Differential equations of motion of a material point in the perpendicular plane to the plane of the gravitating disk

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ABSTRACT

This paper presents an analytical solution of the differential equations of motion of a material point in the plane perpendicular to the plane of the gravitating disk. The differential equations of the problem under study and the applied Gilden's method are described in the works of A. Poincaré. Differential equations refer to nonlinear equations. The analysis of methods for solving nonlinear differential equations was carried out. The methodology of applying the Gilden method to the solution of the differential equations under consideration can be applied in studies of the problem of the motion of celestial bodies in the "disk-material point" system in perpendicular planes. To identify the various properties of the gravitating disk, an analytical review of the state of the problem of the motion of a material point in the field of a gravitating disk is carried out. Summing up the presented review on the problem under study, a conclusion is made. The substantive formulation of the problem is described, which is formulated as follows: the study of the influence of disk-shaped bodies on the motion of a material point and methods for their solution.

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1. INTRODUCTION

The problem of the motion of celestial bodies in the "disk-material point" system, in particular, was of interest to many scientists. The task has its own practical applications. The motion of celestial bodies perpendicular to the plane of the Galaxy, obtaining the trajectory of motion, as well as studying them for stability can be used in Cosmonautics. In addition, the motion of celestial bodies in the perpendicular plane of the near-planetary disks and the equatorial plane of the Earth is very important for studying the mechanical characteristics of the motion of such celestial bodies as a comet, a meteorite [1]-[3].

When setting up numerical experiments, mathematical calculations for modeling the planet Saturn and other planets with rings, such as Uranus, Jupiter, Neptune, also when interpreting the plane of the galaxy and in the space industry, motion around disc-shaped bodies, including self-gravitating ones, is always considered [4], [5]. The purpose of this research is solution of differential equations of motion of a material point in the plane perpendicular to the plane of the gravitating disk. The tasks of this study include the following items:

– Apply the expression for the potential of the gravitating disk. found in previous studies;

- Apply Gilden's method to the investigated system “disk-material point”;
- Present the solution analytically and check the consistency of the results of the mathematical foundations with the components of the classical theory of motion in the galaxy.

2. RESEARCH METHOD

Under celestial bodies, representations of moving cosmic bodies are possible. The mathematical model of the potential of the gravitating disk in the considered case of motion of a material point in the plane perpendicular to the plane of the gravitating disk according to the Legendre formula has the form (1),

$$U(r,R,\theta)=2\pi G\rho\left[\frac{1}{r}\cdot\frac{R^2}{2}-\frac{1}{2}\frac{P_2(\cos\theta)}{r^3}\frac{R^4}{4}+\frac{1}{8}\cdot\frac{P_4(\cos\theta)}{r^5}\cdot\frac{R^6}{2}\right] \tag{1}$$

where is:
R-the radius of the gravitating round thin disk;
r-the distance from the center of the disk to the material point P;
θ-the angle between the radius vector of the material points P and the plane of the disk.

The (1) was obtained in view of the application of the fundamental theory of the potentials of bodies of celestial mechanics, detailed in [6]-[9], as the concept of the force function of the field of attraction, which is also called the potential used. The movement of a material point in the plane perpendicular to the plane of the gravitating disk occurs under the action of gravitational forces. These forces are determined by Newton's law of universal gravitation. To solve differential, the (2) describing the investigated motion:

$$\begin{cases}\ddot{r}-r\dot{\theta}^2=\frac{\partial U(r,\theta)}{\partial r}\\ \frac{d}{dt}(r^2\dot{\theta})=\frac{\partial U}{\partial\theta}\end{cases} \tag{2}$$

the Gilden method described in the works of A. Poincaré [10] is applied.

3. REVIEW OF THE RESEARCH

Nwaigwe [11] describe in detail the relevance of the investigated problem of the resonant relationship, which are perpendicular to the main plane and have an almost periodic force of action directed to the center of symmetry of the Galaxy, which can lead to a star moving away from the plane. Cases in which the stable position of the studied movement is violated is the main problem. This work clarifies the limitations that can be imposed on the shapes of elliptical galaxies from observations of their nuclear disks, and marks the most important moments of rotating stars at large radii in the disk of a galaxy with a rotating central bar. Any barred galaxy will have a ring of these stars. A complete analysis of the importance of this ring requires that the theory of this article be extended to include self-gravity of the disk. But the mechanism discussed here can help us understand the curvatures and corrugations in the gas disks of galaxies, the kinematics of stars in the vicinity of the Sun, and the sharp edges that many stellar disks have.

In the work by Bhandare and Pfalzner [12], the influence of the parabolic passage of another star on the accretion disk around one star was investigated. In direct coplanar approach, the disk is tidally exposed. In this paper, the question of the mutually perpendicular arrangement of the disk and the orbit is considered, and conclusions presented can be applied to models of double star formation.

According to Fujimoto and Tanahashi [13], the free precession of a thin self-gravitating disk of a continuous medium was investigated. The assumptions of Linden Bell that the observed bending of the outer parts of the galactic plane can be explained by the free precession of the Galaxy are confirmed. Large-scale non-circular gas motion is generated for gas at a vertical distance from the galactic equatorial plane. Much attention is paid to the almost parallel motion of the galactic plane, and the instantaneous angular velocity is analyzed when moving perpendicular to the axis of symmetry of the galaxy.

As stated by Ingram and Motta [14], the consequences of the hypothesis that the disks possessed by galaxies have less gravity than stars are investigated. This is explained by the obtained analytical solution of the problem of motion around the galactic disk: the movements outside the plane of the galaxy differ in small fluctuations from the movement of the internal parts. According to Bahcall [15], the solution of two basic Boltzmann and Poisson equations in the perpendicular axis of the galactic disk is obtained. The solutions obtained depend on the ratio of the masses of the celestial objects under consideration. These results have their own application in the distribution of starlight perpendicular to the disks. The conclusions are made by analyzing the obtained exact numerical and approximate analytical solutions.

According to Poggio *et al.* [16], it was assumed that a cosmic fall on galaxies would lead to a reorientation of the angular momentum vectors of disk galaxies. Inside the optical disk, the effect of this reorientation is modeled using a constantly changing axisymmetric representation of the potential. The considered movements are performed at an angle in the plane of the disk. The deviations obtained depend on the radius at which the surface density of the disk is low. The manifestations of the studied effect may be associated with the inclination of the core of the disk of the Milky Way.

Modern studies of extragalactic molecular gas reach the scale of giant star-forming molecular clouds. In the work by Meidt *et al.* [17], a model of three-dimensional gas motions formed by force functions from the gravitational influence of the galaxy was studied and developed. Analyzing the obtained motion models, we can conclude that the galaxy imposes preferential restrictions on the problems of star formation. Monteiro *et al.* [18] are devoted to the main theoretical issues of celestial mechanics: Measurement of the galactic potential, gravity, isothermal expansions, self-consistent solutions for ρ_0, uncertainties of local bulk density, calculation of surface mass density, simple parametrization of plausible functions Kz.

According to Matsakos and Königl [19], the time-dependent scale is determined for the considered cases of a disk having a flat orbital component. If the inclination of the disk exceeds the opening angle of the disk, then the tidal displacement inside the disk is transonic. It was found that the hydrodynamic instabilities associated with the internal shift led to additional scattering, which will allow changing the time scale.

As stated by Aslanov [20], the method of small parameters is applied to construct periodic solutions to the problem being solved, symmetric in shape and structure in the case under consideration. The dynamic compression of the body is taken as a small parameter. The main bodies are axisymmetric, the plane of symmetry of which is perpendicular to the axis of symmetry.

In the work by Huňady *et al.* [21], the results of three experiments are presented, in which oscillatory motions around a flat disk having rotation are studied. These results are characterized by more pronounced frequency response spectra. The essence of the work by Legeza [22] is explained by its difference from other studies, by the fact that the problem under study in previous sources of other authors was interpreted on geometric surfaces of the second order. To obtain an analytical solution, classical methods are used in the work. The functional of time, with the help of which the differential equations of the spatial brachistrochron are constructed, are derived analytically. The research results are illustrated graphically.

Stable accretion disks are considered, and in [23] attention is paid to the role of important elements in the physics of accretion disks. Modeling of the disk made it possible to construct all combinations of viscous and radiation processes. The work by Bu *et al*. [24] presents a classical theoretical material on the interaction between particles depending on the distance. Considered interactions between stars and other gravitating masses in the dynamics of gravitating systems should be decisive. The paper analyzes the properties of individual stars, the motion of stars, solar and lunar eclipses using the laws of Newtonian mechanics.

The work by Tenjes *et al* [25] has a novelty and relevance in that the researchers proposed a new method for the analytical solution of gas-dynamic equations about completely stable motions around the disk galaxy. Limitations on the application of the proposed method within the framework of mechanical and geometric structures and properties of the disk are given. The differential equations of various technological processes and deterministic chaotic systems were deciphered in the research [26]-[29]. The most important is the study of the random nature of chaotic signals and images.

In the work by Siddiki *et al.* [30], [31], concentric rings were chosen for the disk model, and calculations were performed for the moments of motion in the nonlinear mode, it was shown that stable, strongly curved precessing equilibrium states are possible. These equilibrium configurations obey the scaling relation and depend on the disk frequency response. The main result is that due to self-gravity, the considered types of disks retain their state in a temporary mode independently from outside.

A new method for determining the mass density of the galactic disk surface is shown in [32]. To apply the above method, it is necessary to take into account the physical characteristics of this system. The analysis is carried out for more suitable cases of application of this method.

According to Parker [33], the results of a study of the propagation of hydromagnetic waves with low frequency phenomena are presented, and the derivation of the equation that makes up the mathematical model of this problem is obtained. For the results obtained, it is important to take into account self-gravity. As stated by Koh *et al*. [34], a method is described that combines the solution of problems in one system of rotating and stationary disks. The paper describes in detail the comparison of the analytical solution with the obtained numerical implementation.

According to Agapitou *et al*. [35], the electromagnetic properties of a flat thin circular disk consisting of concentric rings were studied, which can lead to the manifestation in classical stars. The solution of nonlinear spiral-like phenomena in protoplanetary disks is described in [36]. The phenomenon leads to a rapid gas propagation in a perpendicular direction to the middle plane of the disk.

The work by Tenorio-Tagle and Bodenheimer [37] is devoted to the study of the interstellar medium, their distribution density in the spiral structures of the Galaxy, taking into account chemical and physical properties. As stated by Falco *et al*. [38], the results of the study of the continuity equation of the flow density by geodesic characteristics and their approximation are presented. After an adequate approximation of the trajectories of individual photons for this problem, approximate solutions of the continuity equation are obtained. The particles in the medium under study, moving towards the center of the disk in a perpendicular direction, receive a large acceleration, which is not almost justified by the relativistic theory.

According to Ghosh and Lamb [39], the results are analyzed, systematized and presented, which consist in the location of the disk depending on the integral magnetic voltage acting on the disk medium. In paper by Merrifield [40], the distribution of the line-of-sight velocities of a disk galaxy with its obverse side was investigated. In the problem under study, the obtained mathematical expressions of the density distribution model of a Galaxy perpendicular to the plane lead to significantly different forms of the observed change in the law of motion depending on time.

4. RESULT AND DISCUSSION

4.1. Role of auxiliary symbols

To solve differential equations, we use the Gilden method described in the works of A. Poincaré [10]. According to the proposed choice of the independent variable A. Poincaré should be taken so that the equations of motion have a form similar to the equations of motion of a material point in the plane of the disk. For this, we accept the conventions of the Gilden method [10] by (3):

$$\frac{d\theta_0}{dt} = \frac{\sqrt{c}}{r^2}, \tag{3}$$

where c is a new constant.

If we take θ_0 as an independent variable, then the second of (2) will be written in the form and instead of the variable t and the polar angle θ using (3) to enter the independent variable θ_0 and the reciprocal distance u, then a transformed system of equations representing the mathematical model is obtain (4) and (5):

$$\frac{d^2\theta}{d\theta_0^2} = \frac{GM}{c} R^2 u \left[\left(\frac{3}{8} - \frac{5}{8} R^2 u^2\right) \cdot \sin 2\theta - \frac{35}{64} R^2 u^2 \sin 4\theta\right] \tag{4}$$

$$\frac{d^2 u}{d\theta_0^2} + u\left(\frac{d\theta}{d\theta_0}\right)^2 - \frac{GM}{c} = -\frac{GM}{c}\begin{bmatrix}\left(\frac{3}{2^4} R^2 u^2 + \frac{45}{2^9} R^4 u^4\right) + \left(\frac{9}{2^4} R^2 u^2 - \frac{25}{2^7} R^4 u^4\right) \cdot \cos 2\theta - \\ -\frac{175}{2^9} R^4 u^4 \cos 4\theta\end{bmatrix} \tag{5}$$

The analogy with the equation of motion of a point in the plane of the disk will become even more obvious [10] if we note that in subsequent calculations θ it will differ little from θ_0. From (5), taking into account the order of smallness of the expression as the perturbing function, a transformed mathematical model of the considered motion is obtained (6).

$$\begin{aligned} &\frac{d^2 u}{d\theta_0^2} + u - \frac{\mu}{c} = -\frac{\mu}{c}\begin{bmatrix}\frac{3}{2^4} R^2 u^2 + \frac{45}{2^9} R^4 u^4 + \\ + \left(\frac{9}{2^4} R^2 u^2 - \frac{25}{2^9} R^4 u^4\right) \cos 2\theta - \\ -\frac{175}{2^9} R^4 u^4 \cos 4\theta\end{bmatrix} + u\left[1 - \left(\frac{d\theta}{d\theta_0}\right)^2\right] \\ &\frac{d^2\theta}{d{\theta_0}^2} = \frac{\mu}{c} R^2 \left[\left(\frac{3}{8} u - \frac{5}{8} \cdot R^2 u^3\right) \sin 2\theta - \frac{35}{64} \cdot R^2 u^3 \sin 4\theta\right] \end{aligned} \tag{6}$$

The choice of the independent variable, which has clear advantages, is not without its drawbacks. The coordinates u and θ are expressed as functions of using θ_0 the equations of system (6), the left-hand sides of which have a simple form $\frac{d^2\theta}{d{\theta_0}^2}$ and $\frac{d^2 u}{d{\theta_0}^2} + u + \frac{\mu}{c}$, and the right-hand sides depend not only on u and θ, but also on θ_0. The variable is related to time t by (3). It remains to choose the first approximation by the Gilden choice in the same way as the Keplerian motion [10]. In this case (7):

$$\left.\begin{aligned}\theta &= \theta_0\\ u &= \frac{\mu}{c} + \alpha\cos\theta_0 + \beta\sin\theta_0\end{aligned}\right\}, \tag{7}$$

where α, β are the constants of integration.

4.2. Obtaining an analytical solution

The differential equation of system (6) after substitution into the right side of the expression for u and θ_0 from (7), expressed through θ_0, can be transformed with the right side in the form of harmonics with coefficients $c_0, c_i, d_i, i = \overline{1,7}$ and can be determined by the following expressions depending on $\frac{\mu}{c}, \alpha, \beta$ (8):

$$\begin{aligned}
c_0 &= -\frac{15}{16}\left(\frac{\mu}{c}\right)^2 R^4\alpha\beta,\\
c_1 &= -\frac{12}{16}\frac{\mu}{c}R^2\beta - \frac{345}{512}\frac{\mu}{c}R^4\alpha^2\beta - \frac{45}{512}\frac{\mu}{c}R^4\beta^3,\\
d_1 &= \frac{3}{16}\frac{\mu}{c}R^2\alpha - \frac{15}{10}\left(\frac{\mu}{c}\right)^3 R^4\alpha - \frac{115}{512}\frac{\mu}{c}R^4\alpha^3 - \frac{135}{512}\frac{\mu}{c}R^4\alpha\beta^2,\\
c_2 &= -\frac{70}{192}\left(\frac{\mu}{c}\right)^2 R^4\alpha\beta,\\
d_2 &= \frac{3}{8}\left(\frac{\mu}{c}\right)^2 R^2 - \frac{5}{8}\left(\frac{\mu}{c}\right)^4 R^4 - \frac{345}{256}\left(\frac{\mu}{c}\right)^2 R^4\alpha^2 - \frac{135}{256}\left(\frac{\mu}{c}\right)^2 R^4\beta^2,\\
c_3 &= \frac{15}{128}\left(\frac{\mu}{c}\right)^3 R^4\beta - \frac{15}{512}\frac{\mu}{c}R^4\alpha^2\beta - \frac{15}{512}\frac{\mu}{c}R^4\beta^3,\\
d_3 &= -\frac{225}{128}\left(\frac{\mu}{c}\right)^3 R^4\alpha - \frac{225}{512}\frac{\mu}{c}R^4\alpha - \frac{225}{512}\frac{\mu}{c}R^4\alpha\beta^2,\\
c_4 &= 0,\\
d_4 &= -\frac{175}{128}\left(\frac{\mu}{c}\right)^2 R^4\alpha^2 - \frac{45}{128}\left(\frac{\mu}{c}\right)^2 R^4\beta^2 + \frac{15}{16}\left(\frac{\mu}{c}\right)^2 R^4\alpha\beta - \frac{35}{64}\left(\frac{\mu}{c}\right)^4 R^4,\\
c_5 &= \frac{105}{128}\left(\frac{\mu}{c}\right)^3 R^4\beta + \frac{225}{512}\frac{\mu}{c}R^4\alpha^2\beta + \frac{65}{512}\frac{\mu}{c}R^4\beta^3,\\
d_5 &= -\frac{105}{128}\left(\frac{\mu}{c}\right)^3 R^4\alpha - \frac{155}{512}\frac{\mu}{c}R^4\alpha^3 + \frac{15}{512}\frac{\mu}{c}R^4\alpha\beta^2,\\
c_6 &= \frac{105}{128}\left(\frac{\mu}{c}\right)^2 R^4\alpha\beta,\\
d_6 &= -\frac{105}{256}\left(\frac{\mu}{c}\right)^2 R^4\alpha^2 + \frac{105}{256}\left(\frac{\mu}{c}\right)^2 R^4\beta^2,\\
c_7 &= \frac{105}{512}\frac{\mu}{c}R^4\alpha^2\beta - \frac{35}{512}\frac{\mu}{c}R^4\beta^3,\\
d_7 &= -\frac{35}{512}\frac{\mu}{c}R^4\alpha^3 + \frac{105}{512}\frac{\mu}{c}R^4\alpha\beta^2.
\end{aligned} \tag{8}$$

Simplifications of the first differential equation of the system (6) taking into account the Gilden choice (7) lead it to an inhomogeneous linear differential equation of the second order (9):

$$\frac{d^2u}{d\theta_0^2} + u = \frac{\mu}{c} - \frac{\mu}{c}\left[\begin{array}{l}\left(\frac{3}{16}R^2 + \frac{9}{16}R^2\cos 2\theta_0\right)\left(\frac{\mu}{c} + \alpha\cos\theta_0 + \beta\sin\theta_0\right)^2 + \\ + \left(\frac{45}{512}R^4 - \frac{25}{128}R^4\cos 2\theta_0 - \frac{175}{512}R^4\cos 4\theta_0\right)\times \\ \times\left(\frac{\mu}{c} + \alpha\cos\theta_0 + \beta\sin\theta_0\right)^4\end{array}\right] + \left(\frac{\mu}{c} + \alpha\cos\theta_0 + \beta\sin\theta_0\right)\left(1 - \left(\frac{d\theta}{d\theta_0}\right)^2\right) \tag{9}$$

similarly, the above description of the transformation for the second differential (6) can be determined by the following expressions depending on $\frac{\mu}{c}, \alpha, \beta$. the coefficients are determined by the following expressions depending on $a_i, b_i, e_i, f_i, i = \overline{1,9}$ (10):

$$\begin{aligned}
e_1 &= 2c_0d_1\frac{\mu}{c} - c_0d_2\alpha + \frac{1}{2}c_0c_1\beta\\
a_1 &= \alpha - \frac{15}{16}\left(\frac{\mu}{c}\right)^2 R^2\alpha - \frac{5}{2^7}\left(\frac{\mu}{c}\right)^4 R^4\alpha + \frac{65}{2^9}\left(\frac{\mu}{c}\right)^2 R^4\alpha^3 - \frac{65}{2^8}\left(\frac{\mu}{c}\right)^2 R^4\alpha\beta^2 + \alpha\sum_{i=1}^{7}\frac{c_i^2+d_i^2}{2i^2} + \\
&\quad \frac{\mu}{c}\sum_{i=1}^{6}\frac{c_ic_{i+1}+d_id_{i+1}}{2i(i+1)} + \frac{1}{2}\alpha\sum_{i=1}^{5}\frac{c_ic_{i+2}+d_id_{i+2}}{2i(i+2)} + \frac{1}{2}\beta\sum_{i=1}^{5}\frac{c_id_{i+2}-c_{i+2}d_i}{2i(i+2)} - \frac{1}{4}\beta c_1d_1
\end{aligned} \tag{10}$$

under these assumptions, the solution can be represented (11):

$$u = u_0 + u' + u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8 + u_9 + u_{10} + u_{11} + u_{12} + u_{13} + u_{14} + u_{15} \tag{11}$$

in (11), each term has a mathematical meaning and represents a solution (12).

$$\begin{aligned} u_0 &= \alpha \cos\theta_0 + \beta \sin\theta_0, \\ u' &= c_0 \frac{\mu}{c} {\theta_0}^2 + a_0\theta_0 + b_0 - 2c_0 \frac{\mu}{c}, \\ u_1 &= \left(-\frac{c_0}{6}{\theta_0}^2 + \frac{c_0 - f_1}{4}\theta_0 + \frac{e_1 + c_0 - 2b_1}{4}\right)\cos\theta_0 + \left(\frac{c_0}{6}\theta_0^2 + \frac{e_1 + c_0}{4}\theta_0 + \frac{2a_1 + f_1 - c_0}{4}\right)\sin\theta_0 \\ u_k &= \left(\frac{e_k}{1-k^2}\theta_0 + \frac{a_k(1-k^2) - 2k\cdot f_k}{(1-k^2)^2}\right)\cos k\,\theta_0 + \left(\frac{f_k}{1-k^2}\theta_0 + \frac{b_k(1-k^2) + 2k\cdot e_k}{(1-k^2)^2}\right)\sin k\,\theta_0, \quad k = \overline{2,\ 8} \\ u_m &= \frac{1}{1-m^2}(a_k \cos m\,\theta_0 + b_k \sin m\,\theta_0), \quad m = \overline{9,\ 15} \end{aligned} \tag{12}$$

According to Gilden's interpretation of the application of the method from [10], it is noted that in subsequent approximations outside the signs of trigonometric functions even higher degrees will be encountered θ_0, that the use of a variable θ_0 does not significantly change the nature of the old methods. When strengthening the requirements of researchers for a variable θ_0, as soon as in the form of an argument of trigonometric functions, it is necessary to resort to other artificial methods. The only advantage given by the Gilden choice θ_0, leaving aside the disadvantages mentioned above, is that the equations of motion become linear or integrable by quadratures [10]. Taking into account that the opposite statement, and the last expressions, a graphical visualization of the obtained analytical solution to the problem of the motion of a material point in a plane perpendicular to the plane of the gravitating disk under certain conditions, the trajectory of motion is built (Figure 1).

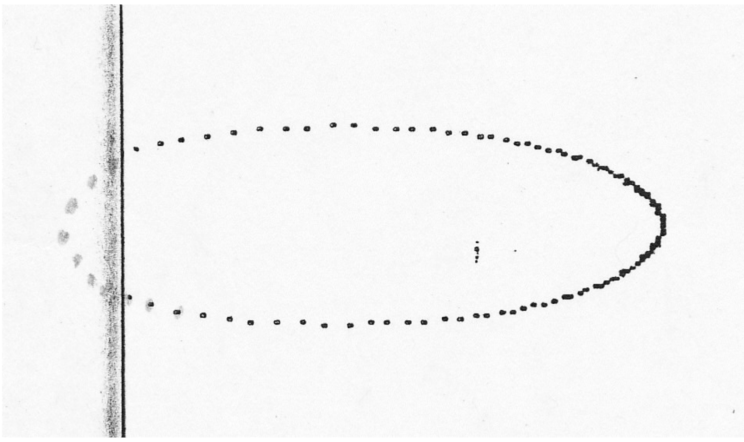

Figure 1. The trajectory of a material points in a plane perpendicular to the plane of the gravitating disk

5. CONCLUSION

The solution of the differential equations of motion of a material point in a plane perpendicular to the plane of the gravitating disk is obtained by the Gilden method, considered as a new method of celestial mechanics in the works of A. Poincaré. The method is based on the idea of constructing the first approximation close to Keplerian motion, thereby reducing the solution of the system of obtained differential equations to the simplest differential equations. Gilden's method for solving the differential equations of motion of a material point in the plane perpendicular to the plane of the gravitating disk, the methodology can be successfully applied in the development of software and numerical modeling of control systems for a spacecraft.

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