assume that the switch is open until it closed at a time, t=0, and then remains permanently closed, producing a "step response" type voltage input [28]. The current, I, begin to flow through the circuit but does not rise rapidly to its maximum value of I_{max} [29]. The modified Euler scheme gives a small error in each calculation step in the electrical circuit equation.

The problem in solving the RL circuit equation is the speed of each scheme takes to complete the simulation. As discussed in the first paragraph, the Euler method can solve the small step size but lacks computational cost [5]. This research will analyze comparing new modified Euler method centroidal polygon (CP) to polygon (P) and harmonic polygon (HP) [22]. The analysis aimed is to study the time taken of the CP scheme's simulation in solving the equation of first order resistor-inductor (RL) circuits. The investigation would evaluate the consistency between the P scheme, HP scheme, and CP scheme. In solving the equation, SCILAB 6.0 software [30]–[34] is used to model the RL circuit equation. Three different RL circuit equations are used for testing to ensure that the CP scheme method can be extended to any value problem. Three differential step sizes h of 0.1, 0.01, and 0.001 [4] are used in the testing. CP shows that it can be used in the electrical application by running the experimenting on the RL circuit equation. The result contributes to the better time taken using CP either in small or higher step size.

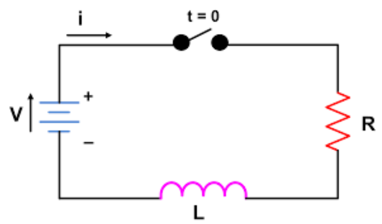


Figure 1. A simple first-order RL circuit

2. METHOD

There are two primary methods to improved Euler, which are the Heun method and the midpoint method. This research used the midpoint method and applied it to the new scheme. This method is called as improved polygon method [35]–[41], whereby the method utilizes a slope estimate at the midpoint of the prediction interval [42]. Thus, this midpoint method is being applied to the new scheme. Figure 2 shows new modified Euler (CP) scheme is developed. This scheme is developed by combining Euler (E) and means (M). The original Euler scheme with the general formula n + 1=y_n + hf (x_n, y_n) is selected as the basis for developing a scheme [43]. Centroidal mean is selected in developing this proposed scheme which is centroidal mean (M). This combination of Euler (E) and mean (M) produces a proposed scheme known as centroidal polygon (CP) (E+M).

Euler (E)
yn+1=yn+hf(xn,yn)
n=1,2,3,….

Mean (M)
$$M=\frac{2((x_n)^2+x_ny_n+(y_n)^2)}{3(x_n+y_n)}$$

Proposed Scheme
E+M = CP

$$\text{CP}=y_n+hf\left(\frac{x_n+(x_n+h)}{2},\frac{y_n+(y_n+f(x_n,y_n))}{2}\right)+\frac{2((x_n)^2+x_ny_n+(y_n)^2)}{3(x_n+y_n)}$$
$$y_{n+1}=y_n+hf\left(\frac{2((x_n)^2+x_n(x_n+h)+(x_n)^2}{3(x_n+x_n+h)},\frac{2((y_n)^2+y_n(y_n+hf(x_n,y_n))+(y_n+hf(x_n,y_n)^2}{3(y_n+y_n+hf(x_n,y_n)}\right)$$

Figure 2. New modified Euler (CP) scheme

In CP scheme, the centroidal mean equation will be applied into the function of $f(x,y)$ which is in the Euler equation. CP scheme implied in (1) and improved the Euler equation based on (2) and (3).

$$y_{n+1}=y_n+\Delta t\, f(x_n,y_n) \quad (1)$$

The improved Euler are using the mean concept. The mean used is centroidal mean where the point that may be considered as a centre of two point as written in (2) and (3).

$$\frac{2x_0^2+x_0x_1+x_1^2}{3x_0+x_1},\frac{2y_0^2+y_0y_1+y_1^2}{3y_0+y_1} \quad (2)$$

$$\frac{2x_0^2+x_0(x_0+h)+(x_0+h)^2}{3x_0+(x_0+h)},\frac{2y_0^2+y_0(y_0+h)+(y_0+h)^2}{3y_0+(y_0+h)} \quad (3)$$

To improve the equation, the CP equation in (3) will be implemented into the (1). The new equation formed are as in (4).

$$\frac{y-y_0}{h}=f\left(\frac{2x_0^2+x_0(x_0+h)+(x_0+h)^2}{3x_0+(x_0+h)},\frac{2y_0^2+y_0(y_0+h)+(y_0+h)^2}{3y_0+(y_0+h)}\right)$$
$$y-y_0=hf\left(\frac{2x_0^2+x_0(x_0+h)+(x_0+h)^2}{3x_0+(x_0+h)},\frac{2y_0^2+y_0(y_0+h)+(y_0+h)^2}{3y_0+(y_0+h)}\right) \quad (4)$$
$$y=y_0+hf\left(\frac{2x_0^2+x_0(x_0+h)+(x_0+h)^2}{3x_0+(x_0+h)},\frac{2y_0^2+y_0(y_0+h)+(y_0+h)^2}{3y_0+(y_0+h)}\right)$$

Euler is more accurate and fast simulate by modify the equation using the midpoints such in (4).

3. RESULTS AND DISCUSSION

The result shows a comparison of the three improved Euler schemes with the time taken to complete the simulation is discussed in this topic. Table 1 shows the RL circuit equation problems used in this experiment. The speed will be tested by simulates the equation in all three circuits with a different step size. There are three sets of RL circuit equations with a different value for voltage (V), resistor (R) and inductor (L). The equation to solve the speed of RL circuit equation [44] come out with (5).

$$\frac{di}{dt}=-\tau i+\left(\frac{V}{L}\right)\text{ where time constant}, \tau=\frac{R}{L} \quad (5)$$

Table 2 shows the results of the time needed for each device to complete the simulation in three schemes (polygon, harmonic-polygon, and centroidal-polygon). All schemes will be evaluated with 0.1, 0.01, and 0.001 as the typical numerical phase scale. Overall, the rapid outcome was given by the centroidal-polygon (CP) method. The other two schemes, the polygon (P) scheme, and the harmonic-polygon (HP) scheme, meanwhile, indicate a lack of speed in the results.

In circuit 1, the CP scheme matched the troubleshooting results because it gave a rapid result compared to the P scheme and HP scheme. As shown in Table 2, for step size 0.1, the CP scheme takes 0.0146682s to complete the simulation compared to 0.4102555s for the P scheme and 0.03222256s for the HP scheme to complete the simulation. It shows that CP could shorten the time to complete the simulation even with a bigger step size. At the step size 0.01, the CP time taken is 0.0169334s to complete the simulation. The other way, for the P scheme and the HP scheme, takes a long time for each scheme which the time taken are 0.5620331s and 0.0464288s. For small step size 0.001, each scheme takes a long time to complete the simulation except CP. As shown in Table 2, to complete the simulation, the CP scheme requires 0.0303234s, P scheme 0.827404s, and HP scheme 0.0514273s to complete the simulation. It slightly different time taken between CP to other two schemes. It shows that CP gives better speed in completing the simulation.

In circuit 2, step size 0.1 shows that CP gets slightly different time taken than P and HP. CP scheme takes 0.014728s to complete the simulation at the small step size compared to P scheme 0.4309688s and HP scheme 0.0360481s to complete the simulation. At step size 0.01, the CP scheme takes 0.0177154s, which is faster than the P scheme at 0.4834862s. In comparison, the HP scheme takes 0.0427233s to complete the simulation. Finally, at a larger step size of 0.1, the simulation time between these three schemes shows a notable difference. As shown in Table 2, the CP scheme takes 0.0396771s. Meanwhile, for HP and P, it takes 0.5092835s and 0.0638495s, respectively, to complete the simulation. It shows that CP gives a better speed for higher or smaller step size cases in completing the simulation.

Lastly, for circuit 3, by comparing the P scheme and HP scheme to complete the simulation, the result shows that time taken for the P scheme is 0.3976792s and HP scheme 0.440234s to complete the simulation. Meanwhile, for the CP scheme, it takes 0.0194034s for a small step size of 0.001. At step size 0.01, the CP method takes 0.0243649s. For both the P scheme and HP scheme, the difference is not too significant in which their time taken is 0.4505656s for the P scheme and 0.0490615s for the HP scheme to complete the simulation. Finally, at a larger step size of 0.1, the CP method takes 0.0362048s. Compare to HP, the time taken is 0.4572861s and 0.380573s for the P scheme to complete the simulation. Even the difference is not slightly different, but again CP still gives better overall speed in solving circuit 3.

Table 1. Set of problem RL circuit equation

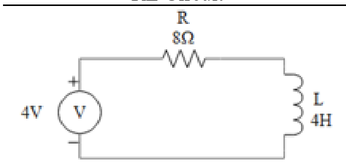
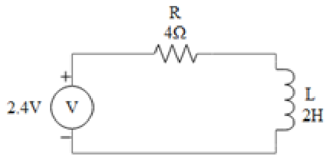
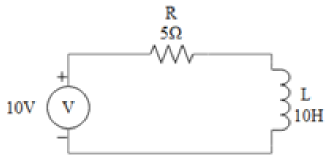
RL Circuit	Value of Voltage, Resistor, Inductor	Equation
	V = 4 V R1= 8 Ω L = 4 H	Equation: $\frac{di}{dt} = -2i + 1$ Exact solution: $0.5\left(1 - e^{-t(2)}\right)$ y(0)=0 ,$0.1 \leq x \leq 0.5$
	V = 2.4 V R1= 4 Ω L = 2 H	Equation: $\frac{di}{dt} = -2i + 1.2$ Exact solution: $0.6\left(1 - e^{-t(0.5)}\right)$ y(0)=0 , $0.1 \leq x \leq 0.5$
	V = 2.4 V R1= 4 Ω L = 2 H	Equation: $\frac{di}{dt} = -0.5i + 1$ Exact solution: $2\left(1 - e^{-t(0.5)}\right)$ y(0)=0 , $0.1 \leq x \leq 0.5$

Table 2. Result for maximum error RL circuit equation problem

Scheme	Centroidal-Polygon			Polygon			Harmonic-Polygon		
Step size	0.1 (s)	0.01 (s)	0.001 (s)	0.1 (s)	0.01 (s)	0.001 (s)	0.1 (s)	0.01 (s)	0.001 (s)
Circuit 1	0.0146682	0.0169334	0.0303234	0.4102555	0.5620331	0.0327404	0.0322256	0.0464288	0.0314273
Circuit 2	0.014728	0.0177154	0.0396771	0.5092835	0.4834862	0.4309688	0.0360481	0.0427233	0.0638495
Circuit 3	0.0194034	0.0243649	0.0362048	0.4505656	0.4572861	0.3976792	0.380573	0.0440234	0.0490615

This research had deduced an outcome to achieve the goal and solve the problem of the first order RL circuit equation. This analysis shows that the result for all phase sizes is directly proportional to the time for circuit 1, circuit 2, and circuit 3. In the first order RL circuit equation, the CP is added to get the result done quickly.

4. CONCLUSION

In this research, the three schemes, namely polygon (P) scheme, harmonic-polygon (HP) scheme, and centroidal-polygon (CP) scheme, are discussed for speed for the time taken in the RL circuit equation. The new scheme results, the CP scheme, are the best scheme to simulate the speed in each step size, h. The result analysis table observed the three different step size h to ensure that the new scheme achieves better speed than the other two schemes. When the scheme is tested into the large step size, the time taken to simulate takes longer than the small step size. Moreover, CP is an alternative scheme in solving the first-order circuit equation and significant in solving the RL circuit equation. To conclude, all the result shows that the speed of time taken to complete the simulation is directly proportional to the step size.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the University College TATI, Universiti Tun Hussein Onn and Universitas Ahmad Dahlan for supporting this collaborative research in the present work, and also would like to thank for the MOHE for FRGS grant (FRGS/1/2019/TK04/TATI/03/1).

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