TELKOMNIKA, Vol. 11, No. 9, September 2013, pp. 4973~4979
ISSN: 2302-4046

Effects of the Colored Pump Noise in a Two-mode Laser

Youlin Xiang*, Haiyun Lin, Jianchun Cai, Zucheng Dai, Yujiao Wang
The Key Laboratory of Optoelectronic Materials and Devices in Yunnan Provincial Higher Education Institutions, Kunming University, Yunnan 650214, China
*Corresponding author, e-mail: youlinxiang@163.com

Abstract

An inhomogeneously broadened two-mode ring laser system with the colored pump noise is considered. The effects of the auto-correlation time, τ, of the colored pump noise on the competition between two modes in the laser system are investigated for the first time by the computer simulation method. The results show that when the laser system works far above threshold, the stationary properties of the laser intensities of two laser modes changes in quite other ways as the value of τ increases. It is obvious that the effects of the colored pump noise on the mode competition between two laser modes. Here does not exist mode competition for enough small value of τ, it increases and then remain unchanged with increasing value of τ.

Keywords: *laser system, colored pump noise, computer simulation, mode competition*

1. Introduction

The two-mode ring laser has attracted a great deal of attention over last decades [1-6]. Its properties such as the intensity correlations [1-3], the first-passage-time problem [4], the backscattering in a laser gyro [5] and a full account of saturation effects [6] have been studied. The reasons that the two-mode ring laser is so attractive are its widespread use [7-9] and there exists the strong competition exhibiting between two counterpropagating traveling-wave modes.

Both theoretical and experimental investigations of the mode competition in a two-mode ring laser system with quantum noise have been published [1, 2]. The results shown that the competition of the two modes for the excited atomic population leads to negative correlation between their intensity fluctuations, whose magnitude depends on the detuning of the laser cavity from the atomic line center, and competition effects prevent the light output in one mode from growing and becoming coherent as the pump parameter increases.

In above works, the fluctuations of the system have been assumed as the white noises with δ correlation. However, allowing for that the fluctuations of the pump of the laser system may not be sufficiently rapid to be representable by a δ-correlated noise, it is more reasonable to model it using a colored noise with auto-correlation time [10]. And it is very interesting to further discuss the effects of the colored pump noise on the laser system.

In present letter, the effects of the auto-correlation time, τ, of the colored pump noise on the stationary statistical properties of laser intensity and mode competition of a two-mode ring laser system are investigated by computer simulation. In Section 2, the laser intensity langevin equations and the procedure of computer simulation of the two-mode laser system are presented. In Section 3, some discussions and conclusions are given.

2. Langevin Equations and Computer Simulation of the Two-mode Ring Laser System

The dimensionless coupled complex electric fields $E_1(t)$ and $E_2(t)$ of a two-mode laser follows Langevin equations [3]:

$$\frac{dE_1}{dt} = (a_1 - |E_1|^2 - \xi |E_2|^2)E_1 + p(t)E_1 + q_1(t), \tag{1}$$

Received February 17, 2013; Revised May 30, 2013; Accepted June 14, 2013

$$\frac{dE_2}{dt} = (a_2 - |E_2|^2 - \xi |E_1|^2)E_2 + p(t)E_2 + q_2(t), \tag{2}$$

Where a_1 and a_2 are the pump parameters of the two modes and ξ is the mode-coupling constant, $p(t) = p_R + ip_I$ is the pump fluctuation, $q_1(t) = q_{1R} + iq_{1I}$ and $q_2(t) = q_{2R} + iq_{2I}$ are the independent quantum noises. These random noise terms are assumed to be zero mean and correlation.

$$\langle q_{ki}(t)q_{kj}(t')\rangle = \left[\delta_{ij} + \lambda(1-\delta_{ij})\right]P\delta(t-t'), \tag{3}$$

And

$$\langle p_i(t)p_j(t')\rangle = \delta_{ij}P'\delta(t-t'), \tag{4}$$

Where $i, j = I, R$ represent the real and imaginary parts and $k = 1,2$. P and P' are the strength of the quantum noise and pump noise, respectively. λ is the cross-correlation coefficient which measures the degree of cross-correlation between $q_{kI}(t)$ and $q_{kR}(t)$, and $|\lambda| \le 1$. Note that we think herein that fluctuation of the complex field can be coherent and the real and imaginary parts of the quantum noise generally originate from a common bath, thus should be cross correlated [11, 12].

By employing $E_k = r_k e^{i\varphi k}$ ($k = 1,2$), Equation (1) can be written in a two-dimensional form:

$$\frac{dr_1}{dt} = (a_1 r_1^2 - \xi r_2^2)r_1 + r_1 p_R(t) + \varepsilon_{r1}(t), \tag{5}$$

$$\frac{d\varphi_1}{dt} = p_I(t) + \frac{1}{r_1}\varepsilon_{\varphi 1}(t), \tag{6}$$

Where

$$\varepsilon_{r1}(t) = q_{1I}(t)\sin\varphi_1 + q_{1R}(t)\cos\varphi_1, \tag{7}$$

$$\varepsilon_{\varphi 1}(t) = q_{1I}(t)\cos\varphi_1 - q_{1R}(t)\sin\varphi_1. \tag{8}$$

Obviously, both $\langle\varepsilon_{r1}(t)\rangle$ and $\langle\varepsilon_{\varphi 1}(t)\rangle$ are non-zero. But a sum form of one non-zero mean part and the other zero mean part for $\langle\varepsilon_{r1}(t)\rangle$ and $\langle\varepsilon_{\varphi 1}(t)\rangle$ can be obtained as follows, respectively:

$$\varepsilon_{r1}(t) = \frac{P}{2r_1}(1 - \lambda\sin 2\varphi_1) + \tilde{\varepsilon}_{r1}(t), \tag{9}$$

$$\varepsilon_{\varphi 1}(t) = -\frac{P}{2r_1}\lambda\cos 2\varphi_1 + \tilde{\varepsilon}_{\varphi 1}(t), \tag{10}$$

Where:

$$\langle\tilde{\varepsilon}_{r1}(t)\tilde{\varepsilon}_{r1}(t')\rangle = P(1 + \lambda\sin 2\varphi_1)\delta(t-t'), \tag{11}$$

$$\langle\tilde{\varepsilon}_{\varphi 1}(t)\tilde{\varepsilon}_{\varphi 1}(t')\rangle = P(1 - \lambda\sin 2\varphi_1)\delta(t-t'), \tag{12}$$

$$\langle\tilde{\varepsilon}_{r1}(t)\tilde{\varepsilon}_{\varphi 1}(t')\rangle = P\lambda\cos 2\varphi_1\delta(t-t'), \tag{13}$$

Where r_1 and φ_1 represent the amplitude and phase of the laser field, respectively.

Inserting Equation (10) into Equation (6), we have:

$$\frac{d\varphi_1}{dt} = -\frac{P}{2r_1^2}\lambda\cos 2\varphi_1 + p_I(t) + \frac{1}{r_1}\varepsilon_{\varphi 1}(t)\cdot \quad (14)$$

Comparing with Equation (6), there appears a new term $-(P/2r_1^2)\lambda\cos 2\varphi_1$ in Equation (14). The phase-locking condition is:

$$\left\langle \frac{d\varphi_1}{dt} \right\rangle = 0, \quad (15)$$

Which leads to:

$$\lambda\cos 2\varphi_1 = 0, \quad (16)$$

And the condition for the phase lock to be stable is:

$$\frac{\partial}{\partial\varphi_1}\left\langle \frac{d\varphi_1}{dt} \right\rangle - \frac{1}{2}\frac{\partial^2}{\partial\varphi_1^2}\left\langle \left(\frac{1}{r_1}\tilde{\varepsilon}_{\varphi 1}(t)\right)^2 \right\rangle < 0, \quad (17)$$

Which leads to:

$$\lambda\sin 2\varphi_1 > 0. \quad (18)$$

Both Equation (16) and inequality (18) lead the laser phase to be locked at a series of values of φ_1 for the non-zero cross-correlation coefficient λ. We call them φ_{1s}, and $\varphi_{1s} = (2n+1)\pi/4$, ($n = \pm 1, \pm 2, \ldots$) and they are determined by:

$$\lambda\sin 2\varphi_{1s} = |\lambda|. \quad (19)$$

Combining Equation (5) and (9) with Equation (19), we have a Langevin equation for the amplitude r_1,

$$\frac{dr_1}{dt} = (a_1 r_1^2 - \xi r_2^2)r_1 + r_1 p_R(t) + \frac{P}{2r_1}(1-|\lambda|) + \tilde{\varepsilon}_{r1}(t)\cdot \quad (20)$$

Through a very similar deriving procedure to the Langevin equation for the amplitude r_2 reads:

$$\frac{dr_2}{dt} = (a_1 r_2^2 - \xi r_1^2)r_2 + r_2 p_R(t) + \frac{P}{2r_2}(1-|\lambda|) + \tilde{\varepsilon}_{r2}(t)\cdot \quad (21)$$

By using the relation $I = r^2$, we have obtained two Langevin equations for the laser intensity I_1 and I_2, respectively:

$$\frac{dI_1}{dt} = 2(a_1 - I_1 - \xi I_2)I_1 + 2I_1 p_R(t) + P(1-|\lambda|) + 2\sqrt{I_1}\tilde{\varepsilon}_{r1}(t), \quad (22)$$

$$\frac{dI_2}{dt} = 2(a_2 - I_2 - \xi I_1)I_2 + 2I_2 p_R(t) + P(1-|\lambda|) + 2\sqrt{I_2}\tilde{\varepsilon}_{r2}(t), \quad (23)$$

Where noise terms $\tilde{\varepsilon}_{rk}(t)$ and $p_R(t)$ are zero mean and their correlation are as follows:

$$\left\langle \tilde{\varepsilon}_{rk}(t)\tilde{\varepsilon}_{rk}(t')\right\rangle = P(1+|\lambda|)\delta(t-t'), \quad (24)$$

$$\left\langle p_R(t)p_R(t')\right\rangle = P'\delta(t-t'), \quad (25)$$

and k = 1,2.

Obviously, $\tilde{\varepsilon}_{rk}(t)$ and $p_R(t)$ are white noises with δ correlation. But allowing for that the fluctuations of the pump of the laser system may not be sufficiently rapid to be representable by a δ-correlated noise, it is reasonable to model it using a colored noise with an auto-correlation time [10]. Now, we assume that the colored noise representing the pump fluctuation is zero mean and exponential correlation as follows:

$$\left\langle p_R(t)\right\rangle = 0, \quad (26)$$

$$\left\langle p_R(t)p_R(t')\right\rangle = \frac{P'}{\tau}e^{-\frac{|t-t'|}{\tau}}, \quad (27)$$

Where τ is the auto-correlation time of the pump noise. Here has a drawback to using colored pump noise, that is the system changes to be nonmarkov process and so has no exactly analytic results. In present article, we employ the computer simulation method to study such properties of the system as the stationary probability distribution $Q_s(I)$ and the mean intensity $\left\langle I_1\right\rangle$, and further discuss the effects of the colored pump noise on the mode competition.

The computer simulation is effective to study our problem in this letter [13] and we have mastered it through the experiences in previous works. The algorithm for computer simulation developed by Fox etc. [14] is used. The details of the simulation procedure are as follows. The starting point was $I_1 = I_2 = 0$ and the time step was 0.001. Five thousand independent realizations were obtained, and a realization was comprised of 100000 different time steps. The values for $Q_s(I)$ and $\left\langle I\right\rangle$ were calculated on the basis of the datum from the 100000 different time steps and 5000 independent realizations. By the law of large numbers, the stationary probability distribution $Q_s(I)$ and the mean intensity $\left\langle I\right\rangle$ can be estimated [13] by:

$$Q_s(I) = \lim_{x\to\infty}\frac{N_1}{N} \quad (28)$$

And:

$$\left\langle I\right\rangle = \frac{1}{N}\sum_{j=1}^{\infty} I_j \quad (29)$$

Respectively, where *N* is the amount of the datum from the 100000 different time steps and 5000 thousand independent realizations, N_1 is the amount of the laser intensity which values is *I* and $\sum_{j=1}^{\infty} I_j$ means sum to all of above datum.

3. Discussion and Conclusion

The results of computer simulation are shown in Figure 1 and Figure 2. Figure 1 displays the stationary probability distributions as a function of the laser intensities for several different values of auto-correlation time of the pump noise τ. Figure 1(a) presents that the curves of stationary probability distribution, $Q_s(I_1)$, of the stronger mode in the two-mode laser system have a peak for very small value of τ, and have two peaks at different zones of the laser intensity for enough large value of τ. In addition, the probability that the laser intensity is much larger than zero gets more and more larger as the value of τ increases. Figure 1(b) shows that the curves of stationary probability distribution, $Q_s(I_1)$, of the weaker mode always have only one peak nearby zero laser intensity with increasing value of τ. So, there has a dramatic difference for the effects of the increasing of value of τ on the stationary probability distributions of two modes and it shows in Figure 1(c), too.

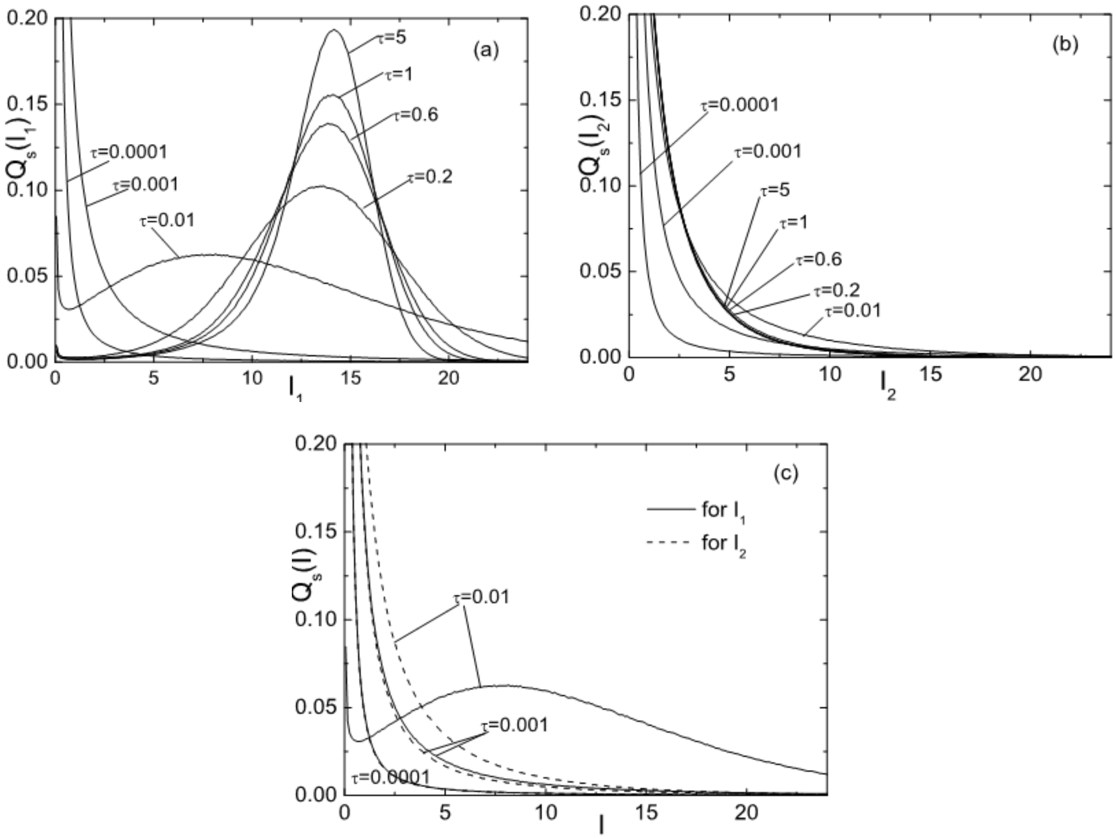

Figure 1. The stationary probability distribution of the laser intensity as a function of the laser intensity for several different values of the auto-correlation time of the colored pump noise τ with $\xi = 1$, $a_1 = 15$, $\Delta a = a_1 - a_2 = 0.8$, $\lambda = 0$, $P = 2$ and $P' = 5$: (a) The stationary probability distribution of the laser intensity $Q_s(I_1)$ gradually shifts to the larger laser intensity zone from the zone near zero intensity as τ increases; (b) The stationary probability distribution of the laser intensity $Q_s(I_2)$ always concentrates on the zone nearby zero intensity as τ increases; (c) The differences between $Q_s(I_1)$ and $Q_s(I_2)$ get larger as τ increases.

The mean laser intensity as a function of the auto-correlation time, τ, of the colored pump noise and the pump parameter a_1 show in Figure 2. It is obvious that the curves of the mean laser intensities of two laser modes coincide each other at and below threshold, however they changes in quite other ways far above threshold. The mean laser intensity of the stronger mode < I_1 > mounts up first and then moves horizontally with increasing value of τ. Meanwhile, the mean laser intensity of the weaker mode < I_2 > slightly increases first and then decreases, then moves horizontally.

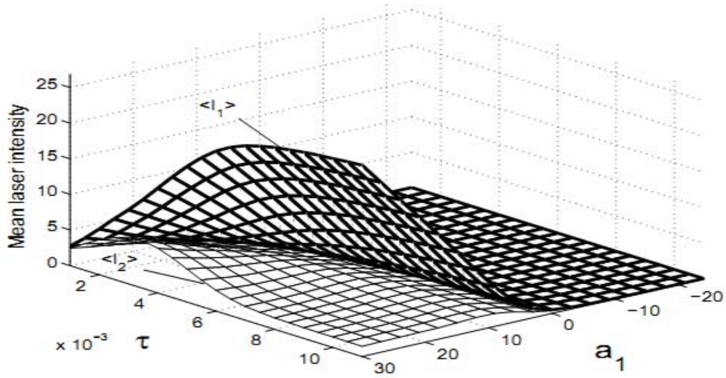

Figure 2. The Mean Laser Intensity as a Function of τ and a_1 with $\xi = 1$, $\Delta a = a_1 - a_2 = 0.8$, $\lambda = 0$, $P = 2$ and $P' = 5$.

From the Figure 1 and 2, some conclusions can be drawn when the laser system works far above threshold as follows:

(1) For very small values of τ, the stationary probability distributions of laser intensity of two modes and the mean laser intensity of two modes are the same, and there hasn't exist mode competition;

(2) Within limits of the value of τ, the stationary probability distributions of laser intensity of the stronger mode gradually shifts to the zone where the laser intensities are larger from the zone nearby zero laser intensity, while that of the weaker mode don't change. At the same time, the difference of the mean laser intensity of two modes gets larger gradually, so the mode competition displays more and more obvious as the value of τ increases;

(3) For enough large values of τ, the difference between two stationary probability distributions of laser intensity and the difference between the mean laser intensities of two modes haven't change obviously, so the mode mode competition keeps unchanged as the value of τ increases.

Acknowledgements

This work supported by the National Natural Science Foundation of China (Grant No.11047192), the Foundation of Yunnan Provincial Science and Technology Department of China (Grant No.2010ZC168), and the Foundation of Kunming University (Grant No.YJL11009).

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