

A heuristic approach to minimize three criteria using efficient solutions

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ABSTRACT

In optimization, scheduling problem is concerning allocations of some resources which are usually limited. These allocations are done in order to fulfil some criterion by performing some tasks or jobs to optimize one or more objective functions. Simultaneous multi-criteria scheduling problem is known as NP-hard optimization problem. Here, we consider three criteria for scheduling a number of jobs on a single machine. The problem is to minimize the sum of total completion time, maximum earliness and maximum tardiness. Every job is to be processed without interruption and becomes available for processing at time zero. The aim is to find a processing order of the jobs to minimize three-objective functions simultaneously. We present a new heuristic approach to find a best overall solution (accepted) of the problem using efficient solutions of some related criteria. We establish a result to restrict the range of the optimal solution, and the lower bound depends on the decomposition of the problem into three subproblems. The approach is tested on a set of problems of different number of jobs. Computational results demonstrate the efficiency of the proposed approach.

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1. INTRODUCTION

One of the well-known issues in manufacturing systems is scheduling [1], [2]. In optimization, scheduling problem is concerning allocations of some resources which are usually limited. These allocations are done in order to fulfil some criterion by performing some tasks or jobs to optimize one or more objective functions in single or multiple machine environment [3]. The single machine scheduling problem has been investigated intensively by researchers in the past fifty years [4]-[7]. Up until the eighties the main focus was on only one criterion. Since total length of schedule depends on minimizing makespan this criterion is vital in scheduling problems [8].

The bicriteria scheduling problem has received great attention from researchers in recent years [9]. The first paper in multi-criteria scheduling problem was addressed by Smith [10]. Vanwassenhove and Gelders extended Smith's problem to find a Pareto set for the simultaneous case [11]. Also, Nelson *et al.* [12] attempted to address the mean flow time, number of tardy jobs, and maximum tardiness simultaneously by using a branch and bound approach. The same approach was found to be useful in solving simultaneous case of regular cost function [13].

Multi-criteria scheduling problems entail two aspects, hierarchical minimization and simultaneous minimization [14]. In the simultaneous case, all efficient schedules are generated and the best solution is chosen. The most important criteria in scheduling problems are total completion time, maximum tardiness, and maximum earliness. These criteria were considered in different conditions for jobs and machines. Kurz and Canterbury [15] used a genetic algorithm to find the set of efficient point for the problem of total completion time and maximum earliness. Abbas [4] introduced an algorithm for many bicriteria scheduling problems on a single machine with release dates. According to Mahnam *et al.* [16] minimized the sum of maximum earliness and tardiness considering unequal release times and idle insert. For preventive maintenance in a single machine Benmansour *et al.* [17] minimized the weighted sum of maximum earliness and tardiness. Pattnaik *et al.* [18] proposed a fuzzy method to solve multi-criteria decision-making problem when there is imprecise and big data which is also proposed by tavakoli *et al.* [19] to solve multi-criteria single-machine scheduling problem. Arik [20] introduced earliness/tardiness with grey processing times and common due date. Since most scheduling problems are NP-hard [21], it is logical to use heuristic methods to find an approximate solution of the problem. Oyetunji and Oluleye [22] used a heuristic approach to minimize the total completion time and number of tardy jobs simultaneously on a single machine with release date. Kramer and Submarian [23] introduced a unified heuristic for a large class of earliness-tardiness scheduling problems.

Here, we consider the problem of scheduling a set of jobs on a single machine to minimize a function of three criteria stated as follows. Each job is to be processed on a single machine which can handle only one job at a time. Associated with job we have processing time and its due date. All jobs are available for processing at time zero. The main concern here is to schedule jobs on a single machine minimizing the sum of total completion time, maximum earliness and maximum tardiness. We propose a new heuristic approach using efficient solutions. Appropriate results are presented to find a best solution of the problem.

2. NOTATIONS AND DEFINITIONS

Here, we give our main notations and definitions:

- $N = \{1, 2, 3, \dots, n\}$.
- I is the set of all schedules.
- π = a permutation schedule.
- p_j = processing time for job j .
- d_j = due date for job j .
- C_j = completion time for job j .
- $\sum_{j=1}^n C_j$ = total completion time for job j .
- $E_j = \text{Max} \{d_j - C_j, 0\}$; the earliness of job j .
- $E_{\max} = \text{Max} \{E_j\}$; the maximum earliness.
- $T_j = \text{Max} \{C_j - d_j, 0\}$; the tardiness of job j .
- $T_{\max} = \text{Max} \{T_j\}$; the maximum tardiness.
- MST (minimum slack times); jobs are sequenced in non-decreasing order of minimum slack times s_j , where $s_j = d_j - p_j$.
- SPT (shortest processing time); jobs are sequenced in non-decreasing order of p_j .
- EDD (Early due date); jobs are sequenced in non-decreasing order of d_j .
- LB = lower bound.
- UB = upper bound.
- Lex = lexicographical.

Definition 1 [24]: For a problem P a schedule $\pi \in I$ is said to be feasible, if it satisfies the constraints of P.

Definition 2 [11]: A feasible schedule π^* is efficient with respect to the criteria (f and g) if $\nexists \pi \in I$ such that $f(\pi) \leq f(\pi^*)$ and $g(\pi) \leq g(\pi^*)$, and at least one of the inequalities is strict.

Definition 3 [25]: For a problem 1 // Lex (f, g) a schedule is feasible if it satisfies the primary criterion f .

Definition 4 [25]: A feasible schedule that minimizes the secondary criterion g for 1 // Lex (f, g) is optimal schedule.

3. FINDING EFFICIENT SOLUTIONS OF BI-OBJECTIVE PROBLEMS

In this section we present two simultaneous bi-objective scheduling problems which are concerned with three criteria, namely $\sum_{j=1}^n C_j$, T_{\max} , and E_{\max} . Each individual criterion has an optimal solution, the

first one is solved by SPT-rule, the second is solved by EDD-rule, where the third one is solved by MST-rule. Also, it is important to mention that the three rules are not usable for all of them.

3.1. $1 // F (\sum_{j=1}^n C_j, T_{max})$ (Minimum of total completion time and maximum tardiness)

The problem is stated as:

$$\left. \begin{array}{l} \text{Min } \sum_{j=1}^n C_j \\ \text{and} \\ \text{Min } T_{max} \end{array} \right\} (p_1)$$

Van Wassenhove and Gelders [9] found all efficient solutions (Pareto set) for (p_1) . They characterized all the efficient solutions by generalizing the algorithm of Smith [10]. The decision maker will choose the appropriate solution according to his case.

3.2. $1 / / F (\sum_{j=1}^n C_j, E_{max})$ (Minimum of total completion time and maximum earliness)

The problem is stated as follows:

$$\left. \begin{array}{l} \text{Min } \sum_{j=1}^n C_j \\ \text{and} \\ \text{Min } E_{max} \end{array} \right\} (p_2)$$

Hoogeveen and Van de Velde [12] introduced an algorithm to find all efficient solutions of (p_2) . They found all the efficient solutions. The decision maker will choose the appropriate solution according to his case.

3.3. Problem formulation for tri-objective problems

We consider the single machine scheduling problem with tri-objective function as the sum of total completion time, maximum earliness and maximum tardiness, denoted by $1 / / \sum_{j=1}^n C_j + E_{max} + T_{max}$. For simplicity, the problem is defined by $1 / / \sum_{j=1}^n C_j + ET_{max}$. A set $N = \{1, 2, \dots, n\}$ of n independent jobs is to be scheduled on a single machine optimizing a given criterion. Job $j \in N$ is to be processed without interruption on a single machine that can handle only one job at a time requiring processing time p_j , and ideally should it be completed at its due date d_j . For a given sequence π of jobs, completion time of job j , $C_{\pi(j)}$, earliness, $E_{\pi(j)}$, and the tardiness, $T_{\pi(j)}$, are given by:

$$\begin{aligned} C_{\pi(1)} &= p_{\pi(1)}, \\ C_{\pi(j)} &= C_{\pi(j-1)} + p_{\pi(j)}, j = 2, \dots, n, \\ E_{\pi(j)} &= \max \{d_{\pi(j)} - C_{\pi(j)}, 0\}, j = 1, 2, \dots, n, \\ T_{\pi(j)} &= \max \{C_{\pi(j)} - d_{\pi(j)}, 0\}, j = 1, 2, \dots, n. \end{aligned}$$

Mathematically, the problem can be stated as shown in:

$$\left. \begin{array}{l} \text{Min } Z = \{ \sum_{j=1}^n C_j(\pi) + E_{max}(\pi) + T_{max}(\pi) \} \\ \text{s. t.} \\ C_{\pi(j)} \geq p_{\pi(j)}, j = 1, \dots, n, \\ C_{\pi(j)} \geq C_{\pi(j-1)} + p_{\pi(j)}, j = 2, \dots, n, \\ E_{\pi(j)} \geq d_{\pi(j)} - C_{\pi(j)}, j = 1, 2, \dots, n, \\ T_{\pi(j)} \geq C_{\pi(j)} - d_{\pi(j)}, j = 1, 2, \dots, n, \\ C_{\pi(j)}, E_{\pi(j)}, T_{\pi(j)}, p_{\pi(j)} \geq 0, j = 1, 2, \dots, n. \end{array} \right\} (p_3)$$

where π_j denotes the position of job j in π . The aim is to find a sequence π which minimizes Z .

3.3.1. Special cases of problem (p_3)

Proposition 1: If $p_j = p, \forall j \in N$, then EDD-rule gives an optimal solution of (p_3) .

Proof: Let S be a sequence that satisfies the EDD-rule. It is clear that $\sum_{j=1}^n C_j$ is optimal for any sequence since $p_j = p, \forall j \in N$, and $T_{max}(S)$ is optimal since S is in the EDD-rule. Now, for each pair of jobs

i and $j \in S$ ($i \leq j$), $d_i \leq d_j$ implies $d_i - p \leq d_j - p$ since p is a constant. Hence, the sequence also satisfies the MST-rule. Consequently, this sequence gives an optimal solution of (p_3) .

Proposition 2: If a schedule S gives the SPT-rule and MST-rule at the same time, then S is an optimal schedule for (p_3) .

Proof: Let a sequence S is satisfy both the SPT and MST-rules and this order gives optimal solution for $\sum_{j=1}^n C_j$ and E_{max} , respectively. Now, it is left to be shown that S is also optimal for T_{max} . Since SPT and MST-rules gives $p_i \leq p_j$ and $d_i - p_i \leq d_j - p_j$, $i, j \in S$, then we have $p_j - p_i \geq 0$ and $d_i - p_i \leq d_j - p_j$, and hence $d_i \leq d_j - (p_j - p_i)$ which implies $d_i \leq d_j$ indicating that the sequence S is also satisfies the EDD-rule. Since the EDD-rule is an optimal order for T_{max} , the sequence S is an optimal solution of (p_3) .

3.4. Design heuristic method - based on efficient solutions

In this section we give some new ideas to build our heuristic method.

Definition 5: A feasible schedule π^* is Pareto optimal, or efficient, with respect to the performance criteria f , g and h , If there is no feasible schedule π such that $f(\pi) \leq f(\pi^*)$, $g(\pi) \leq g(\pi^*)$ and $h(\pi) \leq h(\pi^*)$, where at least two of the inequalities are strict. Let S be a set of all efficient solutions for the problem (p_1) and let S_1 be the set of all efficient solutions for problem (p_2) .

Proposition 3: $S \cap S_1 \neq \emptyset$.

Proof: Since the SPT-rule is an efficient solution for (p_1) , and also, the SPT-rule is an efficient solution for (p_2) , the SPT-rule is one of the common sequences of $(S \cap S_1)$. Note that an analogous proposition for the EDD-rule and the MST-rule does not hold in general.

Proposition 4: If π^* is efficient for (p_1) , then it is also efficient for (p_3) .

Proof: If π^* is efficient, then $\nexists \pi \in \delta$ such that $\sum_{j=1}^n C_j(\pi) \leq \sum_{j=1}^n C_j(\pi^*)$ and $T_{max}(\pi) \leq T_{max}(\pi^*)$ from the definition of efficient solutions. So, for $E_{max}(\pi)$, if $E_{max}(\pi) > E_{max}(\pi^*)$, then π^* remains efficient for (p_5) [from Definition 5].

Proposition (5): If π^* is efficient for (p_2) , then it is also efficient for (p_3) .

Proof: If π^* is efficient, then $\nexists \pi \in \delta$ such that $\sum_{j=1}^n C_j(\pi) \leq \sum_{j=1}^n C_j(\pi^*)$ and $E_{max}(\pi) \leq E_{max}(\pi^*)$ from the definition of efficient solutions. So, for $T_{max}(\pi)$, if $T_{max}(\pi) > T_{max}(\pi^*)$, then π^* remains efficient for (p_3) [from Definition 5].

3.4.1. Lower bound

Lower bound is one of the main factors to find an acceptable solution for a problem. Deriving lower bounds for an NP-hard multi-objective problem is obviously difficult. Here, we will use a decomposition of the problem.

3.4.2. Derive a lower bound

To find a lower bound for problem (p_3) , we decompose the problem into three sub problems (p_4) and (p_5) and (p_6) as shown in:

$$\left. \begin{aligned} \text{Min } Z &= \sum_{j=1}^n C(\pi) \\ \text{s. t.} \\ C_{\pi(j)} &\geq 0, j = 1, 2, \dots, n, \\ C_{\pi(j)} &\geq p_{\pi(j)}, j = 1, 2, \dots, n. \end{aligned} \right\} (p_4)$$

$$\left. \begin{aligned} \text{Min } Z &= E_{max}(\pi) \\ \text{s. t.} \\ E_{\pi(j)} &\geq 0, j = 1, 2, \dots, n, \\ E_{\pi(j)} &\geq d_{\pi(j)} - C_{\pi(j)}, j = 1, 2, \dots, n. \end{aligned} \right\} (p_5)$$

$$\left. \begin{aligned} \text{Min } Z &= T_{max}(\pi) \\ \text{s. t.} \\ T_{\pi(j)} &\geq 0, j = 1, 2, \dots, n, \\ T_{\pi(j)} &\geq C_{\pi(j)} - d_{\pi(j)}, j = 1, 2, \dots, n. \end{aligned} \right\} (p_6)$$

3.5. The main theorem

Define the lower bound as $LB = \sum_{j=1}^n C_j(SPT) + E_{max}(MST) + T_{max}(EDD)$, and the upper bound as $UB = \sum_{j=1}^n C_j(SPT) + E_{max}(SPT) + T_{max}(SPT)$. The relation between the optimal value, say opt., lower bound (LB) and efficient solutions for (p_3) is characterized by the following theorem.

Theorem 1: $\exists r \in N^+$ such that $LB + r = \text{opt.}$ and $r \in [N_1 - 1, N_2 + 1]$, where, $N_1 =$ number of efficient solutions, and $N_2 = (E_{\max}(\text{SPT}) - E_{\max}(\text{MST})) + (T_{\max}(\text{SPT}) - T_{\max}(\text{EDD}))$.

Proof: Since $LB \leq \text{opt.}$, there exists a non-negative integer r such that $LB + r = \text{opt.}$, which proves the first part of the theorem. It remains to show that $r \in [N_1 - 1, N_2 + 1]$ or to show that $N_1 - 1 \leq r \leq N_2 + 1$. Since:

$$LB + r = \text{opt.}, r = \text{opt.} - LB \leq UB - LB = E_{\max}(\text{SPT}) - E_{\max}(\text{MST}) + T_{\max}(\text{SPT}) - T_{\max}(\text{EDD}).$$

So, $N_2 \leq N_2 + 1$. Hence, $r \leq N_2 + 1$. Now, we will prove $N_1 - 1 \leq r$ by mathematical induction on N_1 . Note that N_1 is not known. If $N_1 = 1$, that is, there is only one efficient solution which is SPT-rule as well as MST (not MST-rule), then $r = \sum_{j=1}^n C_j(\text{Opt.}) + E_{\max}(\text{Opt.}) + T_{\max}(\text{Opt.}) - \sum_{j=1}^n C_j(\text{SPT}) - E_{\max}(\text{MST}) - T_{\max}(\text{EDD}) = 0$. Thus, $N_1 - 1 \leq r \leq N_2 + 1$. That is, $r \in [N_1 - 1, N_2 + 1]$, and so the theorem is true for $N_1 = 1$.

If $N_1 = 2$, that is, the number of efficient solutions is two which are the SPT-rule and σ . If the SPT-rule is optimal, then $r = E_{\max}(\text{SPT}) - E_{\max}(\text{MST}) + T_{\max}(\text{SPT}) - T_{\max}(\text{EDD}) \geq 1 = N_1 - 1$. Hence, $N_1 - 1 \leq r \leq N_2 + 1$.

If σ is optimal, then $r = \sum_{j=1}^n C_j(\sigma) + E_{\max}(\sigma) + T_{\max}(\sigma) - \sum_{j=1}^n C_j(\text{SPT}) - E_{\max}(\text{MST}) - T_{\max}(\text{EDD}) \geq 1 = N_1 - 1$. Hence, $N_1 - 1 \leq r \leq N_2 + 1$. So $r \in [N_1 - 1, N_2 + 1]$ and the theorem is true for $N_1 = 2$. If $N_1 = 3$, that is, there are three efficient solutions, the SPT-rule, σ and σ_1 . If the SPT is optimal, then:

$$r = E_{\max}(\text{SPT}) - E_{\max}(\text{MST}) + T_{\max}(\text{SPT}) - T_{\max}(\text{EDD}) \geq 2 = N_1 - 1.$$

hence, $N_1 - 1 \leq r \leq N_2 + 1$. If σ is optimal, then $r = \sum_{j=1}^n C_j(\sigma) + E_{\max}(\sigma) + T_{\max}(\sigma) - \sum_{j=1}^n C_j(\text{SPT}) - E_{\max}(\text{MST}) - T_{\max}(\text{EDD}) \geq 2 = N_1 - 1$. Hence, $N_1 - 1 \leq r \leq N_2 + 1$. Now, if σ_1 is optimal, then:

$$r = \sum_{j=1}^n C_j(\sigma_1) + E_{\max}(\sigma_1) + T_{\max}(\sigma_1) - \sum_{j=1}^n C_j(\text{SPT}) - E_{\max}(\text{MST}) - T_{\max}(\text{EDD}) \geq 2 = N_1 - 1.$$

hence, $N_1 - 1 \leq r \leq N_2 + 1$. Hence, $N_1 - 1 \leq r \leq N_2 + 1$. So $r \in [N_1 - 1, N_2 + 1]$ and the theorem is true for $N_1 = 3$.

Suppose the theorem is true for $N_1 = k$, i.e., the theorem is true for the k efficient solutions $\sigma, \sigma_1, \dots, \sigma_{k-1}$, that is for these k efficient solutions we have $N_1 - 1 \leq r \leq N_2 + 1$. Let $N_1 = k + 1$, that is, there is $k + 1$ efficient solutions $\sigma, \sigma_1, \dots, \sigma_{k-1}, \sigma_k$, if one of the first k efficient solutions $\sigma, \sigma_1, \dots, \sigma_{k-1}$, is optimal, then since the theorem is true for $N_1 = k$, we get $N_1 - 1 \leq r$, and hence $N_1 - 1 \leq r \leq N_2 + 1$, and if the last efficient solution σ_k is optimal, then $r = E_{\max}(\sigma_k) + T_{\max}(\sigma_k) - E_{\max}(\text{MST}) - T_{\max}(\text{EDD}) = E_{\max}(\sigma_k) - E_{\max}(\text{MST}) \geq k = k + 1 - 1 = N_1 - 1$, and thus $N_1 - 1 \leq r \leq N_2 + 1$. So, the theorem is true for $N_1 = k + 1$, which completes the proof.

Corollary 1: If $\sigma \in \Pi$ gives the SPT-rule and the MST-rule at the same time, then $N_2 = 0$.

Proof: From Proposition 2, the sequence σ is also in the EDD-rule, and so $T_{\max}(\text{SPT}) = T_{\max}(\text{EDD})$. Also, $E_{\max}(\text{SPT}) = E_{\max}(\text{MST})$. Therefore, $N_2 = 0$. Corollary 2: If $\sigma \in \Pi$ gives the SPT-rule and the EDD-rule at the same time, then $N_2 = E_{\max}(\text{SPT}) - E_{\max}(\text{MST})$.

Proof: It is clear that if the condition holds, then $T_{\max}(\text{SPT}) - T_{\max}(\text{EDD}) = 0$, and so $N_2 = E_{\max}(\text{SPT}) - E_{\max}(\text{MST})$. The number of efficient solutions N_1 for problem (p_3) is not known, but from Propositions 4 and 5 we can conclude $N_1 \geq \#(S_1 \cup S)$. Therefore, we can minimize the range of r in Theorem 1.

3.6. Modified Smith's algorithm

Smith presented an algorithm to minimize the total completion time subject to no tardy jobs, that is, under the condition $T_{\max} = 0$ [10]. The algorithm solves the multi-objective problem of the lexicographical type. Here, we modify the Smith's algorithm to minimize a function when the value of another function is given. Since problem (p_3) contains three objectives, we let the value of T_{\max} be given and solve the problem according to Algorithm 1.

Algorithm 1. Modified Smith's algorithm

Step (1): Set $R = \sum_{j=1}^n p_j, N = \{1, 2, \dots, n\}, k = n$. Let $T_{\max}(\sigma)$ be given, $\sigma \in \Pi$.

Step (2): Find a job j^* such that $p_{j^*} \geq p_j$, and $R - d_j \leq T_{max}(\sigma)$, j and $j^* \in N$. If there exist a tie order the jobs by the MST-rule. Assign job j^* to position.

Step (3): Set $R = R - p_j$, $N = N - \{j^*\}$, $k = k - 1$. If $k = 0$, then stop. Else, go to Step (2).

3.7. The algorithm

We introduce a heuristic method to find a solution for problem (p_3) . This algorithm depends on the efficient solutions for two related problems. Also, we modified Smith's algorithm to guarantee the best solution for the problem.

Algorithm 2. Heuristic approach for tri-objective using efficient solutions

Step (1): Find efficient solutions for (p_4) , say $S_1 = \{\sigma_1, \sigma_2, \dots, \sigma_k\}$ and for (p_2) , say $S = \{\sigma^*_1, \sigma^*_2, \dots, \sigma^*_{k_1}\}$. Let N_1 be the number of efficient solutions of (p_3) , and $N_2 = (E_{max}(SPT) - E_{max}(MST)) + (T_{max}(SPT) - T_{max}(EDD))$.

Step (2): Compute $z_i = \sum_{j=1}^n C_j(\sigma_i) + E_{max}(\sigma_i) + T_{max}(\sigma_i) \forall i, i = 1, \dots, k$, and compute $z_l = \sum_{j=1}^n C_j(\sigma^*_l) + E_{max}(\sigma^*_l) + T_{max}(\sigma^*_l), \forall l, l = 1, \dots, k_1$. Let z^* be minimum of z_i and z_l , z^* be an upper bound of (p_3) , $N_1 \geq \#(S_1 \cup S)$, $r = N_1 - 1$.

Step (3): Find values of r such that $r \in [N_1 - 1, N_2 + 1]$. If $r > N_2 + 1$ or $LB + r > z^*$, then stop.

Step (4): If there exists a sequence $\pi \in (S_1 \cup S)$ such that $\sum_{j=1}^n C_j(\pi) + E_{max}(\pi) + T_{max}(\pi) = LB + r$, then it is optimal, and go to Step (7). Else $z^{**} = \sum_{j=1}^n C_j(\sigma) + T_{max}(\sigma), \forall \sigma \in (S_1 \cup S)$.

Step (5): If there exists a sequence $\sigma \in (S_1 \cup S)$ such that $z^{**}(\sigma) = LB + r$, then go to Step (6). Else let $r = r + 1$, go to Step (4).

Step (6): Use modified Smith's algorithm for given $T_{max} = T_{max}(\sigma)$. If a solution exists with $E_{max} = 0$, then it is optimal and go to Step (7). Else let $r = r + 1$. Go to Step (4).

Step (7): Stop.

To illustrate the algorithm, we present the following example with 4 jobs as described in Table 1. It can be determined that The SPT schedule is (1,2,3,4), The EDD schedule is (1,3,2,4), The MST schedule is (4,3,1,2). The efficient solution of (p_1) is $S = \{(96,19)\}$, with the schedule (1,2,3,4), and the efficient solutions of (p_2) are $S_1 = \{(96,9), (102,8), (156,5)\}$, with the schedules (1,2,3,4), (3,1,2,4), (4,1,2,3) respectively, and so $S \cup S_1 = \{(1,2,3,4), (3,1,2,4), (4,1,2,3)\}$ as shown in the Table 2.

Table 1. Single machine scheduling problem with four jobs

j	1	2	3	4
p_j	6	8	10	28
d_j	15	19	18	33

Table 2. Efficient solutions of p_1 and p_2

Problem (p_1)	Problem (p_2)	Z	Z ^{**}
(96,19) efficient	(96,9) efficient	124 = Z [*]	115
(102,19)	(102,8) efficient	129	122
(156,34)	(156,5) efficient	195	190

The optimal value of the problem is 124, which corresponds to the schedule (1, 2, 3, 4). Using Algorithm 2, we have $N_1 = \# \{ S \cup S_1 \} = 3$, and so $N_1 \geq 3$, and $N_2 = 4$, therefore $r \in [2,5]$. $LB = 120$. $UB = 124$ and from Algorithm 2, $r = 2$, and so $LB + r = 122$. Since there exists no π such that $\sum_{j=1}^n C_j(\pi) + E_{max}(\pi) + T_{max}(\pi) = LB + r$, we find $z^{**}(\sigma) = \sum_{j=1}^n C_j(\sigma) + T_{max}(\sigma)$. Since there exists no σ such that $z^{**}(\sigma) = LB + r$, we have $r = r + 1$, and go to Step (4). We have $LB + 3 = 123$, and there exists neither π nor σ such that $LB + r = z$ or z^{**} , and thus $r = r + 1$, and go to Step (4). $LB + r = 124 = z$ which gives the optimal value of the problem.

4. COMPUTATIONAL RESULTS

Here, we will introduce our results via computational tests to show the efficiency of the proposed algorithm. The problems were generated as follows: an integer processing time and an integer due date are generated from the uniform distribution. The Algorithm 2 was running and coding it in MATLAB 8.1

(R2013a) and implemented on Intel (R) Pentium (R) CPU 2117U@ 1.80 GHZ, with RAM 4.00 GB personal computer.

5. CONCLUSION

We presented a heuristic algorithm to minimize the sum of total completion time, maximum earliness, and maximum tardiness in a single-machine scheduling. The algorithm depends on a new idea that presented for the first time which depends on the efficient solutions of some related problems. The algorithm is easy to implement and has a simple structure comparing with other used algorithms such as branch and bound algorithm. To evaluate the efficiency of the proposed algorithm, the algorithm was tested on (100) examples, and in all the examples the computational results showed the ability of the algorithm to find the optimal solution for the problem.




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