4742

An Anti Mode Mixing EMD Algorithm for Detecting the Characteristics of Low Frequency Oscillations in Power System

Jianbo Yi^{1*}, Qi Huang¹, Shi Jing¹, Lijie Ding²

¹University of Electric Science and Technology of China, Key Lab of Wide-area Control and Measurement on Power System of Sichuan province, Chengdu, China, 611731 ²Sichuan Electric Power Research Institute, Chengdu, China, 610072 *Corresponding author, e-mail: jimbo_yi@uestc.edu.cn

Abstract

The dynamics of modern interconnected power system is characterized by low frequency oscillations (LFOs) which are produced as results of various disturbances such as changes in loads, tripping of lines, faults, and other discrete events. A data driven empirical mode decomposition (EMD) method is applied to the detection of low frequency oscillation modes from disturbed trajectory with its strong non-stationary signal processing capability, but the mode mixing phenomenon serious impact on the analysis credibility and accuracy of EMD method. In this paper, an anti mode mixing EMD composite algorithm is proposed for detecting the characteristics of LFOs in power system. First, the improved frequency heterodyne method is proposed to increase the spectral distance between adjacent mode components in order to meet the octave resolution requirements. Second, the wavelet singularity detection technology is proposed to determine the adaptive sliding analysis window for each mode, in which there implements the intermittency mixed modes separation and their nonstationary parameters identification. Finally, the analysis result of interconnected grid test case verify that the proposed algorithm can effectively overcome the impact of the mode mixing existed in EMD and improve the characteristics detection accuracy of LFOs characteristics.

Keywords: low frequency oscillations (LFOs), empirical mode decomposition (EMD), mode mixing, frequency heterodyne method, adaptive sliding analysis window

Copyright $\ensuremath{\textcircled{O}}$ 2013 Universitas Ahmad Dahlan. All rights reserved.

1. Introduction

Power system is a typical large-scale system of high complexity. The study of complex dynamic processes governed by nonlinear and nonstationary characteristics is a problem of great importance in the analysis and control of power system. Modern interconnected power system dynamics is characterized by low frequency oscillations (LFOs, usually 0.1-2.5Hz) which are produced as a result of a variety of disturbances such as changes in loads, tripping of lines. faults, and other discrete events. Previous studies on LFO based on small-signal electromechanical dynamic modes are often presented using linear system concepts in [1-2]. A given LFO modal properties are described by its frequency, damping, and shape. Signal processing methods have been proposed to analyze the system dynamic characteristics from system response data which manifest themselves as variations in line flows and generator angle excursions. Fourier signal analysis method in [3] which parameters are identified by Fourier spectrum analysis based on the global Fourier transform. As developing windowed Fourier method, an application-based approach of power system modal identification via wavelet analysis is presented in [4]. The most widely studied modal parametric analysis algorithm is termed Prony method which is presented in [5-7] to estimate small-signal dynamic properties from measured and simulated data. Meanwhile some other similar parameter estimation methods, such as the minimal realization algorithm first introduced in [8], the eigenvalue realization algorithm [9], and the least-mean squares method[10], have been applied to extract the LFO modes. But all above signal analysis and identification methods are based on the basic assumption of stationary signal. In view of the increased dynamic complexity and other nonlinear effects of modern power system, the traditional analysis methods of the LFO are very difficult to cope with the non-stationary characteristics of dynamic power system response signals.

In the past decade, the wide-area measurement system (WAMS) has been widely applied to the interconnected power grids to improve the level of power system dynamic behavior measurement and monitoring. A new nonlinear and non-stationary signal processing method HHT which is composed of EDM and Hilbert transform (HT) is introduced by Huang et al. in [11, 12]. The application of this method to analyze the power flow oscillations is presented in [13], which is a data-driven, non-stationary signal analysis method without stationary signal assumption conditions. Recently, some improved EMD algorithms proposed in [14] and [15] have improved the detection accuracy of the LFOs Characteristics effectively and been able to analyze the measured disturbed trajectories provided by WAMS [16]. However, the frequency of LFO signals are concentrated in the range of 0.1-2.5Hz and the signals exist two nonstationary characteristics: (1) amplitude is a function of change over time and with the damping characteristics; (2) the presence time of each oscillation mode is not sure. In addition, EMD process affected by measurement noise, end effect [17, 18] and frequency resolution is easy to emerge mode mixing phenomena, which cause the intrinsic mode function (IMF) not a single oscillation mode and lose the physical significance. Masking signal method presented in [19] and frequency heterodyne method presented in [20] as two of most effective approach to deal with mode mixing phenomenon in EMD increase the spectral distance of the adjacent mode in order to reach the resolution requirements of algorithm itself [21]. But the determination of the auxiliary signal, high-low-frequency modal flip and pseudo-modal component identification problem is difficult to solve, so to completely eliminate mode mixing phenomenon need in-depth research and improved algorithm.

In this paper, an anti mode mixing EMD composite algorithm is proposed for detecting the characteristics of LFOs in power system. The key issue is to eliminate mode mixing phenomenon in EMD process. So two mode mixing phenomenon including octave and intermittency in EMD algorithm decomposition which are important to the mode identification of measured LFO signals are introduced in Section 2. To separate single pure mode from mixed complex signal, two effective solution methods are proposed in Section 3. The improved frequency heterodyne method is used to increase the spectral distance between adjacent mixed mode components in order to meet the octave resolution requirements. Then the adaptive sliding analysis window technology based on the wavelet singularity detection is proposed to determine each single mode analysis window, in which the intermittency mixed modes do not appear and the octave mixed modes separation process do not product the pseudo mode component owing to IMF high and low frequency flip. Finally, the test case simulates WAMS conditions using Sichuan interconnected grid in RTDS and the test results demonstrate to the effectiveness of proposed algorithm in Section 4.

2. Mode Mixing Phenomenon in EMD

The EMD method is a powerful data-driven, adaptive signal process technique for analyzing nonlinear and non-stationary measured data. The purpose of EMD is to decomposite a composite oscillations signal into the combination of several simple IMFs which are generated by a sifting process. The traditional EMD sifting process sometimes produces energy leakage and damping loss which lead to the IMFs loss of physical significance.

The power system LFO signals can be represented as a finite sum of multi-modal exponentially damped sinusoids signals.

$$s_*(t) = \sum_{i=1}^N A_i e^{\alpha_i t} \cos(2\pi f_i t + \varphi_i) + e(t), \quad t = \frac{k}{f_s} > 0$$
⁽¹⁾

Where $s_*(t)$ is a measured LFO signal, fs is the sampling frequency, t is the product of the kth sampling point and sampling interval. Where N is the number of oscillation modes, and i represents a single oscillation mode, A_i is the amplitude, α_i is the attenuation coefficient, f_i is the frequency, φ_i is the phase, e(t) is the measurement noise.

In [22], G. Rilling and others point out the EMD method has frequency resolution limitations. Using extreme point sampling theorem, if two adjacent oscillation modes can not reach the octave resolution condition, the EMD sifting process will not decompose out the single pure IMF, which is considered as mode mixing phenomenon. Such as using EMD algorithm presented in [23] to deal with a LFO power angle sampling signal including 1.52Hz and 1.22Hz oscillations modes. The IMFs component results and their FFT spectrum are shown in Figure 1.



Figure 1. Octave mode mixing in EMD process, the power angle signal is decomposed into just one effective IMF, and its FFT result still exits two oscillation modes.

The power angle signal contains two mode components. When this two modal frequency ratio range lies in $0.5 < f_1/f_2 < 2$, the EMD process can not distinguish between the two modes as the power angle signal is decomposed into just one effective IMF. Through the IMF FFT frequency spectrum analysis, the IMF still exits two mode components, which phenomenon are defined as octave mode mixing. The LFO signals represent the dynamic behavior of the power system, whose modal constitution is complex and concentrated in the low frequency region, so this situation constantly products the octave mode mixing. Apart from this, another mode mixing sometimes occurs in the case where signals have suddenly changed in time-scale features within analysis window. The power system LFO modes may arise at any time suffered from disturbance and these modes exhibit strong damping characteristics, as an active power oscillations signal example be defined as following shape.

$$x(t) = x_1(t) + x_2(t) = 2\cos(3.04\pi t) + \begin{cases} 4e^{-0.69t}\cos(1.94\pi t + \pi) & t \in [2.9 \quad 8.0] \\ 0 & \text{otherwise} \end{cases}$$
(2)

Where the sampling time lasts 10s and the sampling frequency is 100Hz. The EMD sifting results and each IMF FFT spectrum results are shown in Figure 2.



Figure 2. Intermittency mode mixing in EMD process, the power oscillations signal is decomposed into two effective IMFs, IMF1 exists mode mixing and IMF2 loses a part of damping energy.

It is obvious that the intermittency mode mixing will lead to the EMD algorithm failed. Each IMF can not represent single oscillation mode, and the previous IMF contained mode mixing will affect the others modes sifting. Further, the problem of mode mixing will affect the accuracy and validity of the LFOs non-stationary modal parameters identification, so two mode mixing separation methods, improved frequency heterodyne method and adaptive sliding window technology, will be proposed in the following section.

3. Mode Mixing Separation Method

3.1. Improved Frequency Heterodyne Method

Consider the following example signal contained two adjacent oscillation modes.

$$x(t) = M \cos(2\pi (f_1 + f_2)t) \qquad f_1 > f_2$$
(3)

The frequency heterodyne method utilizes signal modulation principle to change the spectral distance between adjacent mixed modes and achieve the complete EMD decomposition within octave resolution requirements. First, according to the communication modulation principle, a high-frequency carrier signal is added into the original signal, then generate a modulated double sideband signal (DSB).

$$S_M(t) = S(t)e^{j2\pi Ft}$$
(4)

Where the carrier frequency is F, the modulated signal is SM (t), and exists mode mixing $1 < f_1/f_2 < 2$. The essence of the signal modulation is frequency shift modulated signal on the carrier frequency F, so the modulated signal SM (t) contains two new frequency signals f_{m1} , f_{m2} . When the modulated signals satisfy the octave resolution condition | f_{m1}/f_{m2} |>2, the EMD algorithm can decompose the modal component of the modulated signal completely.

In [18], the frequency heterodyne method is presented in detail, in which the carrier frequency F is defined in the range $[f_1, 2f_1-f_2]$. But this method is able to cause the inversion of the high oscillation mode and low oscillation mode as shown in Figure 3(a). If consider the influence of measurement noise, end effect and interpolation error, the EMD process may obtain more Pseudo-IMFs causing difficult to identify the real mode parameters. So in this paper, an improved frequency heterodyne method is proposed to overcome the mode flip problem. First, the carrier frequency F is defined in the range $[2f_2-f_1, f_2]$. As shown in Figure 3(b), the mode mixing signal in the modulation process does not produce frequency flipped IMFs results and the proposed method does not change the IMFs order.



Figure 3. (a) The Frequency Heterodyne Method Leads to the Inversion of High and Low frequency modes. (b) The improver frequency heterodyne dose not change the arrangement order of the modes.

Each sideband of DSB signal contains all information of the modulated signal, so any single sideband signal (SSB) can analyze the signal characteristics. Consider Hilbert transform:

$$s_H(t) = s(t) + jH[s(t)]$$
(5)

The original modulated signal is filtered the upper band signal, then the remained lower SSB signal is expressed as follows:

$$s_{SSB}(t) = \operatorname{Re}\left\{s_{H}(t)e^{-j2\pi Ft}\right\}$$
(6)

Assumed that the SSB signal mode component accord with the octave resolution requirements, the EMD algorithm can obtain the complete IMFs results.

$$s_{SSB}(t) = \sum_{i=1}^{n} c_i(t) + r(t)$$
(7)

Where ci(t) is the single mode IMFs. The IMFs are the modulated mode complements, we need the application of Hilbert transform and the carried signal to restore the original IMFs.

$$c_{si}(t) = -\operatorname{Re}\left\{c_{Hi}(t)e^{j2\pi Ft}\right\}$$
(8)

In practical applications, The LFO signal usually has scant effective analysis data and the analysis method must consider the real-time requirements. Meanwhile, the proposed method should avoid to bring extremely low frequency modes as much as possible, so we set the mode resolution at 0.1Hz and the heterodyne frequency in $[0.8f_2, 0.85f_2]$ has a better effect.

3.2. Adaptive sliding analysis window technology

There is a time uncertainty problem existed in the emergence of the excited LFO modes. Which will lead to the frequency hoping changes of measured signal. At that case, using the EMD algorithm in full analysis window, the dividing characteristics of different modes will be lost and the intermittency mode mixing also exists in IMFs results. Here, an adaptive sliding analysis window technology be proposed to deal with the problem of each mode analysis window partitioning and eliminate the intermittency mode mixing phenomenon.

In the view of signal singularity, when a new LFO mode is excited, the complex measured LFO signal will manifest the singularity and exist some singular points. The singularity detection of signals utilizing the continuous wavelet transform (CWT) which is introduced in [24] can help us determine the time boundaries of excited oscillation modes. The CWT is defined as follows:

$$W_s(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} s(t)\psi^*(\frac{t-b}{a})dt$$
(9)

Where s(t) is the LFO signal, a is a dilation or scale parameter, b is a translation or time shift parameter, and ψ^* (t) $\in L^2(R)$ is complex conjugate of a mother wavelet function. Select has a good oscillation characteristics and smaller vanishing moment mother wavelet function, the |Ws (a, b)| result of CWT will be non-zero dada at the neighborhood of singular points. Consider the following complex LFO signal, which the initial sampling time is set at 0.5s and sampling frequency is 100Hz:

$$x(t) = \begin{cases} e^{-0.69t} \cos(1.04\pi t + \pi/4) & t \in [10 \quad 20] \\ e^{-0.69t} \cos(2.4\pi t + \pi) & t \in [0 \quad 10] \\ 2e^{-0.19t} \cos(3.04\pi t + \pi/8) & t \in [0 \quad 20] \end{cases}$$
(10)

In this paper, choose DB10 wavelet to analyze the signal singularity, which the multiscale decomposition results are shown in Figure 4(a) which reflects the signal singular points information at the high frequency coefficients d1, d2. According to the signal singular points information detected by wavelet, the adaptive sliding analysis windows are set as following rules:

Table 1) The n singular points and two signal boundary points make n+2 child windows split points. Theoretically, there are (n + 2) (n + 1) / 2 child analysis windows, but too short window size can not be effectively for detecting LFO signal by EMD, therefore limit the minimum window not less than 5s.

Table 2) Set the child window boundary expansion 0.5s length in order to reduce the impact of the end effect in EMD sifting process.

Table 3) The analysis order of the each child window is that first analyzes the short-time child window, after continue to analyze the long-term window without according to the order of child windows time markers.

Table 4) The method need predict frequency components in each child window before the EMD process to determine whether contains mode mixing.

Table 5) Using Hilbert spectrum analysis method to identify the oscillation modes characteristic parameters, parameter identification process without considering boundary expansion.



Figure 4. (a) The singular points detection and adaptive sliding analysis window partitioning. (b) The complete FFT frequency spectrum of the LFO signal. (c) The sifting results of 1# window using improved frequency heterodyne method. (d) The sifting results of 2# window direct use of EMD

According to the above rules, set 3 child windows shown in Figure 4(a), 1# child window: 0s to 10.5s, 2# child window: 9.5s to 20s, and 3# child window: 0 to 20s. Before making use of EMD algorithm to sift the signal within the window, we need to roughly analyze the modes composition with FFT method at first. As seen Figure 4(b), 3# window contains three oscillation modes, and 1# window exists octave mode mixing, so the improved frequency heterodyne method should be used to analyze 1# window signal. Set the heterodyne frequency to 0.95Hz, then the modulated signal in 1# window meets the octave resolution conditions and the decomposition result shown in Figure 4(c) contains 0.57Hz and 0.25Hz modes, which are restored as 1.52Hz and 1.2Hz modes. Direct use of the EMD algorithm to sift the signal in 1# window, two mode components of 1.52Hz and 0.52Hz are shown in Figure 4(d).

The adaptive sliding window technique can solve the intermittency mode mixing problem existed in global EMD decomposition and determine the time markers of existence of each oscillation mode, which fully demonstrates the non-stationary characteristics of the LFO signal. The proposed technique will substantially improve the non-stationary parameters identification accuracy of the LFO modes and enhance the resolution performance of EMD method processing the complex modes signals.

An Anti Mode Mixing EMD Algorithm for Detecting the Characteristics of Low... (Jianbo Yi)

4. Algorithm test

To test the feasibility and effectiveness of the proposed anti mode mixing EMD algorithm for detecting the modes parameters of power system complex LFO signal, a study cases about Sichuan interconnected grid are selected as test examples. First, consider the Sichuan interconnected grid simulated in RTDS, and set up WAMS conditions using two PMUs and one data server. The main oscillation modes information are shown in Table 1.

Table 1. The Main Oscillation Modes Information					
Item	Mode parameters	Frequency (Hz)	Damping (%)		
Mode 1	-6.7846 ± j12.0260	1.9140	49.14		
Mode 2	-3.9274 ± j8.4921	1.3516	41.98		
Mode 3	-0.8265 ± j7.6287	1.2141	10.77		
Mode 4	-0.4467 ± j5.3545	0.8522	8.31		
Mode 5	-0.0884 ± j4.6124	0.7341	1.92		
Mode 6	-0.2129 ± j2.6389	0.4200	8.04		

The simulation system is set a three-phase ground fault at outgoing transmission line 0.1s, after 2.5s, a impact load 20% of the total load is set into the system, then set a excitation disturbance at Pubugou generator 8s. Under WAMS experimental conditions, the PMUs are used to measure the LFO signals and set the sampling frequency 100Hz. So, a period of 20s long active power oscillations data measured at transmission line are shown in Figure 5(a).



Figure 5. (a) The measurement data of active power oscillations using PMUs and their singular points, (b) The complete FFT frequency spectrum of the measurement data, (c) The FFT frequency spectrum analysis in 1# window, (d) The sifting results of 1# window using improved frequency heterodyne method, (e) The EMD decomposition results of 3# window

Table 2. Identification Results of the Measured Complex LFO Signal						
	Characteristic parameters					
Modes	Frequency (Hz)	Amplitude (MW)	Attenuation coefficient	Damping (%)	Existence time (s)	
1	1.21	307.82	-0.83	11.21	0-8	
2	0.86	27.11	-0.36	6.65	2.5-8	
3	0.71	28.64	-0.08	2.07	8-20	
4	0.42	94.28	-0.16	4.52	0-20	

Applying the proposed algorithms to detect the characteristic parameters of measured oscillation modes, first detect the singular points and analyze the modal composition of measurement signal which are shown Figure 5(a) and (b). According to singular points results, set 4 adaptive sliding windows, 1# window [2s, 8.5s], 2# window [0s, 8.5s], 3# window [7.5s, 20s], 4# window [0s, 20s]. As seen in Figure 5(c), in 1# window there has three

oscillation modes respectively corresponding to the original signal modes 1, 2 and 4, in which there exits the octave mode mixing. Then employ improved frequency heterodyne method to decomposite oscillation modes in 1# window and the effective heterodyne IMFs are shown in Figure 5(d). The mixed modes are separated successfully, by demodulation technique, the heterodyne IMFs can be restored into single LFO modes. Finally Hilbert spectrum analysis method is used to identify the modes characteristic parameters and comprehensive results are expressed in Table 2.

The comprehensive analysis results show that: 1), the simulation system is inspired 4 LFO modes; 2), there are lower damping lever for each oscillation mode even respective mode damping lower than 5%; 3), the proposed algorithm can determine the existence time of each mode and separate the complex mixed modes; 4), the characteristic parameters identification results of oscillation modes with proposed algorithm meet the example system eigenvalue analysis results consistently; 5), the proposed algorithm has higher effectiveness and real-time analysis capability to detect non-stationary LFO signals in power system.

5. Conclusion

In this paper, an anti mode mixing EMD algorithm for detecting the characteristics of LFOs in power system is studied in a detailed. The proposed algorithm utilizes improve frequency heterodyne method and adaptive sliding window technique to improve the identification accuracy of characteristic parameters of LFO modes. The adaptive sliding window technique based on wavelet singularity detection technology can be used to set each mode existence window and eliminate the intermittency mode mixing in EMD sifting. The improve frequency heterodyne method base on signal modulation principle can be used to separate mixed oscillation modes completely from octave mode mixing LFO signal. Finally, the analysis result of interconnected grid test case verify that the proposed algorithm can effectively overcome the impact of the mode mixing existed in EMD and improve the characteristics detection accuracy of LFOs characteristics. Extension of these results for improving non-stationary signal process methods and the application of these methods to detect the LFO in WAMS will be further studied in future.

Acknowledgments

The authors gratefully acknowledge the support of National Science Foundation of China (No. NSFC-51277022) and New Century Excellent Talent Support Plan (No. NCET-09-0262) and Sichuan Electric Power Research Institute.

References

- [1] DJ Trudnowski. Order reduction of large-scale linear oscillatory system models. *IEEE Transactions on Power Systems*. 1994; 9(1): 451–458.
- [2] JR Smith, F Fatehi, CS Woods, et al. Transfer function identification in power system applications. *IEEE Transactions on Power Systems*. 1993; 8(3): 1282–1290.
- [3] Ma Chao, Wang Huizheng, Meng Yongping. Research on integrated monitoring algorithm of low frequency oscillation. *Power System Technology (in China).* 2009; 33(3): 1-8.
- [4] M Bronzini, S Bruno, M De Benedictis, M La Scala. *Power system modal identification via wavelet analysis.* IEEE Conference on Power Tech. 2007: 2041–2046.
- [5] JF Hauer. Application of Prony analysis to the determination of modal content and equivalent models for measured power system response. *IEEE Transactions on Power Systems*. 1991; 6(3): 1062-1068.
- [6] Zhu Wei, Tang Yingjie, Zhou Youqing, et al. Identification of power system low frequency oscillation mode based on improved Prony algorithm. *Power System Technology (in China).* 2009; 33(5): 44-47.
- [7] DJ Trudnowski. Estimating electromechanical mode shape from synchrophasor measurements. *IEEE Transactions on Power Systems*. 2008; 23(3): 1188-1195.
- [8] N Zhou, JW Pierre, RW Wies. Estimation of low-frequency electromechanical modes of power systems from ambient measurements using a subspace method. Proceedings of the North American Power Symposium. 2003.
- [9] D Trudnowski, J Pierre, N Zhou, J Hauer. Performance of three mode-meter block-processing algorithms for automated dynamic stability assessment. *IEEE Transactions on Power Systems*. 2008; 23(2): 680–690.

- [10] RW Wies, JW Pierre, DJ Trudnowski. Use of least-mean squares (LMS) adaptive filtering technique for estimating low-frequency electromechanical modes in power systems. Proceedings of the IEEE Power Engineering Society General Meeting. 2004: 1863–1870.
- [11] N Huang, et al. The empirical mode decomposition and the Hilbert spectrum for nonlinear and nonstationary time series analysis. Proc. R. Soc. Lond. A. 1998; 45: 903–995.
- [12] NE Huang, MWu, S Long, et al. A confidence limit for the empirical mode decomposition and Hilbert spectral analysis. Proc. R. Soc. Lond. A. 2003; 45(9): 2317-2345.
- [13] AR Messina, V Vittal. Nonlinear non-Stationary analysis of inter-area oscillations via Hilbert spectral analysis. *IEEE Transactions on Power Systems*. 2006; 21(3): 1234-1241.
- [14] Senroy N, Suryanarayanan S, Ribeiro PF. An improved Hilbert-Huang method for analysis of timevarying waveforms in power quality. *IEEE Transactions on Power Systems*. 2007; 22(4): 1843–1850.
- [15] Shi Pingan, Wei Jia. Measured data processing method for relative motions between two side-by-side ships. *Telkomnika, Indonesian Journal of Electrical Engineering*. 2013; 11(1): 73-82.
- [16] AR Messina, V Vittal, D Ruiz-Vega. Interpretation and visualization of wide-area PMU measurements using Hilbert analysis. *IEEE Transactions on Power Systems*. 2006; 21(6): 1763–1771.
- [17] Yang, Zhengyi Yin, Aijun. Dynamic stability enhancement of power systems using neural-network controlled static-compensator. *Telkomnika, Indonesian Journal of Electrical Engineering*. 2012; 10(1): 9-16.
- [18] Wang Xiaofeng, Pan Hao. Study of HHT end effect suppression based on RBF extension. International Journal of Advancements in Computing Technology. 2012; 4(13): 369-377.
- [19] Deering R, Kaiser JF. *The use of masking signal to improve empirical mode decomposition.* Proc. IEEE Int. Conf. Acoustics, Speech Signal Processing. 2005: 485–488.
- [20] Senroy N, Suryanarayanan S. Two techniques to enhance empirical mode decomposition for power quality applications. *IEEE PES General Meeting*. 2006: 1–6.
- [21] Dina Shona Laila, AR Messina, Bikash C Pal. A refined Hilbert–Huang transform with applications to inter-area oscillation monitoring. *Transactions on Power Systems*. 2009; 24(2): 610-620.
- [22] G Rilling, P Flandrin. One or two frequencies? The empirical mode decomposition answers. *IEEE Transaction on Signal Process.* 2008; 56(1): 85–95.
- [23] G Rilling, P Flandrin, P Goncalves. On empirical mode decomposition and its algorithms. IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing. 2003: 27-33.
- [24] TU C, HWANG W, HO J. Analysis of singularities from modulus maxima of complex wavelets. IEEE Transactions on Information Theory. 2005; 51(3): 1049-1062.