# Pareto Optimal Reconfiguration of Power Distribution Systems with Load Uncertainty and Recloser Placement Simultaneousely Using a Genetic Algorithm Based on NSGA-II 

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#### Abstract

Reconfiguration, by exchanging the functional links between the elements of the system, represents one of the most important measures which can improve the operational performance of a distribution system. Besides, reclosers use to eliminate transient faults, faults isolation, network management and enhance reliability to reduce customer outages. For load uncertainty a new method based on probabilistic interval arithmetic approach is used to incorporate uncertainty in load demand that can forecast reasonably accurate operational conditions of radial system distribution (RDS) with better computational efficiency.In this paper, the optimization process is performed by considering power loss reduction along with reliability index as objective functions. Simulation results on radial 33 buses test system indicate that simultaneous optimization of these two issues has significant impact on system performance.


Keywords: power distribution systems, reconfiguration, load uncertainty, genetic algorithms, multiobjective optimization, pareto optimality, recloser placement

## 1. Introduction

The most important measures which can improve the performance in the operation of a distribution system are: (i) reconfiguration of the system, exchanging the functional links between its elements (system/network/feeder reconfiguration problem); (ii) variation and control of the reactive power flow through the system (optimal reactive power dispatch problem), using bank capacitors, power generators, etc.; (iii) variation and control of the voltage by using onload tap-changers for power transformers (by using automatic voltage regulators); and (iv) changing the operating scheme of the parallel connected power transformers, etc.This paper focuses on optimization through the reconfiguration of power distribution systems.

The reconfiguration problem is one of the multi-criteria optimization types, where the solution is chosen after the evaluation of some indices (e.g., active power losses, reliability indices, branch load limits, voltage drop limits, etc.), which represent multiple purposes. These criteria can be grouped in two different categories: (i) objective functions: criteria that must be minimized; and (ii) constraints (restrictions): criteria that must be included within some bounds. On the other hand, the criteria are incompatible from the point of view of measurement units and are often conflicting. Moreover, some criteria can be (or it is important for them to be) modeled, at the same time as objectives and constraints. For instance, the active power losses must be minimized but we can simultaneously impose a maximal acceptable value (constraint). Thus, in order to solve the problem, first of all, a proper model has to be chosen. The problem of optimization through the reconfiguration of a power distribution system, in terms of its definition, is a historical single objective problem with constraints. Since 1975, when Merlin and Back [1] introduced the idea of distribution system reconfiguration for active power loss reduction, until nowadays, a lot of researchers have proposed diverse methods and algorithms to solve the reconfiguration problem as a single objective problem. The most frequently used one is the main criterion method ( $\varepsilon$-constraint) where the problem is defined in the following conditions: a main criterion is chosen, concomitantly indicating acceptable values for the other criteria. Usually, active power losses are adopted as the main criterion [1-7]. This approach has a major
weakness because there is more than one index that must be taken into account in the optimization process and, without any prior information about the different criteria, choosing the acceptable value can be problematic. Additionally, this approach considers load uncertainty. On the other hand, some authors have studied this problem using aggregation functions, converting the multi-objective problem into a single objective one that assumes a (weighted or not) sum of the selected objective functions [8-10]. The major difficulty in this kind of problem consists in the incompatibility of different criteria. To create a global function, all criteria must be converted to the same measurement unit; a frequently used method is to convert them into costs, which is usually a tricky and often inaccurate operation. In addition, subjectivity appears, caused by the introduction of weighting factors for different criteria. Thus, the existence of a model that could take into consideration more objective functions and constraints at the same time is of great interest. To eliminate the subjectivity and rigidity of the classic methods, the authors propose an original approach to formulate this problem using the Pareto optimality concept that defines a dominate relation among solutions.

Recloser use to eliminate transient faults, faults isolation, network management and enhance reliability indices. A recloser is a device with the ability to detect phase and phase-toearth overcurrent conditions, to interrupt the circuit if the overcurrent persists after a predetermined time, and then to automatically reclose to re-energized the line. Optimum switch placement has carried out with QEA-based algorithm to improve customer service reliability [11]. A new composite objective function of investment cost and reliability on optimum placement of line switch is presented [12]. Simulated annealing optimizing algorithm have discussed [13].

In this paper an original method, aiming the optimization through the reconfiguration of distribution systems and recloser placement simultaneously are proposed which can improve reliability index and reduce power losses of the distributionnetwork. The novelty of the method consists in:

1) The criteria for optimization are evaluated on active power distribution system.
2) The original formulation of the optimization problem, as a Pareto optimal one, with two objective functions (active power losses and system average interruption frequency index).
3) The probabilistic distribution-based interval arithmetic approach is used to incorporate uncertainty in load demand.
4) An original genetic algorithm (based on NSGA-II) to solve the problem (as a Pareto optimal one) in a non-prohibitive execution time.

All the simulations are carried out in MATLAB software. The rest of the paper is organized as follows: section 2 highlights problem formulation with the criteria for optimization.Section 3, 4 represent Pareto optimality problem formulation and solving method. Genetic operators are introduced in section 5. In section 6 best comprise solution will be discussed. The simulation results are illustrated in section 7 and finally concluding remarks are drawn in section 8.

## 2. Problem Formulation

### 2.1. Load Uncertainty Modeling

Assuming that the real and reactive power load vary as per Gaussian distribution

$$
\begin{equation*}
\mathbf{f}\left(\mathbf{x}_{\mathbf{i}}\right)=\frac{1}{\sigma \sqrt{2 \pi}} \mathbf{e}^{-1 / 2} \frac{\left(\mathbf{x}_{\mathbf{i}}-\mu\right)^{2}}{\sigma^{2}} \tag{1}
\end{equation*}
$$

Where $\pi$ is constant, $\sigma$ is the standard deviation, and $\mu$ is themean value. $x_{i}$ is the normalized real or reactive load at the ithbus. For real and reactive load

$$
x_{i}=\frac{P L(i)}{P L_{r}(i)} \text { and } x_{i}=\frac{Q L(i)}{Q L_{r}(i)}
$$

Where $P L_{r}(i)$ and $Q L_{r}(i)$ are rated (nominal) real and reactive load at ith bus. Let the degree of belongingness (membership) for the real and reactive load at all of the buses be represented by $\alpha_{P L}(k)$ and $\alpha_{Q L}(k)$, respectively, where $k$ is the number of degree of belongingness. From the characteristic curve shown in Figure 1, the mean value of the normalized real and reactive load is 1.0 for the degree of belongingness 1.0.

$$
\mu=\frac{\mathbf{P L}(\mathbf{i})}{\mathbf{P L}_{\mathbf{r}}(\mathbf{i})}=1.0 \text { for } \alpha_{\mathbf{P L}}(\mathbf{k})=1.0 \text { and } \mu=\frac{\mathbf{Q L}(\mathbf{i})}{\mathbf{Q L}_{\mathbf{r}}(\mathbf{i})}=1.0 \text { for } \alpha_{\mathbf{Q L}}(\mathbf{k})=1.0
$$

Equation (1) can be written as

$$
\begin{equation*}
\alpha_{\mathbf{P L}}(\mathbf{k})=\mathbf{f}\left(\frac{\mathbf{P L}(\mathbf{i})}{\mathbf{P L}_{\mathbf{r}}(\mathbf{i})}\right)=\frac{1}{\sigma \sqrt{2 \pi}} \mathbf{e}^{-\left[\left(\mathbf{P L}(\mathbf{i}) / \mathbf{P \mathbf { P L } _ { \mathbf { r } } ( \mathbf { i } ) ) - \mu ] ^ { 2 } / 2 \sigma ^ { 2 }}\right.\right.} \tag{2}
\end{equation*}
$$

For $\mu=1.0$ and $\alpha_{\text {PL }}(k)=1.0$. From (2), we get $\sigma=0.398$. Using $\sigma=0.398$ and $\mu=1.0$, (2) results in

$$
\begin{equation*}
\frac{\mathbf{P L}(\mathbf{i})}{\mathbf{P L}_{\mathbf{r}}(\mathbf{i})}-1= \pm\left[\frac{-\ln \left(\alpha_{\mathbf{P L}}(\mathbf{k})\right)}{\pi}\right]^{1 / 2} \text { for } \frac{\mathbf{P L}(\mathbf{i})}{\mathbf{P L}_{\mathbf{r}}(\mathbf{i})} \neq 1.0 \tag{3}
\end{equation*}
$$

A similar equation can be derived for the reactive load. In the above expression, $\alpha_{P L}(k)$ is the degree of belongingness and it can take any value between $\alpha_{P L}(k) \cong \alpha_{P L m a x}(k) / N_{L}$ to $\alpha_{P L}(k)=$ $\alpha_{\text {PLmax }}(k)$ for the specified number of intervals $N_{L}$ where $N_{L}$ is the number of point of linearization of the curve (Figure 1 ) and $\alpha_{\text {PLmax }}(k)$ is the maximum possible degree of belongingness. Let the right-hand side (RHS) of (3) be symbolically represented as $\left[-\ln \alpha_{P L}(k) / \pi\right]^{1 / 2}=\delta_{k}, k=1,2,3, \ldots$, $N_{L}$. Thus, (3) can be rewritten as $P L(i) / P L_{r}(i)=1 \pm \delta_{k}$, where the $\pm$ sign results in the following lower and upper limit of the real power load at ith bus:

$$
\begin{equation*}
\mathbf{P L}_{1}(\mathbf{i})=\mathbf{P L}_{\mathbf{r}}(\mathbf{i})\left[1-\delta_{\mathbf{k}}\right] \text { and } \mathbf{P L}_{\mathbf{u}}(\mathbf{i})=\mathbf{P L}_{\mathbf{r}}(\mathbf{i})\left[1+\delta_{\mathbf{k}}\right] \tag{4}
\end{equation*}
$$

Similar analysis results in the following lower and upper limit of reactive load at ith bus:

$$
\begin{equation*}
\mathbf{Q L}(\mathbf{i})=\mathbf{Q} \mathbf{L}_{\mathbf{r}}(\mathbf{i})\left[1-\delta_{\mathbf{k}}\right] \text { and } \mathbf{Q} \mathbf{L}_{\mathbf{u}}(\mathbf{i})=\mathbf{Q L}_{\mathbf{r}}(\mathbf{i})\left[1+\delta_{\mathbf{k}}\right] \tag{5}
\end{equation*}
$$

For the presentation of the results, linearization is conducted atthree points that result in three distinct load intervals (regions), which are as follows:

$$
\begin{align*}
& \mathbf{D}_{1} \rightarrow\left\{\mathbf{P L}_{\mathbf{r}}(\mathbf{i}), \mathbf{P L}_{\mathbf{r}}(\mathbf{i})\right\}, \text { i.e.,for } \mathbf{k}=1 \\
& \mathbf{D}_{2} \rightarrow\left\{\mathbf{P L}_{\mathbf{r}}(\mathbf{i})\left[1-\delta_{2}\right], \mathbf{P L}_{\mathbf{r}}(\mathbf{i})\left[1+\delta_{2}\right]\right\}, \text { i.e., for } \mathbf{k}=2  \tag{6}\\
& \mathbf{D}_{3} \rightarrow\left\{\mathbf{P L}_{\mathbf{r}}(\mathbf{i})\left[1-\delta_{3}\right], \mathbf{P L}_{\mathbf{r}}(\mathbf{i})\left[1+\delta_{3}\right]\right\}, \text { i.e.,for } \mathbf{k}=3
\end{align*}
$$

Equation (6) reflects that $D_{1}, D_{2}$, and $D_{3}$ are certainly in boundform and, hence, an interval arithmetic operation has to be performed to incorporate these variations in conventional load flow.


Figure 1. Gaussian distribution of load demand

### 2.2. Interval Arithmetic

Let $M$ and $N$ be two interval numbers with a supporting interval [ $m_{1}, m_{2}$ ] and $\left[n_{1}, n_{2}\right]$, respectively. If, in particular, $m_{1}=m_{2}=m$, the interval number $M$ reduces to the real number $m$ $=[\mathrm{m}, \mathrm{m}]$, which is called a point interval or singleton. Rules for addition, subtraction, multiplication, and division are defined as:

$$
\begin{align*}
& \mathbf{M}+\mathbf{N}=\left[\mathbf{m}_{1}+\mathbf{n}_{1}, \mathbf{m}_{2}+\mathbf{n}_{2}\right]  \tag{7}\\
& \mathbf{M}-\mathbf{N}=\left[\mathbf{m}_{1}-\mathbf{n}_{2}, \mathbf{m}_{2}-\mathbf{n}_{1}\right]  \tag{8}\\
& \mathbf{M} * \mathbf{N}=\left[\min \left(\mathbf{m}_{1} * \mathbf{n}_{1}, \mathbf{m}_{1} * \mathbf{n}_{2}, \mathbf{m}_{2} * \mathbf{n}_{1}, \mathbf{m}_{2} * \mathbf{n}_{2}\right),\right. \\
& \left.\max \left(\mathbf{m}_{1} * \mathbf{n}_{1}, \mathbf{m}_{1} * \mathbf{n}_{2}, \mathbf{m}_{2} * \mathbf{n}_{1}, \mathbf{m}_{2} * \mathbf{n}_{2}\right)\right]  \tag{9}\\
& \frac{\mathbf{M}}{\mathbf{N}}=\mathbf{M} * \mathbf{N}^{-1} \tag{10}
\end{align*}
$$

Where $\mathrm{N}^{-1}=\left[1 / \mathrm{n}_{2}, 1 / \mathrm{n}_{1}\right]$ with $0 \notin\left[\mathrm{n}_{1}, \mathrm{n}_{2}\right]$. However, for the purpose of power flow analysis, calculations involving complex number, rather than real numbers are needed. Hence, basic interval numbers in [14] are used.

### 2.3. Interval Power Flow Analysis

The basic power flow analysis method used in this work is essentially the backward/forward sweep power flow algorithm and given in [15]. The uncertainties are considered varying as [16]. The degree of belongingness was obtained as [17] for the specified number of intervals $N_{L}$. For different degree of belongingness ( $\alpha$-cuts), is estimated for all $k$. Using the value of $\delta_{k}$, the intervals are obtained over which real and reactive loads are varying using (4) and (5), respectively, in the form of lower and upper bound.

### 2.4. Active Power Losses ( $\Delta P$ )

Active power losses represent the most important criterion and cannot be ignored in reconfiguration problems [1-10]. In order to evaluate this criterion it is necessary to perform the load flow calculus. As mentioned before the most recommended approaches are backward/forward sweep based algorithms and used in this article. Due to load uncertainty and
three active power loss intervals generated by different $\alpha_{\text {PL }}(k)$, weighted sum for total power loss calculation were used. The basic concepts are:

$$
\begin{aligned}
& W_{1}=1, W_{2}=0.2, W_{3}=0.6 ; W_{\text {total }}=W_{1}+W_{2}+W_{3} ; \\
& \text { PLoss_kW }=\left(W_{1} / W_{\text {total }}^{*}\right)^{*} \operatorname{Ploss}\left(\alpha_{P L}(k)=1\right)+\left(W_{2} / W_{\text {total }}\right)^{*} \operatorname{mean}\left(\operatorname{Ploss}\left(\alpha_{P L}(k)=0.2\right)\right)+\left(W_{3} / W_{\text {total }}\right)^{*} \\
& \text { mean }\left(\operatorname{Ploss}\left(\alpha_{P L}(k)=0.6\right)\right) .
\end{aligned}
$$

### 2.5. Reliability of the Distribution System

The essential attributes of interruptions in the power supply of the customers are the frequency and duration. While duration is predominantly influenced by the distribution system structure (radial, meshed, weak meshed) and the existing automations, the frequency is mainly influenced by the adopted operational configuration; it can be minimized by the suitable choice of the effective configuration and optimal recloser placement. In other words, through reconfiguration and optimal recloser placement, we can improve those reliability indices which refer to the interruption frequency [18]. Otherwise, the reliability of a distribution system can be considered from two different angles:

1) Reliability of a particular customer: e.g., the average number of interruptions to the power supply. This index can represent a possible objective and/or constraint in the optimization problem (because some customers can impose maximal/minimal limits in their supply contracts).
2) Reliability of the entire supply system: e.g., the number of interrupted customers per year [18], system average interruption frequency index (SAIFI) [19] (defined as: total number of customer interruptions longer than 3 minutes per total number of customers served).

Knowing the failure rates at the level of each supplied node (load point), we can estimate the SAIFI using the relationship (in a similar form to the one given in [20]):

$$
\begin{equation*}
\text { SAIFI }=\frac{\sum_{i=1}^{n} \lambda_{i} \cdot \mathbf{N}_{\mathrm{i}}}{\mathbf{N}} \tag{11}
\end{equation*}
$$

where $N$ represents the total number of customers served; $N_{i}$ is the total number of customers supplied from node $i ; n$ is the number of load nodes of the system; $\lambda_{i}$ is the total failure rate of the equivalent element corresponding to the reliability block diagram at the level of node i [year ${ }^{-1}$ ].
With a simple example, the method of calculation of the indicators described. Figure 2 shows a sample radial distribution system of four lines A, B, C and D with four load points 1, 2, 3, and 4. Hypothetical reclosers R1, R2, R3 have been placed in appropriate locations. If $X_{i}=0$, it means the absence and $X_{i}=1$, means presence of reclosers. Unavailability and failure rate of load point can be calculated in the following approach.


Figure 2. Sample radial distribution system

First, unavailability to each of lines $A, B, C$, and $D$ can be calculated.

According to the network structure shown in Figure 2. Load point 1, in the following scenarios are without electricity:

$$
\begin{array}{ll}
U_{A}=\lambda_{A} r_{A} & U_{B}=\lambda_{B} r_{B} \\
U_{C}=\lambda_{C} r_{C} & U_{D}=\lambda_{D} r_{D} \tag{12}
\end{array}
$$

1) $A$ fault on the line $A$.
2) A fault on the line B in the absence of $R_{1}\left(X_{1}=1\right.$ or $\left.X_{1}=0\right)$.
3) A fault on the line C in the absence of $R_{1}, R_{2}\left(\mathrm{X}_{1}, \mathrm{X}_{2}=1\right)$.
4) A fault on the line D in the absence of $R_{1}, R_{3}\left(\mathrm{X}_{1}^{\prime} . \mathrm{X}_{3}^{\prime}=1\right)$.

Therefore load point 1 unavailability can be formulated in (13).

$$
\begin{equation*}
u_{1}=U_{A}+U_{B} \cdot x_{1}+U_{C} \cdot x_{1} \cdot x_{2}+U_{D} \cdot x_{1} \cdot x_{3} \tag{13}
\end{equation*}
$$

In this equation $X^{\prime}$ is complimentary to $X\left(X^{\prime}=1-X\right)$.
Similarly, unavailability of load points 2,3 , and 4 can be formulated as equations (14-16).

$$
\begin{align*}
& u_{2}=U_{A}+U_{B}+U_{C}+U_{D} \cdot x_{3}^{\prime}  \tag{14}\\
& u_{3}=U_{A}+U_{B}+U_{C} \cdot x_{2}^{\prime}+U_{D} \cdot x_{3}^{\prime}  \tag{15}\\
& u_{4}=U_{A}+U_{B}+U_{C} \cdot x_{2}^{\prime}+U_{D} \tag{16}
\end{align*}
$$

Failure rate for load point 1 to 4 can be accessed by replacing with line unavailability. Thus, reliability indices can be calculated.

### 2.6. Other Criteria

1) Node Voltages (Vi): Basically, each voltage r.m.s. value of the network nodes must be framed within the allowable limits.
2) Branch Load Limits through Lines ( $\mathrm{l}_{\mathrm{ij}}$ ): a typical constraint on the reconfiguration problem.
3) Safeguard of power supplies for all customers: The attached graph of the electric system should be connected (a tree or a forest).

Configuration of the Distribution System: Generally, electrical distribution systems are operated in radial configuration. This condition can be expressed as follows:

$$
\begin{equation*}
\sum_{i j \in E} \alpha_{i j}=\mathbf{n}-\mathbf{p} \tag{17}
\end{equation*}
$$

Where $\alpha_{i j}$ is a binary variable, representing the status of a tie line ( 0 -open, 1 -closed); $n$ is the number of electric system nodes; $E$ is the set of power system lines (branches) and $p$ is the number of connected components. In graph theory terms, for a system with one source ( $p=1$ ) we are talking about an optimal tree and for a system with more than one feeder ( $p>1$ ) we are talking about an optimal forest with a number of trees (connected components) equal to that of source nodes.

## 3. Pareto Optimality Problem Formulation

The criteria presented above are not unique, but we consider them to be the most important ones. Taking into account these criteria, we can begin to perceive the real dimensions of the problem. These criteria are incompatible from the point of view of measurement units and can be grouped in two different categories: objective functions and constraints (restrictions). In Pareto optimization, the central concept is named non-dominated solution. This solution must satisfy the following two conditions: (i) there is no other solution that is superior at least in one objective function; (ii) it is equal or superior with respect to other objective function values. Usually, the solution is not unique and consists of a set of acceptable optimal solutions (Pareto
optimal). The set of Pareto solutions forms the Pareto front associated with a problem. Figure 3 presents a possible Pareto front for the optimization problem based on two objectives ( $\Delta \mathrm{P}$ and SAIFI). The Pareto front allows an informed decision to be made by visualizing an extensive range of options since it contains the solutions that are optimal from an overall standpoint.


Figure 3. A Pareto front for a bi-objective reconfiguration problem

As a Pareto optimal multi-objective problem, we propose the following form:
Objective function

$$
\begin{align*}
& \min [\Delta \mathbf{P}, \text { SAIFI }]  \tag{18}\\
& \mathbf{V}_{\mathbf{i}}^{\min } \leq \mathbf{V}_{\mathbf{i}} \leq \mathbf{V}_{\mathbf{i}}^{\max } \\
& \mathbf{I}_{\mathbf{i j}} \leq \mathbf{I}_{\mathbf{i j}}^{\max }  \tag{19}\\
& \sum_{\mathrm{ij} \in \mathbf{E}} \alpha_{\mathrm{ij}}=\mathbf{n}-\mathbf{p}
\end{align*}
$$

## 4. Problem Solving

### 4.1. Genetic Encoding

The initial populationis generated using the branch-exchange heuristic algorithm presented in [3]. In this implementation, the representation using the branch lists was chosen because a power system node is only linked with a small part of the other nodes (it results in a rare graph, i.e., the associated matrix contains many zero elements). Consequently, the graph associated with the electric network can be described by a matrix with 2 lines and mcolumns (where $m$ is the number of the branches), each column indicating the two ends of a branch. This matrix does not contain zero elements. Therefore, using the representation via the branch lists, a binary codification of the problem (binary chromosome with fixed length) can be obtained. Binary values of the chromosome will indicate the status of every electric line: 0-open, 1closed. Figure4a exemplifies the graph (which indicates the network topology) attached to a distribution system, represented by branch lists ( $\alpha$ and $\beta$ ), and the binary attached chromosome g (system/grid encoding).


| $\alpha:$ | 1 | 2 | 1 | 5 | 6 | 1 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\beta:$ | 2 | 3 | 5 | 6 | 3 | 4 |


a:


| 1 | 2 | 1 | 5 | 6 | 1 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 3 | 5 | 6 | 3 | 4 | 3 |

Figure 4. A power distribution system: (a) the branch lists of the attached graph ( $\alpha$ and $\beta$ ) and the attached chromosome (g); (b) Branch lists ( $\alpha$ and $\beta$ ) obtained by decoding the chromosome a.

### 4.2. Genetic Decoding

The operation scheme of the system will be obtained by making the preservation of the corresponding branch value equal 1 (in operation). For instance, by decoding the chromosome a, the radial operation scheme will be obtained (with corresponding $\alpha$ and $\beta$ lists) (Figure 4b). Using this codification, we have a population that consists of a set of chromosomes of type a. By decoding each chromosome, a particular configuration will be obtained and its performance can be tested.

## 5. Genetic Operators

### 5.1. Selection

The goal of the selection operator is to assure more chances to replicate for the best chromosomes of a population. The selection is performed taking into account the fitness of the chromosomes. The most used selection methods are Monte Carlo and tournament. For this multi-objective optimization problem, the author has used the ecological niche method [21].

### 5.2. Crossover

Choosing the number and position of crossover points for the crossover operator depends on the system topology. If these points are selected in an inadequate mode we will obtain "bad" chromosomes: (i) un-connected systems with isolated nodes; or (ii) connected systems with loops (meshed). In order to reduce the number of these cases, we propose that the number of cut points be equal to $\mathrm{CN}-1$. CN represents the cyclomatic number (the number of fundamental circuits/loops) corresponding to the attached graph: $\mathrm{CN}=\mathrm{m}-\mathrm{n}+\mathrm{p}$ (where m is the number of branches, $n$ is the number of nodes/vertices and $p$ is the number of connected components).

### 5.3. Mutation

One of the two conditions in order to have a tree or a forest is to assure n-p closed branches (in operation), as in (17). A radial configuration cannot be converted to another radial one by simply altering the value of a chosen gene. Therefore, we use this operator only in the case when, by performing the crossover operator, non-radial configurations are obtained. Thus, if in a chromosome there are more or fewer than n-p genes equal to 1, the mutation operator randomly replaces the excess/insufficiency of genes equal to 1 (in order to have n-p genes equal to 1 ).

### 5.4. Inversion

The second condition in order to have a tree or a forest is to have a connected graph (for a tree) or a graph with connected components (for a forest). Thus, this operator makes some branch-exchanges (each inversion between two genes of a chromosome behaves as a branch-exchange), repairing existing non connected graph chromosomes (which are not
connected but which have $n-p$ genes equal to 1) and increases the diversity of a population. In our algorithm, this is an intensively used operator after performing crossover and mutation

## 6. Best Comprise Solution using Fuzzy Set Theory

A multiobjective optimization algorithm generates the non-dominated set of solutions known as the Pareto-optimal solutions. The decision maker who is in this context the power system operator may have imprecise or fuzzy goals for each objective function. To aid the operator in selecting an operating point from the obtained set of Pareto-optimal solutions, fuzzy set theory is applied to each objective functions to obtain a fuzzy membership function $\mu$ fi as follows [22]:

$$
\mu_{i}^{k}=\left\{\begin{array}{cc}
1 & f_{i} \leq f_{i}^{\min }  \tag{20}\\
\frac{f_{i}^{\max }-f_{i}^{k}}{f_{i}^{\max }-f_{i}^{\min }} & f_{i}^{\min }<f_{i}<f_{i}^{\max } \\
0 & f_{i} \leq f_{i}^{\max }
\end{array}\right.
$$

The best non-dominated solution can be found when Equation (21) is a maximum where the normalized sum of membership function values for all objectives is highest:

$$
\begin{equation*}
\mu^{k}=\frac{\sum_{i=1}^{N} \mu_{f_{i}}^{k}}{\sum_{k=1}^{M} \sum_{i=1}^{N} \mu_{f_{i}}^{k}} \tag{21}
\end{equation*}
$$

Where Mis the number of non-dominated solutions and Nis the number of objective functions.

## 7. Simulation Results

The Pareto front allows an informed decision to be made by visualizing an extensive range of options since it contains the solutions that are optimal from an overall standpoint. The implementation was adapted in order to work with two objectives ( $\Delta \mathrm{P}$ and SAIFI). Thus, the user can choose (in a flexible mode) $\Delta \mathrm{P}$ as the objective function and some criteria as constraints (voltage deviation, loads limits through lines, etc.) or a vector objective function with $\Delta P$ and SAIFI as variables (at the same time) and other criteria as constraints. The stopping criterion of the algorithm is an imposed maximum number of generations. Results were obtained for 100 runs and 50 population. The stopping criterion of the algorithm is an imposed maximum number of generations. Table 1, presents single-objective (active power losses). The evolution of the active power losses along the searching process is presented in Figure 5.

Performing reconfiguration for the test system [3] as a Pareto problem, the operating configurations are obtained by optimizing two criteria: $\Delta \mathrm{P}$ and SAIFI (Table 2).

The proposed algorithm has obtained a Pareto front with ten solutions (Figure 6). In this case, the first non-dominated solution was obtained from initial population. After the first generation, the algorithm found the second non-dominated solution (the Pareto front contains two solutions). The searching process continued and the third non-dominated solution was found in generation 2 (at the end of generation 2, the Pareto front contains three solutions). The searching process continued until generation 8, where the eighth and final non-dominated solution was found. In the end, the Pareto front contains six non-dominated solutions.


Figure 5. The evolution of the active power losses along the searching process

Table 1. Results for Single-Objective Reconfiguration

| Configuration | Open branches (tie lines) | Active power losses (kW) |
| :---: | :---: | :---: |
| Base case | $8-21,9-15,12-22,18-33,25-29$ | 202.7 |
| MOReco | $7-8,9-10,14-15,28-29,32-33$ | 139.55 |

Table 2. Results for Pareto Reconfiguration with two Objectives

| Configuration | Open branches (tie lines) | Active power losses | SAIFI | Recloser placement |
| :---: | :---: | :---: | :---: | :---: |
| Initial population | 7-8, 9-10, 12-13, 27-28, 18-33 | 149.58 | 1.42 | 3-4, 17-18 |
| generation 2 | $\begin{gathered} 7-8,9-10,12-13,28-29 \\ 18-33 \end{gathered}$ | 146.33 | 1.5 | 3-4, 29-30 |
| generation 4 | $\begin{gathered} 7-8,9-10,12-13,28-29 \\ 32-33 \end{gathered}$ | 145.73 | 1.51 | 3-4, 30-31 |
| generation 5 | $\begin{gathered} 7-8,9-10,14-15,27-28 \\ 18-33 \end{gathered}$ | 145.01 | 1.63 | 3-4, 10-11, 3-23 |
| generation 6 | $\begin{gathered} 7-8,9-10,14-15,28-29 \\ 18-33 \end{gathered}$ | 141.76 | 1.71 | $\begin{gathered} 3-4,10-11,13-14 \\ 20-21 \end{gathered}$ |
| generation 8 | $\begin{gathered} 7-8,9-10,14-15,28-29 \\ 32-33 \end{gathered}$ | 140.1 | 1.73 | $\begin{gathered} 3-4,10-11,3-23 \\ 12-22 \end{gathered}$ |



Figure 6. The Pareto front

The proposed algorithm tries to exploit the fundamental properties of a distribution system, i.e., to have a radial configuration in operation. It tries to generate just radial configurations (by using the branch exchange heuristic procedure to generate the initial population and by crossover operator). By choosing the number of cut points equal to the cyclomatic number -1 , usually, other valid chromosomes are obtained, increasing the diversity of the population. This implementation does not ensure just valid chromosomes because in some cases, non-valid chromosomes (non radial configurations) are obtained. However, in combination with the mutation, this disadvantage is transformed into an important advantage because the diversity of the population is substantially increased and new zones from the research space are explored. Not ultimately, the implemented inversion operator, applied a random number of times to the chromosomes, expands the search space enough in order to find a good solution in a reduced number of generations.

## 8. Conclusion

Reconfiguration represents one of the most important measures which can improve the performance in the operation of a distribution system. Optimization through the reconfiguration (or optimal reconfiguration) of a power distribution system is not a new problem but still represents a difficult one and nowadays has new valences. Besides active power losses, the average number of interruptions to the power supply represents an essential criterion which must be taken into consideration in the optimization problem. Interval arithmetic-based distribution load flow is reported to evaluate the effects of the input uncertainties. The probabilistic variation of load uncertainty is linearized at multipoint to find bounded intervals. The criteria for optimization have been evaluated on active power distribution systems.

The original formulation of the optimization problem, as a Pareto optimal one, with two objective functions (active power losses and system average interruption frequency index) ensures an objective and robust solution. Thus, the weak points of the classic methods proposed in literature can be eliminated.

Usually, the existing reconfiguration methods used nowadays either demand prohibitive execution times or result in non-optimal solutions (in the case of most common heuristics). The authors propose an original genetic algorithm (based on NSGA-II) to solve the problem (as a Pareto optimal one) in a non-prohibitive execution time. The comparative tests performed on active test system has demonstrated the accuracy and promptness of the proposed algorithm.

Reclosers have played a fundamental role in improving the reliability of distribution systems. Therefore, it is essential to determine optimal number and location of reclosers to enhance network reliability. Results show reconfiguration can affect both power loss reduction and optimal recloser number.

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