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Prediction of Electric Power Consumption based on the Improved GM (1, 1)

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Abstract

Based on the electric power consumption data in 2001-2010, this paper discusses GM (1, 1) model and its improved model in the application of power consumption forecasting. Due to the traditional Grey Model itself has certain defects, we grouped the original sequence according to the degree of deviation first, and then combined with nonlinear GM (1, 1, α) to improve the traditional GM (1, 1) model. Through the relative error testing and the posterior testing, this paper made a comparative analysis to the traditional GM (1, 1) model and the improved GM. Example of Beijing shows that the improved model had good accuracy; it had a good application value in the actual prediction system.

Keywords: *grey model group, nonlinear GM (1, 1, α), posterior testing, electric power consumption*

1. Introduction

With the sustained and rapid growth of the national economy, electricity supply impact on economic development and people's normal life directly, it has become a restrictive factors for sustainable development of the national economy [1]. In order to ensure the adequate power supply to avoid its impact on social stability and investment environment, electric power investment must be arranged in advance based on the socio-economic development, and electric power supply should strive to avoid electricity shortfall. Therefore, it is particularly important to do "power first" and establish early warning for the electrical energy consumption [2-3].

There are many methods to predict the electrical energy consumption [4-6]. Gray system method has been widely used as its advantages of less raw data, simple principle, regardless of distribution, easy operation and etc. GM (1, 1) is the traditional prediction method of Grey System, this model is restricted in the application due to it is a deviation index model actually [7-8]. This paper introduces the traditional GM (1, 1) model as well as the reasons for its deviation, and then puts forward the prediction model to eliminate the deviation and improve the accuracy, the steps are as follows: group the original sequence in accordance with the deviation first, and then combined with nonlinear GM (1, 1, α) model to improve the traditional GM (1, 1) model. Example shows that the improved model has widespread use value and higher accuracy than the traditional model.

2. The Improved Nonlinear GM (1, 1, α)

2.1. The Reason of Traditional GM (1, 1)'s Deviation

From GM (1, 1) modeling steps we can see that grey derivative is dealed with difference, that is $\frac{dx^{(1)}}{dt}|_{t=k} = x^{(1)}(k) - x^{(1)}(k-1)$. The white background value of grey derivative uses the formula $z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$ instead of the assuming sequence, that is $x^{(1)} = (x^{(1)}(1), x^{(1)}(2)....x^{(1)}(n))$. $x^{(1)} = (x^{(1)}(1), x^{(1)}(2)....x^{(1)}(n))$ are the discrete points on the index $y = x^{(1)}(t)$, we know from the Lagrange mean value theorem that there is a point ξ_k in (k-1, k), it makes $x^{(1)}(k) - x^{(1)}(k-1) = \frac{dx^{(1)}}{dt}|_{t=\xi_k}$, $(k-1 < \xi_k < k)$. Due

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to the sequence $x^{(1)}$ is monotonous, so $x^{(1)}(\xi_k) = \lambda_k x^{(1)}(k-1) + (1-\lambda_k)x^{(1)}(k)$, $\lambda_k \in (0,1)$. Therefore, when we build GM (1, 1), grey derivative $x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1)$ is the derivative of ξ_k. It's background values should be $x^{(1)}(\xi_k) = \lambda_k x^{(1)}(k-1) + (1-\lambda_k)x^{(1)}(k)$, while replaced the background values by $0.5(x^{(1)}(k) - x^{(1)}(k-1))$ is just the special case. When it is not abiding the white index law coincidence, obviously not the best. So, improve the traditional GM (1, 1) combined with nonlinear GM (1, 1, α) model can more scientific, it also can improve the prediction accuracy.

2.2. Improved GM (1, 1)

The traditional grey prediction model itself has some defects, it have certain restrictions in the application. This paper groups the original sequence according to the deviation degree, and then improve the traditional GM (1, 1) combined with nonlinear GM (1, 1, α) can effectively improve the accuracy of the gray model forecasting results. Modeling steps of improved model group are as follows:

1. Caculate the grade ratio of the original data.

Assume the original sequence is $x^{(0)} = (x^{(0)}(1), x^{(0)}(2)...x^{(0)}(n))$, $x^{(1)}$ is the sequence whose generation is accumulated by $x^{(0)}$. We call:

$$\sigma(k+1) = \frac{x^{(0)}(k+1)}{x^{(0)}(k)} \tag{1}$$

is the grade ratio in k point, and it rules $x^{(0)}(2) = x^{(0)}(1)$.

2. Calculate the grade ratio and its mean error sequence.

Suppose the grade ratio sequence is $\sigma = (\sigma(1), \sigma(2)....\sigma(n))$, if there has $\forall k, \sigma(k)$ is a constant, he points of $x^{(0)}$ are all on an exponential curve. Solutions of GM(1,1) build the index curve, predicted values and actual values are equal. While generally, sequence of the grade ratio is not a fixed constants in constant situation, we suppose:

$$c = \frac{1}{n}\sum_{i=1}^{n} \sigma(i) \tag{2}$$

$r(i) = c - \sigma(k)$, the grade ratio and its mean error sequence is r=(r(1),r(2)…r(n)).

3. Groupe the original data according to the degree of deviate.

Divide the original data into m groups ($m < n$), select m+1 threshold $\gamma(i), (1 \le i \le m+1)$ meet: $\min_{1\le i\le n}\{r(i)\} = r(1)\langle r(2)\langle ...\langle r(m+1) = \max_{1\le i\le n}\{r(i)\}$.

Make the error $r(i) \in [\gamma(j), \gamma(j+1)]$ into one group, and then get m set of data, correspond the grade ratio σ to the original sequence, and get the group of $x^{(0)}$.

$$x_1^{(0)} = (x^{(0)}(k_{11}), x^{(0)}(k_{12}), ..., x^{(0)}(k_{1s_1})) \tag{3}$$

$$x_2^{(0)} = (x^{(0)}(k_{21}), x^{(0)}(k_{22}), ..., x^{(0)}(k_{2s_2})) \tag{4}$$

$$x_m^{(0)} = (x^{(0)}(k_{m1}), x^{(0)}(k_{m2}), ..., x^{(0)}(k_{ms_m})) \tag{5}$$

In above formulas, $\sum_{j=1}^{m} s_j = n$, i=1,2,…m, j=1,2,…,s m .

Since c is a constant, so they are the spaced sequences,then build nonlinear GM (1, 1, α) model for the sequences.

4. Determine the optimal value of α for each sequence.

The divided difference of $x^{(1)}$ is d2(i), d2(i)>0. Calculate the derivative of the white form is:

$$\frac{d^2x^{(1)}(t)}{dt^2}+a\alpha(x^{(1)}(t))^{\alpha-1}\frac{dx^{(1)}(t)}{dt} \quad (6)$$

When t=k, the first and second order difference quotient of $x^{(1)}$ instead of $\frac{d\ x^{(1)}(t)}{dt}$ and $\frac{d^2x^{(1)}(t)}{dt^2}$, $z^{(1)}(k)$ replace the nonlinear term, we can get formula (7):

$$x^{(0)}(k)-x^{(0)}(k-1)=-a\alpha[\frac{1}{2}(x^{(1)}(k)+x^{(1)}(k-1))]^{\alpha-1}x^{(0)}(k) \quad (7)$$

When t=k+1, do as the above steps can get formula (8):

$$x^{(0)}(k+1)-x^{(0)}(k)=-a\alpha[\frac{1}{2}(x^{(1)}(k+1)+x^{(1)}(k))]^{\alpha-1}x^{(0)}(k+1) \quad (8)$$

Taking the logarithm after the two equations divided:

$$\ln[\frac{x^{(0)}(k+1)-x^{(0)}(k)}{x^{(0)}(k)-x^{(0)}(k-1)}.\frac{x^{(0)}(k)}{x^{(0)}(k+1)}]/\ln[\frac{x^{(1)}(k+1)+x^{(1)}(k)}{x^{(1)}(k)+x^{(1)}(k-1)}]+1 \quad (9)$$

Put k=2,3,…,n-1 into the above formula, we can get n-2 different α, that is αk. Make $g(\alpha)=\sum_{k=2}^{n-1}(\alpha-\alpha_k)^2$, and take $g(\alpha)$ minimum, α is the optimal undetermined constant value.

5. Determin model parameters a and b.

The differential equations with clear form is:

$$\frac{dx^{(1)}(t)}{dt}+\hat{a}x^{(1)}(t)^{\alpha}=\hat{u} \quad (10)$$

Mark $B=\begin{pmatrix} -[x^{(1)}(1)+\frac{1}{2}x^{(1)}(2)]^{\alpha} & 1 \\ -[x^{(1)}(2)+\frac{1}{2}x^{(1)}(3)] & 1 \\ \cdots & \cdots \\ -[x^{(1)}(n-1)+\frac{1}{2}x^{(1)}(n)] & 1 \end{pmatrix}, Y_n=\begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \ldots\ldots \\ x^{(0)}(n) \end{pmatrix}$,

Parameters a, u are calculated by:

$$[a,b]^T=(B^TB)^{-1}B^TY_n, \quad (11)$$

6. Solution of the model.

Make $x^{(1)}(t) = f(t)$,white form become into this formula

$$\frac{df(t)}{dt}=-af(t)^{\alpha}+b, \quad (12)$$

$$f_{t=0} = f(0).$$

7. Model test.

The improved GM (1, 1) also need to test by posterior difference and residual, specific, steps is same to traditional GM (1, 1).

3. Case Analysis

This paper uses Matlab 7.1 to forecast the electricity consumption data in the years 2001-2010, original data are as follows:

Table 1. Electric Power Consumption

Year	Consumption(10^8kw/h)
2001	400
2002	440
2003	468
2004	513
2005	571
2006	612
2007	667
2008	690
2009	739
2010	810

1. Calculate grade ratio and it's mean error, consequence are as follows:

Table 2. Grade Ratio and It's Mean Error

Serial Number	$\sigma(k)$	$\gamma(k)$
1	1	-0.073
2	1.1	0.027
3	1	-0.013
4	1.1	0.027
5	1.11	0.037
6	1.07	-0.003
7	1.09	0.017
8	1.03	-0.043
9	1.07	-0.003
10	1.1	0.027

2. Groupe the original sequence.

Get the groups of grade ratio sequence by choosing different threshold.That is γ_i =-0.07,0,0.04,and then get the groupes of original sequence:

$$x_1^{(0)} = (x^{(0)}(1), x^{(0)}(3), x^{(0)}(6), x^{(0)}(8), x^{(0)}(9)),$$
$$x_2^{(0)} = (x^{(0)}(2), x^{(0)}(4), x^{(0)}(5), x^{(0)}(7), x^{(0)}(10))$$

3. Determine the optimal α.

After calculated, α1 =1.03, α2 =1.07.

4. Determine the parameters a and b.

According to the traditional GM (1, 1) solving steps,we can get a1=-0.0745,b1=399.8497,using matlab7.1we get the fitting values of the model 1 are as follows:

$\hat{x}_1 =$（414, 481, 601, 698, 751, 839, 922, 1038）, fitting values of the model 2 are as follows: $\hat{x}_2 =$（446, 518, 558, 647, 810, 854, 956, 1129）.

5. Precision inspection

Test results is showed in Table 3 and Table 4.

Table 3. Precision of Model 1

Serial Number	$x^{(0)}(k)$	$\hat{x}^{(0)}(k)$	$\varepsilon(k)$
1	400	414	14
3	468	481	13
6	612	601	11
8	690	698	8
9	739	751	12
11		839	
12		922	
13		1038	

Table 4. Precision of Model 2

Serial number	$x^{(0)}(k)$	$\hat{x}^{(0)}(k)$	$\varepsilon(k)$
2	440	446	6
4	513	518	5
5	571	558	13
7	667	647	20
10	810	810	0
11		854	
12		956	
13		1129	

Prediction results shown in Table 5.

Table 5. Forecasting Results of Electric Power Consumption

Year	Actual Value (10^8kW/h)	Predicted Value of Traditional GM(1,1)				Predicted Value of Improved GM(1,1)			
		Predicted Value(10^8kW/h)	Residual Value	Relative Value(%)	Accuracy(%)	Predicted Value(10^8kW/h)	Residual Value	Relative Value(%)	Accuracy(%)
2001	400	432	32	8	92	414	14	3.5	96.5
2002	440	456	16	3.6	96.4	446	6	1.4	98.6
2003	468	500	32	6.8	93.2	481	13	2.8	97.2
2004	513	542	29	5.7	94.3	518	5	0.1	99.9
2005	571	589	18	3.2	96.8	558	13	2.3	97.7
2006	612	619	7	1.1	98.9	601	11	1.8	98.2
2007	667	638	29	4.3	95.6	647	20	3	97
2008	690	655	35	5.1	94.9	698	8	1.2	98.8
2009	739	723	16	2.2	97.8	751	12	1.6	98.4
2010	810	758	52	6.4	93.6	810	0	0	100
2011		842				[859,874]			
2012		913				[922,956]			
2013		998				[1038,1129]			
Mean accuracy		95.35				98.23			
Posterior Difference Ratio		0.42				0.38			

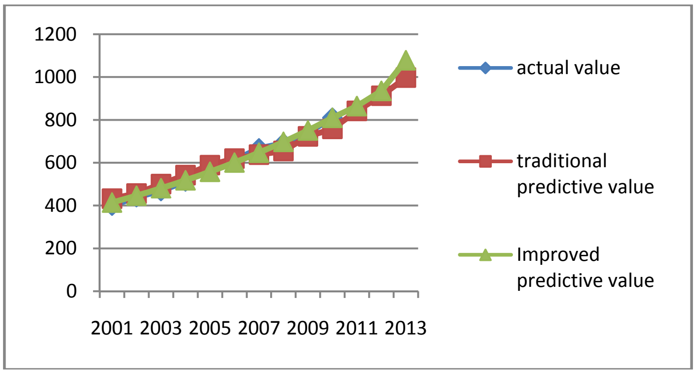

Figure 1. Electric Power Consumption

Table 5 shows that the prediction error of traditional GM (1, 1) model is bigger, it's average accuracy is 95.35%, posterior difference ratio is 0.42, while predicted results of the improved GM (1, 1) shows that it's average accuracy is 98.23%, and the posterior difference ratio is 0.38, it's fitting degree is better than traditional GM (1, 1), and the prediction accuracy is higher. Through the analysis, the improved GM (1, 1) model can get a better prediction.

Figure 1 can analysis the fitting degree to power consumption of traditional prediction model and improved model more intuitively, it can also analysis the future trend of the electric power consumption. With the economic rapidly growth, income level increased significantly, its electric power consumption also has a steady growth trend, and hasn't appear larger peak value.

4. Conclusion

Through the analysis of traditional GM (1, 1) and the improved model in actual forecast we can see that improved GM (1, 1) has greater advantages than traditional model in fitting original data and predicting the future data. Analysis and predict the electric power consumption accurately have a strong guiding significance for understanding people's quality of life, building economical society, amplifying the domestic demand and promoting economic development.

The improved GM (1, 1) which is this paper gived can response electric power consumption trend reasonably; it provides a scientific and effective method for forecasting the electric power consumption.

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