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Hybrid basis vector based underdetermined beamforming algorithm in optimized antenna reconfiguration

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Article Info	ABSTRACT
	Optimized positioning of antenna to obtain the best beam forming solution is adopted in this research. Non-uniform linear array-based beamforming algorithms have the challenge of placing the array of antennas in positions that would implement best beamforming outputs. This paper attempts to obtain the optimized beam forming by tuning the sparse Bayesian learning based algorithm. The parameters used for tuning involve choosing the hybrid basis vector for creating the steering vector while at the same time developing the optimized position of the antennas. Basis vectors are the building blocks of the steering vector developed for the beamforming algorithm that finds the angle of arrival in antennas. Reconfiguration of antennas is carried out using particle swarm optimization (PSO) algorithm and the basis vectors are generated using two different ways. One by cumulating similar basis vectors and another by cumulating two different basis vectors. The performance of accurate detection of angle of arrival in the beamforming algorithm is analyzed and results are discussed. This basis vector and antenna distance optimization is adopted on the sparse Bayesian learning paradigm. Performance evaluation of these optimizations in the algorithm is realised by validating the mean square error (MSE) versus signal to noise ratio (SNR) graphs for both the cumulative basis vector and hybrid basis vector cases.

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1. INTRODUCTION

Angle of arrival (AOA) estimation is a part of channel estimation procedure while receiving the signals in the wireless communication paradigm. Beam forming approaches are used for AOA estimation in antenna communication systems. Accuracy in which the estimation of AOA is performed, highlights the performance of the antenna communication implementation. Direction of the signal approaching the antennas are important in both the radar and sonar applications [1]-[3]. Traditional methods like music [1] and estimation of signal parameters via rotational invariant techniques (ESPRIT) [2] are capable of finding the AOA of N-1 signals while N element uniform linear array is used. While the number of sources that are resolved is less than the number of sensors used then the problem is defined as underdetermined AOA estimation discussed in [4]-[6]. These underdetermined approaches increase the degree of freedom (DOF) by creating a virtual array [5]. Completely defined non-uniform linear array and the signal received on that array is used to generate the virtual array. A covariance matrix is generated from the non-uniform linear array and the received data. Vectorizing that covariance matrix gives the virtual array. Such an array called the

minimum redundancy array (MRA) [7] is generated introducing the aperture that is maximum possible for N sensors. Design of MRAs cannot be predicted although a MRA for N≤17 sensors are created in [8]. Reducing the number of the sensors for the underdetermined environment is discussed in research publications [9]-[25]. Larger aperture virtual array or co-prime array are used for the AOA estimation for an underdetermined system with nonlinear uniform linear array (NULA) with the use of a lesser number of sensors [5], [25]. Minimum hole array (MHA) [9] is another method for virtual sensors or arrays in the underdetermined system. Since there is no closed form method for the AOA estimation an extensive search method is used for the physical arrays. Sparse arrays are the recent ways of improving the search method in the underdetermined system. Nested arrays as discussed in [5] concatenate two uniform linear arrays (ULAs) to resolve N square AOA from N physical sensors. NULA structures with coprime arrays obtained by interlacing two ULAs having intersensor spacing of M and N [10], where M and N are coprime. M+N-1 sensors are needed to resolve MxN sources. Similarly more targets using lesser sensors can be resolved using the coprime array with displaced subarrays (CADiS) [11]. Different AOA estimation algorithms are defined [12]-[22] that uses lesser sensor and sparse arrays for resolving higher number of signals are detailed. Non circular signals used for the AOA estimation algorithm [14]-[16] exhibited better performance while reducing the noise [14]. Sparse Bayesian learning based algorithms are implemented for the multiple measurement vector (MMV) sparse signal recovery problem [26]. Extension of this SSR algorithm is done by block sparse signal recovery problem in [27]-[29] by considering the temporal correlation of sources. Off grid error based algorithms for AOA estimation are introduced in [30]. Bayesian model which defined the off grid error as the prior is defined. Non negative prior information is incorporated for the sparse Bayesian learning algorithm [31].

To enhance the accuracy of AOA estimation, parametric changes are made in the non negative sparse Bayesian learning (NNSBL) algorithm. This paper attempts increasing the accuaracy of AOA estimation by replacing the overcomplete basis vector of NNSBL algorithm by the cumulative and the hybrid basis vector. Antenna reconfiguration using the particle swarm optimization (PSO) algorithm is also applied for better performance. The obtained results are compared using mean square error (MSE) v/s signal to noise ratio (SNR) and MSE v/s snapshot plots. Section 2 deals with the implementation details of the methodology carried out. Section 3 discusses optimization algorithm implemented for antenna reconfiguration and section 4 deliberates on the result and discussion of the implementation and followed by conclusion and reference.

2.   HYBRID BASIS VECTOR BASED UNDERDETERMINED AOA ESTIMATION
Non negative sparse Bayesian learning (NNSBL) for underdetermined AOA estimation is implemented with the hybrid basis vector based algorithm.

2.1. Sparse representation: introduction
Since the research is based on basis pursuit denoising (BPDN) based sparse representation direction of arrival (DOA) estimation methods an introduction about this method is made. The advancement in the existing DOA estimation algorithm in sparse representation paradigm is applied and compared with the traditional method. The BPDN based DOA estimation algorithm thus developed in the existing literature is advanced with the cumulative basis vector and hybrid basis vector-based implementation.

2.2. Signal model
Omnidirectional antennas with M elements are placed in a non-uniform linear array which are located at different distances [0, d_1,… d_{M-1}], which denotes distance between the reference location and different antennas. This distance is the integral multiples of half the wavelength. Improvement of convergence in any sparse representation problem is improved by increasing the degree of freedom (DOF). DOF considered in the omnidirectional antenna array is the difference co-array defined as:

$$\Omega = \left\{ d_{m_1} - d_{m_2} \right\}_{m_1 = 0,1,……M-1,; m_2 = 0,1,….M-1}$$

For M antennas Ω provides more DOFs. Considering that N far-filed sources uncorrelated in nature is falling on M antennas. The narrow band sources is defined by $S_k(t), k = 1,2, …, N$ which impinges on antenna arrays. The proposed implementation calculates the DOA estimation with spatially white Gaussian noises as the channel for all the M antennas denoted by $n_m(t), m = 0,1, ….,M - 1$. The snapshots of the signal with noise is defined as:

$$x(t) = As(t) + n(t) \tag{1}$$

Array received vector x(t), signal from the transmitting source **s**(t) and the noise in the channel n(t) for the tth snapshot is denoted in (1). The steering vectors of all the N sources are consolidated in the manifold matrix A.

$$A = [a(\theta_1), a(\theta_2), \cdots, a(\theta_N)]$$

Where the steering vector a(θ_n), n=1,2,$\cdots$,N, corresponding to the nth incident signal is defined as a(θ_n) = $[1, v(d_1,\theta_n), \cdots, v(d_{M-1},\theta_n)]^T$, phase component $v(d,\theta)$ is defined as v(d$_m$, θ) = $exp\left[-j2\pi\left(\frac{d_m}{\lambda}\right)sin\theta\right]$, and $\{\cdot\}^T$ denotes the transpose. It is considered that the signal and the noise are uncorrelated and thus the covariance matrix is formulated as defined in (2).

$$R_x = E\{x(t)x^H(t)\} = A diag\ (\sigma1^2, \sigma2^2, \cdots, \sigma_{N^2})A^H + \sigma_{n^2}I_M, \tag{2}$$

The uncorrelation between the source and the noise is denoted in (2) by introducing multiple variances σ_1^2, σ_2^2...., σ_N^2 corresponding to N sources. Expectation E $\{\cdot\}$ for the component x(t)x^H(t) defines the covariance matrix. The identity matrix I$_M$ with size M x M. Vectorizing the (2), creates the virtual array from covariance matrix. The vectorization involves Khatri Roa (KR) product in the (3).

$$Y \cdot = \cdot vec \cdot (R_x) = vec\ (AR_sA^H) + \cdot \sigma_n\ ^2vec\ (I) = \cdot (A^* \circledast A)g \cdot + \cdot \sigma_n^2 I_M \tag{3}$$

In (3) KR product $\circledast$, conjugate transpose $\{\cdot\}^H \cdot$g=[σ_1^2, σ_2^2,$\cdots$, σ_N^2]^T denotes source variance vector, 1_m=[e$_1^T$,e$_2^T$,$\cdots$,e$_M^T$]^T with e$_m$vector being zeros excluding the mth entry which is 1. $\underline{A}$=(A$^*\circledast$A), is the virtual array manifold matrix. The virtual manifold matrix $\underline{A}$ consisits of N virtual steering vectors with $\underline{a}(\theta_n)$ = $a^*(\theta_n)\otimes a(\theta_n)$, n=1,2,$\cdots$,N, where $\otimes$ indicates the Kronecker product. Distinct entries of a$^*(\theta)\otimes$a(θ) increases the DOF of the DOA estimation problem. The provided data is the sample covariance matrix while in reality defined as $\hat{R}_x = \sum_{t=1}^{T} X(t)X^H(t)/T$. As the incident signals are defined as circularly symmetric Gaussian distribution. A asymptotic complex Gaussian distribution results as the residual error of covariance matrix. The residual error is defined in (4).

$$\hat{Y} - Y = vec(\hat{R}_x) - vec(R_x) \sim CN\left(0, \frac{1}{T}R_x^T \otimes R_x\right) \tag{4}$$

Let $\tilde{R}_x = R_x^T \otimes R_x/T$, and by using (3) and (4) is transformed to be:

$$\hat{Y} \sim CN(\underline{A}g + \sigma_n^2 1_M, \tilde{R}_x) \tag{5}$$

In (5) defines the BPDN formulation for DOA estimation. The sparse solution space is identified in the sample grid represented as $\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$. The sample grid or the sparse solutions space spans range of all possible incident directions of the signal. Thus the (5) is converted to the following (6).

$$\hat{Y} \sim CN(\Phi w + \sigma_n^2 1_M, \tilde{R}_x) \tag{6}$$

Matrix Φ acts as the overcomplete dictionary of the DOA estimation problem. All the direction in the grid θ is utilized in Φ and acts as the basis vector for matrix $\underline{A}$. The non negative sparse matrix W contains ones where actual DOA is present and zeros in all other positions. N being the non-negative Gaussian distribution defined in [32].

2.3. Sparse Bayesian modeling
Sparse Bayesian learning (SBL) discussed in [32] is considered as nonnegative. Due to which the BPDN problem is considered as the real valued problem, incorporating the positive source variance. It is discussed in [32] that if the incident signals follow a circular-symmetric Gaussian pattern, the positive source variance is converted to Gaussian distribution with real values. Thus (6) is rewritten as (7):

$$P(\hat{y}|w, \sigma_n^2) = N(\tilde{\Phi}w, \underline{R}), \tag{7}$$

where:

$$\tilde{y}(\sigma_n^2) = [Re(\hat{y}^T) - \sigma_n^2 \cdot 1_M^T, Im(\hat{y}^T)]^T, \quad \tilde{\Phi} = [Re(\Phi)^T, Im(\Phi)^T]^T$$

and

$$\underline{R} = \frac{1}{2}[Re(\tilde{R}_x) - Im(\tilde{R}_x); Im(\tilde{R}_x)Re(\tilde{R}_x)]$$

Traditional SBL uses l_1-norm as the objective for sparse learning problem. NNSBL, uses the Laplacian prior distribution in place of l_1-norm. The prior distribution is defined in (8):

$$\mathrm{P}(w|\lambda) = \frac{\lambda^N}{2^N} exp\ (-\lambda \parallel w \parallel_1) \tag{8}$$

In (8) is rewritten considering that w is a nonnegative vector and given in (9).

$$p(\lambda) = \lambda^N exp\ (-\lambda \sum_{i=1}^N w_i), \quad w_i \geq 0,\ \mathrm{i} = 1, 2, \cdots, \mathrm{N} \tag{9}$$

Bayesian framework starts with the prior distribution. The solution for the sparse problem starts with this prior and develops a posterior distribution. If the prior distribution that is defined in (9) does not appear to be conjugate of the conditional distribution of the observed data, the non-negative Laplace prior is developed. The marginal prior of the 'w' is defined as a laplace distribution while the hyper prior for the hyper parameter λ is given as a gamma distribution:

$$P(\lambda; v) = \Gamma(\lambda|v, v) \tag{10}$$

Definition of the gamma probability distribution function is given as $\Gamma(a, b) = b^a \lambda^{a-1} exp\ exp(-b\lambda)/\Gamma(a)$. Where v is the hyperparameter, v which defines the set of constant values $v \rightarrow 0$, called as the Jeffrey's hyper prior. Another prior for the variance value σ_n^2, is considered as a noninformative distribution to complete the Bayesian model:

$$p(\sigma_n^2) \propto 1, \quad \sigma_n^2 > 0 \tag{11}$$

All the distributions defined for different variables and hyperparameters are combined to obtain a joint PDF to form the Bayesian model defined in (12).

$$p(w, \gamma, \lambda, \sigma_n^2, \hat{y}) = p(w, \sigma_n^2)P(w|\gamma)p(\gamma|\lambda)p(\lambda)p(\sigma_n^2) \tag{12}$$

With the developed Bayesian model the Bayesian inference and the solutions are obtained to estimate the DOA of the given signal. Once the Bayesian model is ready with all the priors combined, the posterior has to be obtained in order to infer from the signal. Thus, an expectation maximization algorithm is adopted to find the solution.

2.4. Cumulative basis vector

The NNSBL based DOA estimation algorithm uses the matrix Φ that acts as the overcomplete dictionary. This overcomplete matrix is generated using usually a Gaussian basis vector. This basis vector is advanced in the proposed algorithm to make it a cumulative basis vector implementation. In NNSBL DOA estimation during a to searching a or to generate manifold matrix processing time to using multi basis vector using more than one basis vector using is a cumulative basis vector Using cumulative basis vector for generating manifold matrix increases the AOA estimation accuracy. Gaussian basis vector used in finding manifold matrix $\phi(x)$ is given by:

$$\Sigma_{i=1}^{\infty} \phi^T(x)\phi(x') \tag{13}$$

2.5. Hybrid basis vector and optimization

In this paper, a hybrid basis vector means using two different basis vectors, one is Gaussian and other is hyperbolic tangent basis vector is used to find manifold matrix. The dot product in the infinite dimensional space transforms the Gaussian basic vector into the Gaussian function of the distance between points in the data space. The angle between the basis vectors is small in the vector space, if two points in the data space are nearby:

$$k(x, y) = exp\left(-\gamma ||x - y||^2\right) \tag{14}$$

Hyperbolic tangent basis vectors owe their popularity to neural networks, which, traditionally use the hyperbolic tangent activation function $tanh(\alpha(x.x') + c)$. The hyperbolic tangent of a dot product with fixed linear scaling provides a basis vector based manifold matrix A. Adjusting parameter α, equilibrium constraint c intercept constant. The overcomplete basis vector that is used in the NNSBL algorithm is changed with the cumulative and the hybrid basis vector and the performance is checked and compared.

PSO is the bio-inspired algorithm that is developed by formulating the behavior of the birds searching its prey [27]. The independent variables of the objective functions are assumed as the birds and the objective function is analogous to the act of the bird finding the prey. The objective function in this implementation is the MSE between the DOA estimated from the algorithm and the actual DOA of the incident signals. PSO based reconfiguration is done by changing the distance between the antennas. The formulation defining the antenna reconfigured DOA estimation using Multibasis vector is developed. Distance between the antenna is optimised using the PSO algorithm, considering the MSE as the minimization parameter. The MSE is defined by calculating the difference between the actual DOA and the DOA estimated. This paper introduces more stochastic nature in this formulation using the particle swarm optimization that reconfigures the antenna, search for the best distance between the antennas. The PSO is a bio-inspired algorithm that is mathematically models the activity of the bird flock that tries to find the food in the swarm. Mathematically the bird in the real world is emulated as a particle in the search space. As the bird gains knowledge from the neighbouring bird and by its own reference similarly each particle generated gains the knowledge on the convergence. The bird moves faster with higher acceleration while the bird is far away from the food. Similarly, the particle moves faster in the search space and with higher movement when it is far from convergence. Near the convergence space the particle moves slower and with lesser movement. Particles in PSO are the sample space of solutions. These solutions are optimized using the objective functions, which is primarily the cause of the formulation. In this paper the objective function is the mean square error that is between the actual DOA angle and the estimated angle. The objective function is given by (15).

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 \qquad (15)$$

Here n is the number of signals incident on the antennas. Y_i is the actual angle of arrival for the ith signal and $\hat{Y}_i$ is the estimated angle of arrival. Optimization converges towards the minimization of this mean square error. With a lesser number of parameters, the convergence is possible using the PSO algorithm. The distance between the antennas is the parameter that is populated and searched. Position of the PSO particles varies from one position to another by adding the velocity function that drives the particles towards the convergence. Velocity function is added with the particles to move from one position to another and check for convergence in the new position. In each iteration, the particles will move towards the convergence. Usually, 20 to 30 particles are generated in the first iteration and in each iteration, the movement will create the same number of particles. This iterative process converges towards the MSE minimization. The velocity values get added to the particle position values to obtain the new positions of the particles. These particles get the idea about the neighboring particles and the particles that are near to the convergence point from the velocity function defined in (16).

$$vel_1 \leftarrow wvel_1 + c_1 r_1 (Pbest - x_1) + c_2 r_2 (Gbeat - x_1) \qquad (16)$$

vel_1 is the velocity function which is formed by the use of the current position values $x_{1,}$, constants C_1, C_2 and r_1, r_2, Pbest is the best particle in the previous iteration and Gbest is the best particle for all the iterations carried out. Constants C_1 and C_2 are chosen to be integer 2. While, r_1, r_2 are randomly generated:

$$x_1 new \leftarrow x_1 + vel_1 \qquad (17)$$

The added sum of the previous position values and the velocity values produces the new position values to further the process of search [29]. 'w' is the inertia weight control or the rate at which the velocity varies and chosen between 0.4 to 0.9. The pseudo code of the PSO optimized algorithm is as given in the following:
- Initialize population (Distance of antennas are populated (total six variable is populated))
- Evaluate the populated particles (i.e distance for the lowest MSE (objective function))
- Find the best fit value and corresponding set of distances (Gbest),
- Repeat
- Find the velocity values from (16)
- Find new particles by (17)
- Find the best fit value (MSE) and corresponding set of distances (Pbest)
- Update the Gbest values Stop when total number of iterations is completed

In this paper as the reconfiguration is developed by the use of PSO the distance between the antenna are populated in order to get the optimal placement of the antenna until a minimized MSE occurs for the angle of arrival estimation. The DOA estimation is improved by two means in this implementation. One is the cumulative/hybrid basis vector implementation in the creation in the manifold matrix and the other one is the reconfiguration of the antenna positions that improves the DOA estimation still further.

3.   RESULTS AND DISCUSSIONS

A novel antenna reconfigurable DOA estimation problem is solved using the PSO algorithm. A cumulative and hybrid basis vector framework is developed on the NNSBL algorithm and optimized for better DOA estimation using PSO. MATLAB based simulation is developed for the PSO based reconfiguration on the cumulative basis vector based NNSBL and hybrid basis vector based NNSBL DOA estimation. The antenna signal configuration for the proposed algorithm is as shown in the Table 1.

Table 1. Antenna signal configuration for proposed DOA estimation algorithm

Details	Configuration
Number of Antennas	6
Antenna Array type	Non-uniform
Angle Range	-π/3 to π/3
Min to Max degrees	-70 to 70
Carrier frequency	280Hz
Propagation velocity	360
Interval of angle Searching	1
Angles of source signals	-54.8, -28.6 -9.2, 10.5 31.4, 56.7

Cumulative basis vector based manifold matrix thus developed for the DOA estimation is as given in the Figure 1. As per the Table 1 there are six signals thus there are six 'A' matrix. The source signal that is generated which incidents on the antenna is as depicted in Figure 2. The signal is generated with the carrier frequency of 280 Hz.
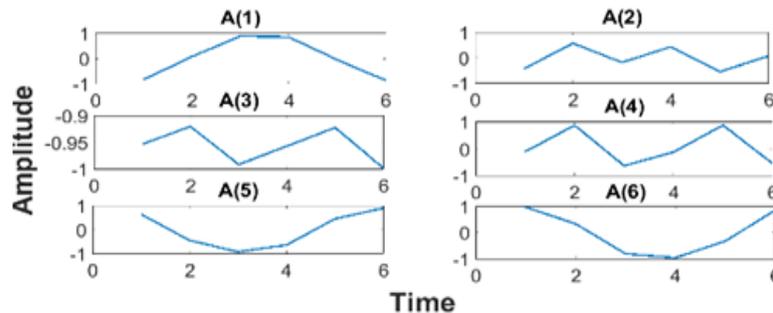

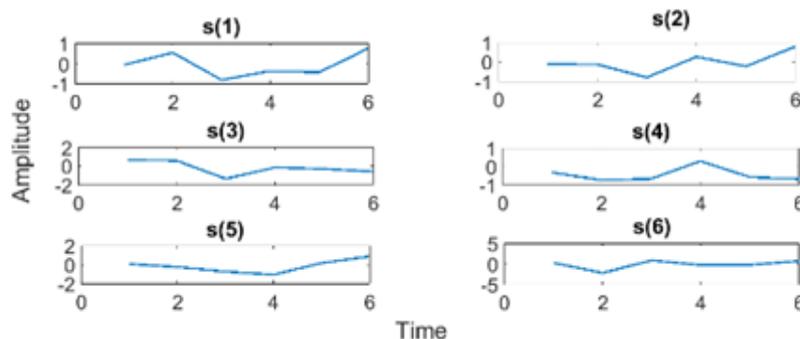Figure 1. Cumulative basis vector-based manifold matrix

Figure 2. Source signal incident on antenna

The noise is added with the source signal which is the white gaussian noise and the DOA estimation process is started. The cumulative basis vector based NNSBL (CBVNNSBL) method is compared with hybrid basis vector based NNSBL (HBVNNSBL) and found that the MSE is competitive with the HBVNNSBL method. From Table 2 it can be observed that the MSE is reduced in the HBVNNSBL compared to the CBVNNSBL method.

Table 2. Antenna signal configuration for proposed DOA estimation algorithm

	MSE vs SNR PSO CBVNNSBL and PSO NNSBL		
Sl.No	SNR	MSE PSO CBVNNSBL	MSE PSO HBVNNSBL
1	-10	1.0799	0.9785
2	-8	0.1985	0.0989
3	-6	0.1259	0.0273
4	-4	0.0355	0.02338
5	-2	0.0342	0.02123

It can be observed from the Table 2 that the MSE is almost zero for positive SNR values when PSO CBVNNSBL and PSO HBVNNSBL methods are used. The reconfiguration of the antenna has given better results than the advanced basis vector method. Figure 3 also confirms that the HBVNNSBL method is competitive to CBVNNSBL methods taken for discussion.

For the same configuration the SNR vs Snapshots analysis is carried out. The MSE values obtained for variation in the snapshots are observed. It can be observed from the Table 3 that the MSE is almost zero for snapshot values when PSO-HBVNNSBL and PSOCBVNNSBL methods are compared. The reconfiguration of the antenna has given better results for PSO HBVNNSBL than the PSO CBVNNSBL method. Figure 4 confirms that the PSOHBVNNSBL method is competitive PSO CBVNNSBL.
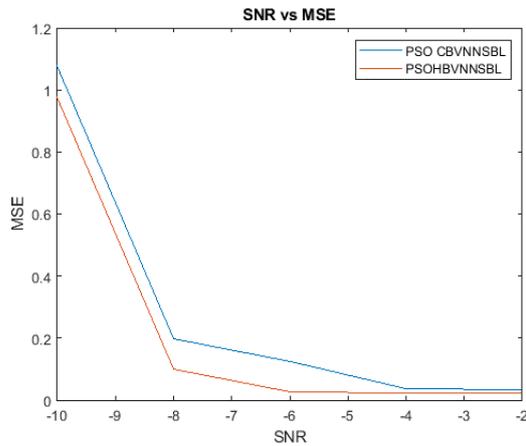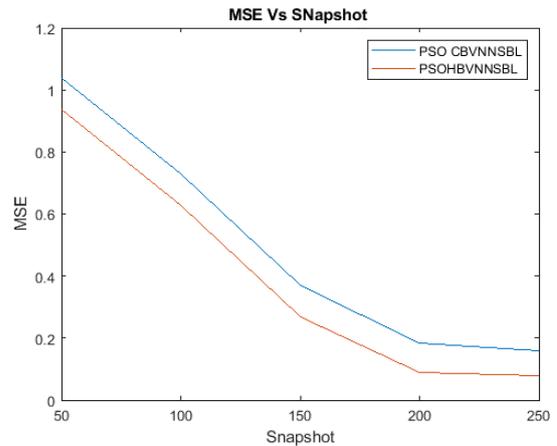

Figure 3. MSE vs SNR (PSO CBVNNSBL, PSOHBVNNSBL)

Figure 4. MSE vs snapshots (PSO-CBVNNSBL, PSOHBVNNSBL)

Table 3. MSE versus snapshots (PSO CBVNNSBL vs PSOHBVNNSBL with varying snapshots)

	MSE vs snapshot PSO CBVNNSBL and PSOHBVNNSBL		
Sl.No	Snapshot	MSE PSO CBVNNSBL	MSE PSO HBVNNSBL
1	50	1.0380	0.9376
2	100	0.7298	0.6269
3	150	0.3680	0.2689
4	200	0.1850	0.0876
5	250	0.1586	0.0787

4. CONCLUSION

From the results and discussion, the objective of minimization of MSE for the antenna reconfiguration by optimizing the distance between them is satisfactorily performed well. The competitiveness of the proposed algorithm with the CBVNNSBL and HBVNNSBL algorithm is evident from

the results thus obtained. Antenna reconfiguration using PSO algorithm on the DOA estimation of the signals on a non-uniform linear array is developed. MSE as the optimization parameter the antenna reconfiguration is formulated as a Meta heuristic optimization problem. Distance between the antennas is considered as the independent variable in the parameter optimization problem thus developed. The hybrid basis function based NNSBL method with the proposed antenna reconfiguration method with PSO algorithm showed better performance in the DOA estimation method proposed. The results are found to be satisfactory.

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