
A Watermarking Method Based on Optimization Statistics

Xiuhu Tan¹, Defa Hu²

¹Journalism Department, Chengdu Sport University, Chengdu 610041, Sichuan, China

²Department of Information, Hunan University of Commerce, Changsha 410205, Hunan, China
Corresponding author, e-mail: 252441934@qq.com¹, hdf666@163.com²

Abstract

This paper presents a robust image watermarking scheme based on optimization Statistics. This method aims to select a feature space, which has the greatest robustness against various attacks after watermarked. We separately obtain the feature spaces through calculating the statistical property of the digital image. Passing spaces decomposing and reconstructing of the feature spaces, constructing the embedding matrix, we obtain that the robustness of the approach lies in hiding a watermark in the subspace that is the least susceptible to potential modification; and realize the optimization statistics of the embedding watermark. Through analysis and constraint the conditions of subspace, the algorithm we proposed can obtain a high detection probability and security, a low false alarm probability. Experimental results show that the proposed scheme is robust against a kind of attacks.

Keywords: digital watermark, feature space, optimization statistics, security and robustness

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1. Introduction

In the past decades, digital watermarking techniques have received much attention in the community for a variety of applications, such as copyright protection, authentication, quality evaluation [1], etc. Digital image watermarking can be divided into three categories [2]: robust watermarking, semi-fragile watermarking and fragile watermarking. In addition, as to the energy of embedded watermark, balance must be made between the security and robustness of digital watermark. However, a more exact understanding is that, because the watermark embedder fully understands the host image prior to watermark detection, the host image should not be simply understood as a pure strong noise.

The time-frequency representation is used to select the dynamic regions of the host signal where the presence of watermark is difficult to perceive. Data rate, robustness and embedded visual imperceptibility are necessary issues for consideration during algorithm design of every robust digital image watermark. In addition, as to the energy of embedded watermark, balance must be made between the safety and robustness of digital watermark. For the detection of digital watermark, a host image is often understood as a strong noise, and watermark detection is like the detection of weak signal against the background of strong noise. However, a more exact understanding is that, since the watermark embedder fully understands the host image prior to watermark detection, the host image should not be simply understood as a pure strong noise [3]. However, a more exact understanding is that, since the watermark embedder fully understands the host image prior to watermark detection, the host image should not be simply understood as a pure strong noise [4]. Therefore, the watermark embedder should utilize the features of host image to embed the watermark. Most previous research methods also focus on that. Nevertheless, according to the signal detection theory, watermark detection resembles optimal consistency detection in a noise channel with known features to select the optimal transmission signal [5].

In this algorithm, analyzing the statistical features of random errors of embedded watermark, and conducting space decomposition and reconstructing the statistical features of random errors, the watermark is embedded into the reconstructed space so that the error energy can be minimally mapped into the reconstructed space.

2. Mathematical Problem

In order to build up the asymmetric watermark embedding algorithm, we propose and prove the following deductions.

Assumption 1: Set matrix $W_1, W_1 \in R^{L \times g}$, if $L \leq g$ any $rank(W_1) = L$, namely W_1 is full –rank matrix. To the random unitary matrix $U, U \in R^{L \times L}$, there is always the existing of unitary matrix $V, V \in R^{g \times g}$, and V is satisfied the following equation:

$$UW_1 = W_1V^T \quad (1)$$

And if $L \geq g$ and $rank(W_1) = g$, namely W_1 is full –column matrix. To the random unitary matrix $V, V \in R^{g \times g}$, there is always the existing of unitary matrix $U, U \in R^{L \times L}$, and V is satisfied the following equation:

$$W_1V = U^TW_1 \quad (2)$$

And it is unique existence of the matrix V in equation (1) and U in equation (2), if the condition of $L - g = \pm 1$ or $L = g$ is satisfied.

Demonstration: Suppose W_1 matrix $rank(W_1) = g_1$, conduct SVD division to W_1 and obtain the following equation:

$$W = U_w S_w V_w^T \quad (3)$$

Both $U_w \in R^{g_1 \times g_1}$ and $V_w \in R^{g_2 \times g_2}$ are unitary matrixes, diagonal matrix $S_w \in R^{g_1 \times g_2}$, as $rank(W_1) = g_1$, so the main diagonal of S_w is the positive number greater than zero, namely full column rank. Select the unitary matrix $U_{w1} \in R^{g_1 \times g_1}$ arbitrarily, suppose $U_{w1} = (a_1, a_2, \dots, a_{g_1})$, $a_i (i = 1 \dots g_1)$ is the row coordinate of $g_1 \times 1$. For $V_{w1}^T = (b_1, b_2, \dots, b_L, \dots, b_{g_2})$, $b_i (i = 1 \dots g_2)$ is the $g_2 \times 1$ row coordinate. Suppose V_{w1}^T row coordinate meets the following equation:

$$\begin{cases} b_i = (a_i^T, 0 \dots, 0)_{g_2 \times 1}^T & \text{if } 1 \leq i \leq g_1 \\ b_j^T b_i = 0 (j \neq i), b_j^T b_j = 1 & \text{if } L \leq j \leq g_2, 1 \leq i \leq g_2 \end{cases} \quad (4)$$

Then the following equation comes into existence:

$$U_{w1} S_w = S_w V_{w1} \quad (5)$$

Then conducts the simple matrix transforms, the existence of (5) equals the arbitrary selection of unitary matrix U , then there is the unitary matrix V^T meeting the following equation:

$$UW_1 = W_1V^T \quad (6)$$

Similarly, we could prove that when W_1 matrix $rank(W_1) = g_2$, (2) comes into existence.

Citation of expansion theorem of matrix:

Let the matrix $X_1 = (x_{i,j})_{m \times n}$ expand X_1 according to the following formula:

$$cs(X_1) = (x_{11}x_{21}\dots x_{m1}x_{12}x_{22}\dots x_{m2}\dots x_{mn})^T \quad (7)$$

It is the expanded X_1 column.

3. Analysis of Watermark Error Energy

Due to the unpredictability and randomness of such operations as noise and image processing, the watermark detection of one image is just like detecting a sample in a random signal [6]. For a watermark embedder, the watermark and the host image are known, so it is necessary for the watermark embedder to learn about and utilize the statistical features of image errors of embedded watermarks and select the optimal mode so that the error energy can be minimally mapped into the embedded watermark.

3.1. Analysis of Statistical Features of Image Error of Embedded Watermark

For obtaining the statistical features of image error of embedded watermark, we embed in advance the watermark W_0 in the time domain into the host image I to obtain the image I_w . By various image operations, such as adding noise and conducting attacks to I_w , one group of images (as the random process I_w^1) are obtained to calculate and obtain the mathematic expectation I_p of this group of images. Expand I_w and I_p as the matrix column to obtain the column vector $cs(I_w)$ and $cs(I_p)$ of $N \times 1$ dimensions. Let $\vec{\varepsilon}$ be the random error vector.

$$\vec{\varepsilon} = cs(I_w^1) - cs(I_w) \quad (8)$$

Taken expectation and covariance matrixes from the above formula to obtain:

$$E(\vec{\varepsilon}) = E(cs(I_w^1) - cs(I_w)) = E(cs(I_w^1)) - E(cs(I_w)) = cs(I_p) - cs(I_w) \quad (9)$$

$$\text{cov}(\vec{\varepsilon}) = E((cs(I_w^1) - cs(I_p))(cs(I_w^1) - cs(I_p))^T) = \begin{bmatrix} d_{11}d_{12}\dots d_{1N} \\ d_{21}d_{22}\dots d_{2N} \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ d_{N1}d_{N2}\dots d_{NN} \end{bmatrix} \quad (10)$$

Where, $\text{cov}(\vec{\varepsilon})$ is the covariance matrix of $\vec{\varepsilon}$, and the main diagonal element $d_{ii} \geq 0$ ($i = 1, 2, \dots, N$) is the variance corresponding to each dimension of the random error vector $\vec{\varepsilon}$. $E(\vec{\varepsilon})$ is the direct-current component of error, and $\text{cov}(\vec{\varepsilon})$ is the covariance between components of the random error vector $\vec{\varepsilon}$.

3.2. Decomposition of Error Space

We first analyze the direct current component of error. $E(\vec{\varepsilon})$ is the column vector of $N \times 1$ dimensions. According to matrix knowledge [7], it is known that $E(\vec{\varepsilon})$ can be linearly expressed by the orthogonal space of $N \times N$ dimensions; that is to say, it can be linearly expressed by the orthogonal basis of N units of $N \times 1$ dimensions (there are many orthogonal bases of such units). However, what we concern more is to find the orthogonal basis ($0 < m_1 < N$) of units of $N - m_1$ dimensions so that $E(\vec{\varepsilon})$ can form orthogonal intersection with the space spread by the orthogonal basis of units of $N - m_1$ dimensions; that is to say, the

projection of the vector $E(\vec{\varepsilon})$ at the orthogonal basis of units of $N - m_1$ dimensions is zero. When the watermark is embedded into the space spread by the orthogonal basis of units of $N - m_1$ dimensions, the energy of error direct current component $E(\vec{\varepsilon})$ projected at the embedded watermark is zero. For this purpose, let the matrix B be [8],

$$B = E(\vec{\varepsilon}) \times E(\vec{\varepsilon})^T \quad (11)$$

The matrix B is a symmetric matrix. Conduct singular value decomposition (SVD) to the matrix B to obtain the unit orthogonal matrix U (the matrix B is a symmetric matrix, and the two orthogonal matrixes after SVD are the same),

$$B = U \Lambda U^T = U \begin{bmatrix} \lambda_{11} & 0 & \cdots & 0 \\ 0 & \lambda_{22} & \cdots & 0 \\ \vdots & \cdots & \ddots & \cdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}_{N \times N} U^T \quad (12)$$

Λ is a diagonal matrix, and $\lambda_{11} \geq \lambda_{22} \geq \cdots \geq 0$. Suppose that there are m_1 singular values greater than zero, and that the space spread by the unit orthogonal basis of the first m_1 columns in the corresponding U is X_1 , then the space spread by the unit orthogonal basis of the $N - m_1$ dimensions corresponding to the other $N - m_1$ zero singular values is Y_1 , and X_1 and Y_1 are orthogonal complement space to each other. If the watermark is embedded into the Y_1 space, then the energy of error direct current component $E(\vec{\varepsilon})$ projected at the embedded watermark is zero. Next, we analyze the other components of error. $\text{cov}(\vec{\varepsilon})$ contains the variance (alternating current energy) between the same component of $\vec{\varepsilon}$ as well as the covariance between different components, equal to the perturbed distribution of error of zero mean $\vec{\varepsilon}$ (without direct current component) at the space of $N \times N$ dimensions. Conduct singular value decomposition (SVD) to $\text{cov}(\vec{\varepsilon})$ to obtain the unit orthogonal matrix V ($\text{cov}(\vec{\varepsilon})$ is a orthogonal matrix).

$$\text{cov}(\vec{\varepsilon}) = V \Lambda_1 V^T = V \begin{bmatrix} \sigma_{11} & 0 & \cdots & 0 \\ 0 & \sigma_{22} & \cdots & 0 \\ \vdots & \cdots & \ddots & \cdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}_{N \times N} V^T \quad (13)$$

Λ_1 is a diagonal matrix, and $\sigma_{11} \geq \sigma_{22} \geq \cdots \geq 0$. Suppose that there are m_2 ($0 < m_2 < N$) singular values greater than zero, and that the space spread by the unit orthogonal basis of m_2 dimensions in the corresponding V is X_2 , then the space spread by the unit orthogonal basis of the $N - m_2$ dimensions corresponding to the other $N - m_2$ zero singular values is Y_2 , and X_2 and Y_2 are orthogonal complement space to each other. According to Formula (7), it is known that, if the watermark is embedded into the Y_2 space, then the energy of the zero mean vector $\vec{\varepsilon}$ (without direct current component) projected at the embedded watermark is zero.

3.3. Space Reconstruction

We first analyze for simultaneously making the energy of error direct current component $E(\vec{\varepsilon})$ and the energy of error perturbed component $\text{cov}(\vec{\varepsilon})$ projected at the embedded watermark are zero, we reconstruct the space for watermark embedding. Let the matrix C be,

$$C = [X_1 X_2] \quad (14)$$

$C \in R^{N \times (m_1 + m_2)}$, and the rank of C is:

$$\max(m_1, m_2) \leq \text{rank}(C) \leq m_1 + m_2 \leq N \quad (15)$$

Where, $\max(m_1, m_2)$ is the greater value between m_1, m_2 . According to Formula (9) and Formula (10), it is known that $\text{rank}(B)$ and $\text{rank}(\text{cov}(\vec{\varepsilon}))$ are usually small; that is to say, m_1 and m_2 are small. Then, $m_1 + m_2 \leq N$ (if the numerical value of $m_1 + m_2$ is big, the threshold of singular value of Λ and Λ_1 can be raised to make $m_1 + m_2 \leq N$). According to formula (15), solve the following formula:

$$C^T x = 0 \quad (16)$$

Where, $x \in R^{N \times 1}$ dimension column vector. Solve Equation (10) to obtain the unit orthogonal solution vector of $N - \text{rank}(C)$ dimensions. Suppose that the space spread by the unit orthogonal solution vector of $N - \text{rank}(C)$ dimensions is Y , and that the corresponding unit orthogonal matrix $G \subset R^{N \times (N - \text{rank}(C))}$, then X and Y are the subspaces of space Z and the space Y and space X are orthogonal complement space to each other. If we embed the watermark to the space Y , then the energy of error direct current and error perturbed component projected at the embedded watermark is zero, achieving the statistically optimal embedding of watermark.

4. Optical Digital Watermarking Algorithm based on Space Reconstruction

As for original image $I \in R^{m \times n}$, carry out DCT2 [9] transformation to I to get A , $A \in R^{m \times n}$ and $\text{cs}(A) \in R^{m \times 1}$. As for I , embed the digital watermarking in the time domain and perform various operations (such as rotation, fuzziness and sharpening) to get a group of images. Calculate the mathematic expectation $\text{cs}(A_p)$ of this group of images in DCT2 domain to get the random error vector $\vec{\varepsilon}_1$. Carry out space decomposition and space reconstruction to the random error vector $\vec{\varepsilon}_1$ to get Z space. Divide Z into two subspaces, space Y and orthogonal complement space X . Set matrix $G_1, G_1 \in R^{m \times Lg}$ ($L \leq m, g \leq n$), to be orthogonal basis of any Lg unit of Y space. G_1 constructs cryptographic key, watermark embedding matrix. G_2^T and $\text{cs}(W_1)$ are public keys, watermark detection matrix.

4.1. Space Reconstruction Embedding and Extraction of Watermark

As for watermark $W, W \in R^{l \times g}$, embed W into A through the following formula, and get:

$$\text{cs}(A_w) = \text{cs}(A) + k_1 \times G_1 \times \text{cs}(W) \quad (17)$$

Where, k_1 represents coefficient of embedding strength. Meanwhile, to obtain high signal to noise ratio (SNR), regulate positive number k_1 to make the watermark embedding strength $k_1 \|cs(W)\|$ to be as high as possible. However, $k_1 \|cs(W)\|$ should not obviously influence the visual effect of the image. Then produce the image with watermark I_w through DCT2 reconstruction. Set: take any q unit orthogonal basis from Z space again to construct matrix $P, P \in R^{mn \times q}$, which should satisfy the following formula:

$$P^T \times G_1 = 0_{q \times Lg} \quad (18)$$

$0_{q \times Lg}$ is $q \times Lg$ all-zero matrix. Construct the matrix G_2 as per the following [10]:

$$G_2 = G_1 + P \times P_1 \quad (19)$$

Among which $P_1 \in R^{q \times Lg}$, and $G_2^T cs(A_w) = 0$, thus blind extraction is realized. Public key $(cs(W), G_2^T)$ is for detection. Set the obtained image feature to be A_w^* . The watermark detection can be done with the public key through the following formula.

$$\begin{aligned} (cs(A_w^*) - cs(A)) &= (G_1 + P \times P_1)^T \times G_1 \times cs(W^*) = (G_1^T + P_1^T \times P^T) \times G_1 \times cs(W^*) \\ &= G_1^T \times G_1 \times cs(W^*) + P_1^T \times P^T \times G_1 \times cs(W^*) = E_{lg \times lg} \times cs(W^*) + P_1^T \times 0_{q \times Lg} \times cs(W^*) \\ &= cs(W^*) + 0_{Lg \times 1} = cs(W^*) \end{aligned} \quad (20)$$

Then we do relevant judgment. Though the attacker can analyze out the space division, X space and Y space, he cannot calculate out the embedded matrix G_1 from the public keys G_2 and $cs(W)$. Thus he cannot remove the embedded watermark, and the safety of the embedded watermark is ensured.

4.2. Detection Probability and False Alarm Probability

Use relevant judgment for detection function. As for detected image I_g , perform DCT2 transformation to I_g to get A_g , and make A_g through G_2^T projection and get:

$$G_2^T \times cs(A_g - A) = c \times cs(W) + n \quad (21)$$

c represents the coefficient of vector $I_g - I$ passing through G_2^T projection, and G_2^T represents error. Set the judgment function to satisfy the following conditions,

$$\begin{cases} \|n\| \ll \|cs(W)\| & \text{if } cs(A_g - A) \text{ through } G_2 \text{ project including } cs(W) \\ |c| \rightarrow 0 & \text{if } cs(A_g - A) \text{ through } G_2 \text{ project no including } cs(W) \end{cases} \quad (22)$$

Use the relevant detection function,

$$sim(cs(W), G_2^T cs(A_g - A)) = \frac{|cs(W)^T (G_2^T cs(A_g - A))|}{\|cs(W)\| \|G_2^T \times cs(A_g - A)\|} \quad (23)$$

Detection probability: if $I_g - I$ contains W after passing through G_2^T projection, and from Formula (22), get,

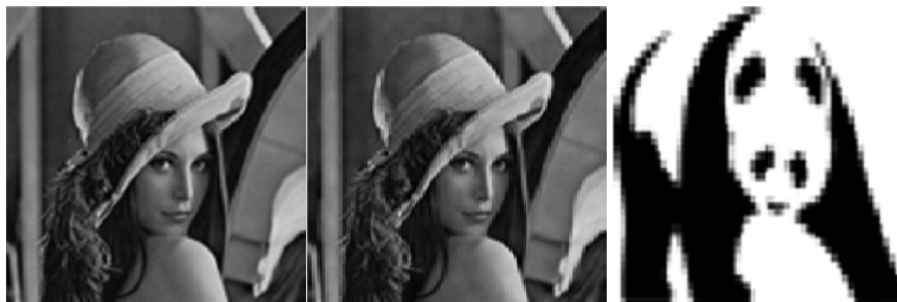
$$\begin{aligned} \text{sim}(cs(W), G_2^T(cs(A_g - A))) &= \text{sim}(cs(W), ((1+c)cs(W) + n)) \\ &\approx \frac{|1+c| |cs(W)^T cs(W)|}{|1+c| \|cs(W)\| \|cs(W)\|} = 1 \end{aligned} \quad (24)$$

False alarm probability: if $I_g - I$ does not contain W after passing through G_2^T projection, and from Formula (22), get:

$$\begin{aligned} \text{sim}(cs(W), G_2^T(cs(A_g - A))) &= \text{sim}(cs(W), (c \times cs(W) + n)) \\ &\approx \frac{|c| |cs(W)^T cs(W)|}{|c| \|cs(W)\| + \|cs(W)\| \|n\|} \approx 0 \end{aligned} \quad (25)$$

5. Simulated Test

The pixel of every host image selected in the test is 256×256 , including people and animals, like Lena and the monkey. Through image operations to every image embedded into the watermark, 143 changed images are obtained. After DCT2 transformation is conducted to the 143 images one by one, the mathematic expectations of the 143 images are calculated. Obtain the difference value between the mathematic expectation and the image embedded in the watermark, and take the 32×32 numerical values at the upper left in the difference value matrix (i.e. both vertical and horizontal directions are 1×32), which include 1,024 different frequencies in total. Expand their column and obtain the random error vector. Calculate the mathematic expectation and covariance of the random error to obtain the direct current component and covariance matrix of random error. The direct current component and covariance matrix of random error receive space decomposition and space construction to obtain the reconstructed space Z . Therefore, we select the dimensions of Z space to be 1,024 dimensions and the feature vector of 900 dimensions with corresponding small characteristic values to be Y space. The 30×30 panda image is selected to be the watermark embedded into the Y space, forming a watermark-containing image, shown in Figure 1.



(a) Original image, (b) Image embedded with watermark, (c) Extracted watermark

Figure 1. Images used in the Experiment

After images with and without watermarks receive various image operations and the relevant tests, the detection probability and false alarm probability are obtained, as shown in Figure 2. Figure 2 show that this algorithm has a high detection probability and a low false alarm probability. Besides, the algorithm receives the test of copy attack, i.e. the watermark is embedded into another image after it is estimated in Figure 1(b). The test results show that,

when the threshold value range is in the interval between 0.5 and 0.6 (the threshold value has been very low), the detection probability is only around 0.03, at the level of false alarm. This proves our previous analysis: this algorithm can effectively defend copy attacks. We conducted blind attack and robustness tests to the watermark-containing image, and the obtained test results are in Table 1. The test results prove our theoretical analyses. Due to the page limit, only some main index figures for the test are listed.

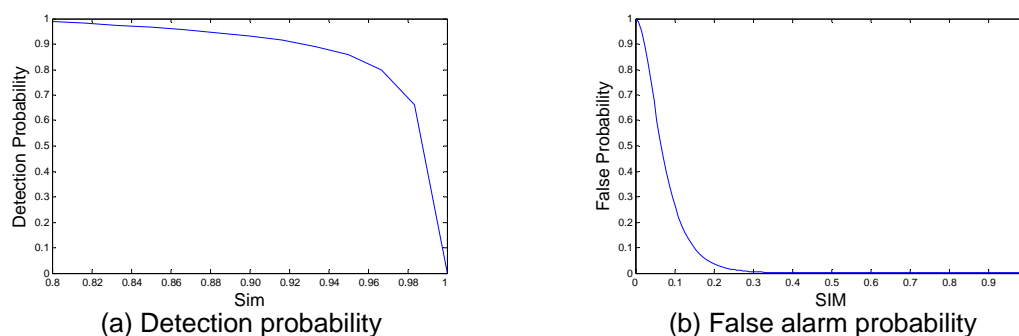


Figure 2. Detection Probability and False Alarm Probability

Table 1. The Performance of Test and Blind Attack Test

Attack Type	Sim Mean	Sim Variance
Gaussian Additive Noise (Mean 0, Variance 0.01)	0.9917	0.0065
Salt & pepper Noise (Noise Density 0.02)	0.9997	0.0007
Speckle Noise (Mean 0, Variance 0.04)	0.9913	0.0095
Possion Noise	0.9997	0.0002
Median Filtering (medfilt2)	0.9999	0.0001
Jpeg Compression	0.9999	0.0001
Rotation (2^0)	0.9973	0.0041
Mosaic Attack	0.9949	0.0053
Sharpening	0.9999	0.0003
Collage (10)	0.9915	0.0017
Fuzziness	0.9997	0.0006

6. Conclusion

Both theoretical analysis and experimental results show that the robust watermarking method based on space reconstruction is a feasible watermarking method, with high safety and robustness.

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