TELKOMNIKA, Vol. 11, No. 9, September 2013, pp. 4871~4877
ISSN: 2302-4046

LQG Control of Unmanned Aerial Vehicles under Communication Constraints

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Abstract

This paper addresses the LQG control problem for unmanned aerial vehicles (UAV) with time delays. In particular, the case with data-rate limitations is considered. It is shown in our results that the data rate of the communication channel has important effects on the control performances. It is derived that there exists a tradeoff between the data rate and the LQG cost. A quantization, coding, and control scheme is proposed to stabilize the unstable plant. Simulation results show the validity of the proposed scheme.

Keywords: *unmanned aerial vehicles, LQG control, time delays, data rates*

1. Introduction

In the past few years, unmanned aerial vehicles (UAV) have attracted recurring interests due to their wide applications in industrial automation, intelligent transportation, and national defense. Such systems are often viewed as networked control systems (NCS) where the sensors, the controllers, and the plant are connected via a digital, wireless communication network [1-5].

Networked control has received a lot of attention in recent years. This raises many new challenges to control schemes for unmanned aerial vehicles. Related study of networked control may be traced back to [6-9].

Borkar and Mitter [10] introduced the problem of LQG control under communication constraints. Furthermore, Tatikonda, Sahai, and Mitter [11] examined the LQG performance over both noisy and noiseless channels. Liu [12] addressed the stabilization problem for unmanned air vehicles over digital and wireless communication channels with time delay.

In this paper, we consider a class of networked control problems for unmanned aerial vehicles, and address the LQG control under data-rate limitations. In particular, we design the quantizer, encoder, decoder, and controller to satisfy some given control performances of the unstable plant. Furthermore, we also consider the case with time delays, and present the bounds on the optimal LQG cost. It states that there exists a tradeoff between the data rate of the channel and the control performances.

2. Problem Formulation

We consider a class of networked control problems which arise in the coordinated motion control of unmanned aerial vehicles. The unmanned aerial vehicle (UAV) evolves in discrete-time according to:

$$X(k+1) = FX(k) + GU(k) + DW(k) \quad (1)$$

Where $X(k) \in R^n$ is the plant state process, $U(k) \in R^m$ is the control input, and $W(k) \in R^l$ is the process disturbance. *F, G,* and *D* are known constant matrices with appropriate dimensions. Furthermore, we make the following assumptions:

N_1: The states of the UAV are reachable and observable;

Received February 18, 2013; Revised May 28, 2013; Accepted June 10, 2013

N_2 : Notice that the stable part in the UAV does not play any key role on the condition for stabilization. Thus, we assume that all the eigenvalues of the system matrix F have magnitudes >1;

N_3: In the UAV, the sensors and the controller are geographically separated and connected by a wireless, digital communication network with the channel propagation delay d ;

N_4 : The initial condition *X*(0) and the disturbance *W*(*k*) are unknown random variables. *W*(0), $\cdots$, *W*(*k*) are viewed as independent and identically distributed Gaussian random variables. Furthermore, we assume that $E\|X(0)\|^2 < \varphi_X < \infty$ and $E\|W(k)\|^2 < \varphi_W < \infty$ hold.

For the UAV, our main task here is to present a quantization scheme, a coding strategy, and a control policy which can stabilize the unstable plant in the mean square sense.

$$\lim\ \sup_{k\to\infty}\ E\|X(k)\|^2 < \infty \tag{2}$$

Furthermore, an optimal LQG cost under communication constraints can be obtained in the same time. The LQG cost is give by

$$J = \lim \sup_{T\to\infty} \frac{1}{T} E\left[\sum_{k=0}^{T-1} X'(k)QX(k) + U'(k)SU(k)\right] \tag{3}$$

Where $Q \in \mathrm{R}^{n\times n}$ and $S \in \mathrm{R}^{m\times m}$ are symmetric positive definite.

We examine the inherent tradeoffs between the LQG cost and the date rate of the communication channel, and discuss the effect that the date rate has on the control performances of the UAV.

3. Quantization, Coding, and Control Schemes

Our task is to design the quantizer, encoder, decoder, and controller to satisfy some given control objectives. In particular, we will derive bounds on the achievable LQG cost in the following section. In this section, we discuss the structure of quantization, coding, and control schemes for linear discrete-time plants.

Notice that the matrix $F'F$ is real symmetric matrix. Thus, we may find a real orthogonal matrix $N \in R^{n\times n}$ which can diagonalize $F'F$. Namely, we have $F'F = N'\Phi^2 N$ with $\Phi := diag\ [\sigma_1, \ldots, \sigma_n]$. Here, let σ_i denote the *i*th singular value of the system matrix $F (i = 1, \ldots, n)$. Then we may define the transformed state as:

$$\overline{X}(k) := NX(k).$$

Let $\hat{X}(k)$ denote the estimate of the plant state $X(k)$. We may also define the transformed value of $\hat{X}(k)$ as:

$$\tilde{X}(k) := N\hat{X}(k).$$

Since both the encoder and the decoder have access to the control signals, we may define the prediction value of the plant state $\breve{X}(k)$ as:

$$\breve{X}(k) := \begin{cases} \sum_{i=0}^{d-1} F^{d-i-1}GU(k-d+i) + F^d \hat{X}(k-d), & when \quad d < k, \\ 0, & when \quad 0 \le k < d. \end{cases}$$

Let $\dot{X}(k)$ denote the centroid of the uncertain region of the plant state. Then, we may implement a quantized state feedback control law of the form:

$$U(k) = K\breve{X}(k). \tag{4}$$

Thus, we have:

$$\dot{X}(k) := F\hat{X}(k-1) + GK\breve{X}(k-1), \quad \vec{X}(k) := N\dot{X}(k).$$

Then, we define the differential value as:

$$Y(k) := \overline{X}(k) - \vec{X}(k).$$

Here, $Y(k)$ is unknown to the decoder. We will quantize it, encode it, and transmit it over a digital, wireless communication channel. The decoder may compute the estimate value of the plant state once the decoder receives the information of $Y(k)$.

Let $\hat{Y}(k)$ and $C(k)$ denote the quantization value and the quantization error, respectively. Clearly, we have

$$Y(k) = \hat{Y}(k) + C(k).$$

We consider a memoryless channel, and construct a more general encoder by solving an optimization problem [13].

$$C = \sup_{P(Y(k))} I(Y(k), \hat{Y}(k)) \tag{5}$$

Where the maximization is over the probability distribution of $Y(k)$. Here, $I(Y(k), \hat{Y}(k))$ represents the mutual information [13]. Then, we may compute code alphabet according to C.

4. LQG Control under Data-Rate Constraints

In this section, we discuss the classical LQG control problem under communication constraints, and present the optimal LQG cost. It is well known that, more information available at the decoder will lead to better LQG cost. However, there exists the inherent tradeoff between the data rate and the LQG cost. Now, we derive the lower bound on the data rate for the achievable performance.

First, we give the following lemma which comes from [13].

Lemma 1: Let $z \in R$ denote a random variable and $\hat{z}$ denote an estimate of *z*. Define *R*(*D*) as the information rate distortion function between $\hat{z}$ and *z*. The expected distortion constraint is defined as $d \in R^{+}$. Given $D \geq E\left(z - \hat{z}\right)^2$, there must exist a quantization and coding scheme if the data rate *R* satisfies the following condition:

$$R > R(D) \geq \frac{1}{2}\log_2 \frac{\sigma^2(z)}{D} \quad \text{(bits/sample)} \tag{6}$$

Where $\sigma^2(z) = E\left(z - Ez\right)^2$.

Proof: See [13].

Then, we have the following results.

Theorem 2: Consider the closed-loop system (1) with the time delay *d*. Let *R* denote the data rate of the channel. Then, the optimal state control law is given by the certainty equivalent control gain.

$$K = -(G'PG + S)^{-1}G'PF \tag{7}$$

Where P is available by solving the following equation:

$$P = F'(P - PG(G'PG + S)^{-1}G'P)F + Q. \quad (8)$$

The optimal LQG cost is given by:

$$J \le \frac{2^R}{2^R - |F|}\left(\left\|P^{\frac{1}{2}}F^d D\right\|^2 + \sum_{i=1}^{d-1}\frac{|F|}{2^R}\left\|Q^{\frac{1}{2}}F^{d-i-1}D\right\|^2\right)\varphi_W. \quad (9)$$

Proof: For the closed-loop system (1), we may obtain

$$\begin{aligned} X(k+1) &= FX(k) + GU(k) + DW(k) \\ &= F(X(k) - \hat{X}(k)) + [F\hat{X}(k) + GK\breve{X}(k)] + DW(k) \\ &= FN'C(k) + \dot{X}(k+1) + DW(k). \end{aligned}$$

This means that:

$$E\|X(k+1)\|^2 = tr(\Phi^2\Sigma_{C(k)}) + tr(\Sigma_{\dot{X}(k+1)}) + tr(D'D\Sigma_W).$$

For a given $\varepsilon \in (0,1)$, we design a coding scheme under the data-rate limitation such that:

$$\varepsilon E\|X(k)\|^2 = tr(\Phi^2\Sigma_{C(k)}) + tr(\Sigma_{\dot{X}(k+1)}) \quad (10)$$

Holds. Then, we have:

$$E\|X(k+1)\|^2 = \varepsilon E\|X(k)\|^2 + tr(D'D\Sigma_W) \le \varepsilon^k\varphi_X + \frac{1-\varepsilon^k}{1-\varepsilon}\|D\|^2\varphi_W.$$

Thus, it follows that:

$$\limsup_{k\to\infty} E\|X(k)\|^2 < \frac{1}{1-\varepsilon}\|D\|^2\varphi_W < \infty.$$

Furthermore, we set:

$$D_i = \begin{cases} \dfrac{\varepsilon}{\sigma_i^2(F)}\sigma^2(y_i(k)), & when \quad \sigma_i^2(F) > \varepsilon, \\ 0, & when \quad 0 < \sigma_i^2(F) \le \varepsilon. \end{cases}$$

Then, it follows from Lemma 1 that:

$$\sum_{i=1}^{n}\sigma^2(y_i(k)) = \sum_{i=1}^{n}\frac{\sigma_i^2(F)}{\varepsilon}\sigma^2(c_i(k)).$$

Thus, we obtain:

$$\begin{aligned} R &> \frac{1}{2}\sum_{i=1}^{n}\log_2\frac{\sigma^2(y_i(k))}{D_i} = \frac{1}{2}\sum_{i=1}^{n}\log_2\frac{\sigma^2(y_i(k))}{\frac{\varepsilon}{\sigma_i^2(F)}\sigma^2(y_i(k))} \\ &= \frac{1}{2}\sum_{i=1}^{n}\log_2\frac{\sigma_i^2(F)}{\varepsilon} = \log_2\frac{|F|}{\varepsilon}. \end{aligned}$$

Namely, we have:

$$\varepsilon < \frac{|F|}{2^R}. \quad (11)$$

Notice that:

$$\breve{X}(k) = F^d \hat{X}(k-d) + \sum_{i=0}^{d-i-1} GU(k-d+i).$$

Then, we have:

$$\breve{X}(k+1) = F\breve{X}(k) + GU(k) + F^d N'\hat{Y}(k-d+1).$$

As stated in [11], the optimal steady state control law is a linear gain of the form:

$$U(k) = K\breve{X}(k)$$

Where:

$$K = -(G'PG + S)^{-1} G'PF.$$

Here, P is available by solving the following equation:

$$P = F'(P - PG(G'PG + S)^{-1} G'P)F + Q.$$

The optimal LQG cost is obtained by:

$$\lim\sup_{T\to\infty} \frac{1}{T} E\left[\sum_{k=0}^{T-1} \breve{X}'(k)\breve{X}(k) + U'(k)SU(k)] = tr(P\Sigma_{\overline{W}})\right] \quad (12)$$

With:

$$\Sigma_{\overline{W}} := \lim\sup_{T\to\infty} \frac{1}{T}\left[\sum_{k=0}^{T-1} F^d N'\Sigma_{\hat{Y}(k-d+1)} N(F^d)'\right].$$

Notice that:

$$X(k) = \hat{X}(k) + NC(k) = \breve{X}(k) + \sum_{i=1}^{d} F^{d-i} N'\hat{Y}(k-d+i) + N'C(k) \cdot$$

Thus, we have:

$$\begin{aligned} J &= \lim\sup_{T\to\infty} \frac{1}{T} E\left[\sum_{k=0}^{T-1} X'(k)QX(k) + U'(k)SU(k)\right] \\ &= \lim\sup_{T\to\infty} \frac{1}{T} E\left[\sum_{k=0}^{T-1} \breve{X}'(k)Q\breve{X}(k) + U'(k)SU(k)\right] \\ &+ \lim\sup_{T\to\infty} \frac{1}{T} E[\sum_{k=0}^{T-1} \hat{Y}'(k-d+i)\Phi^{d-i} NQN'\Phi^{d-i}\hat{Y}(k-d+i)] \\ &+ \lim\sup_{T\to\infty} \frac{1}{T} E[\Sigma_{k=0}^{T-1} C'(k)NQN'C(k)]. \end{aligned}$$

Substitute (12) into the equation above, and obtain:

$$J \le \frac{1}{1-\varepsilon}\left(\left\|P^{\frac{1}{2}}F^{d}D\right\|^{2} + \sum_{i=1}^{d-1}\varepsilon\left\|Q^{\frac{1}{2}}F^{d-i-1}D\right\|^{2}\right)\varphi_{W}.$$

Then, it follows from (11) that:

$$J \le \frac{2^{R}}{2^{R}-|F|}\left(\left\|P^{\frac{1}{2}}F^{d}D\right\|^{2} + \sum_{i=1}^{d-1}\frac{|F|}{2^{R}}\left\|Q^{\frac{1}{2}}F^{d-i-1}D\right\|^{2}\right)\varphi_{W}.$$

□

Remark 3:

(1) Theorem 2 states that, the data rates have important effects on the LQG cost of the unstable plant. Clearly, there exists the tradeoff between the data rate and the control performance. Namely, more data rates will lead to better control performances.

(2) It is shown in our results that, the time delays have important effects on the LQG cost of the plant too. Notice that the more time delays will lead to worse control performances.

5. Numerical Example

In this section, we consider a class of networked control problems for unmanned air vehicles (UAVs). Here we present a numerical example to illustrate the proposed quantization, coding and control scheme. Here, we consider an open-loop unstable system as follows:

$$X(k+1) = \begin{bmatrix} 3.323 & 1.142 & 0.637 \\ -0.312 & 4.17 & 0.235 \\ 0.243 & 0.637 & 5.32 \end{bmatrix} X(k) + \begin{bmatrix} 2.245 \\ 0.245 \\ 1.237 \end{bmatrix} U(k) + 1.27W(k).$$

Let $X(0) = [4000, 1000, -4000]'$ and $\phi_W = 200$. A quantized feedback control policy of the form:

$$U(k) = K\breve{X}(k)$$

is employed. We compute the controller gain $K = [2.374, \; 2.453, 1.843]$. In order to illustrate the effects of the data rate and time delay on the LQG cost, we first set $R = 60\,\text{bits/s}$ and the time delay d=5. A corresponding simulation is given in Figure 1.

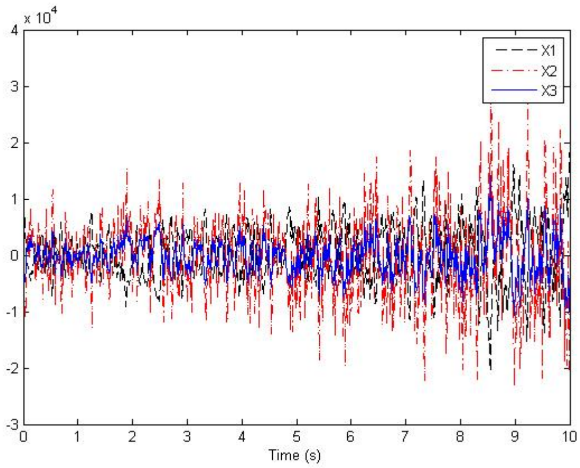

Figure 1. The Responses of System States when $R = 60\ \text{bits/s}$

Clearly, there exists no quantization, coding and control scheme to stabilize the system if the data rate is smaller than the lower bound given in Theorem 2. In order to stabilize the unstable plant, we increase the data rate of the channel, and set $R = 240\,\text{bits/s}$. A corresponding simulation is given in Figure 2. It is shown that the quantization, coding and control scheme can stabilize the unstable system if the data rate *R* is greater than the lower bound.

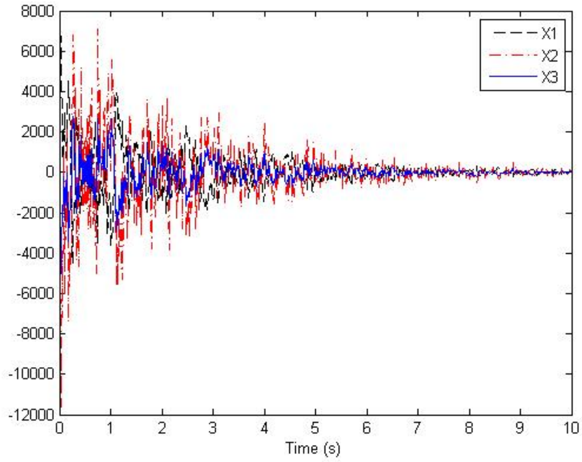

Figure 2. The Responses of System States when $R = 240\,\text{bits/s}$

6. Conclusion

In this paper, we considered the LQG control under data-rate limitation. This problem arises when the controller and the plant are connected via a digital, wireless communication channel with data-rate limitations. It was shown in our results that there exists a tradeoff between the data rate, the time delays, and the control performance. The simulation results have illustrated the effectiveness of the proposed scheme.

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