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Uniform Error Bounds for Reconstruct Functions from Weighted Bernstein Class

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Abstract

Errors appear when the Shannon sampling series is applied to reconstruct a signal in practice. This is because the sampled values may not be exact, or the sampling series may have to be truncated. In this paper, we study errors in truncated sampling series with localized sampling for band-limited functions from weighted Bernstein class. And we apply these results to some practical examples.

Keywords: *Shannon expansion (SE), localized sampling (LS), truncation error (TE), weighted berstein space (WBS)*

1. Introduction

Since Shannon introduced the sampling series in the context of communication in the landmark paper [1], the Shannon sampling theorem has been playing a fundamental role in signal processing. However, signal and sampled values are always not exact in real life, so there will arise some errors when applying the Shannon sampling theorem to practical signals. Until now, these errors have been widely studied in [2–9]. In this paper, we will study the truncation error when the functions are band-limited functions from weighted Bernstein space.

Let $L_p(R)$ be the space of all p -th power Lebesgue integrable functions on R equipped with the usual norm.

$$\|f\|_{L_p(R)} := \begin{cases} \left(\int_R |f(t)|^p \, dt\right)^{1/p}, & 1 \le p < \infty \\ \operatorname*{ess\,sup}_{t \in R} |f(t)|, & p = \infty \end{cases}$$

For convenience, we denote $\|\cdot\|_{L_p(R)}$ by $\|\cdot\|_p$.

For any positive number Ω , we say an entire function h is of exponential type Ω provided that for every $\varepsilon > 0$ there exists a positive number c , such that for all complex number $z \in C$, we have the bound:

$$|h(z)| \le c \exp\left((\Omega + \varepsilon)|z|\right)$$

We denote by $E_\Omega(C)$ the space of all entire functions of exponential type Ω . Let $B_\Omega(R)$ be the subset of $E_\Omega(C)$ which are bounded on R . Let $B_\Omega^p(R)$ denotes the Bernstein class [10] of entire functions of exponential type at most Ω whose restriction to R is in $L_p(R)$. In view of Schwartz's theorem every function $f \in B_\Omega^p(R)$ is band-limited to $[-\Omega, \Omega]$, that is:

$$B_\Omega^p(R) := \left\{ f \in L_p(R);\ \operatorname{supp} \hat{f} \subset [-\Omega, \Omega] \right\}$$

where $\hat{f}$ is the Fourier transform of f in the sense of distribution. In the special case $p = 2$, it reduces to the Paley-Wiener theorem [10].

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Let $B^p_{\Omega,\delta}(R)$ be weighted Bernstein class which is defined by:

$$B^p_{\Omega,\delta}(R) := \left\{ f : f(\cdot)\left(1+|\cdot|^{\delta}\right) \in B^p_{\Omega}(R), \delta > 0 \right\}$$

We organize this paper as follows. In section 2, we study the truncation error for univariate case. In section 3, we give some practical examples. In what follows, we often use the same symbol A, $A(\alpha)$ for possibly different positive constants. These constants are independent on N, Ω, and $A(\alpha)$ is dependent on α.

2. Truncation Errors of Univaria-te Shannon Sampling Expansion

The famous Whittaker-Kotelnikov-Shannon sampling theorem states that every function $f \in B^p_{\Omega}(R)$ can be completely reconstructed from its sampled values taken at instances $\{k/\Omega\}_{k\in\mathbf{Z}}$ [2–4]. In this case the representation of f is given by:

$$f(t) = \left(S_{\Omega} f\right)(t) := \sum_{k\in\mathbf{Z}} f\left(\frac{k}{\Omega}\right) \sin c(\Omega t - k), \quad (1)$$

Where $\sin c(t) = \sin \pi t/(\pi t)$, $t \neq 0$, and $\sin c(0) = 1$. Series (1) converges absolutely and uniformly on R.

Whittaker-Kotelnikov-Shannon's expansion requires us to know the exact values of a signal f at infinitely many points and to sum an infinite series. In practice, we use the finite sum $\sum_{k=-N}^{N} f\left(\frac{k}{\Omega}\right)\sin c(\Omega t - k)$ as an approximation to $f(t)$. In this way, we define the truncation error as follows:

$$\left| f(t) - \sum_{k=-N}^{N} f\left(\frac{k}{\Omega}\right) \sin c\left(\Omega t - k\right) \right|$$

Now we introduce the error modulus:

$$\Delta_{\Omega}(f,\lambda) := \sup\left|\lambda_k f(\cdot + k/\Omega) - f(k/\Omega)\right|, \ \Omega > 0$$

Where $\lambda = \{\lambda_k\}$ is any sequence of continuous linear functionals $C_0(R) \to R$ with $C_0(R)$ being the Banach space consisting of all continuous functions defined on R and tending to zero at infinity. We write Δ for $\Delta_{\Omega}(f,\lambda)$ if no confusion arises. The error modulus Δ provides a quantity for the quality of a signal's measured sampling values. When the functionals in λ are concrete, we may get some reasonable estimates for Δ. The sampling series with the measured sampled values is:

$$\left(S^{\lambda}_{\Omega} f\right)(t) := \sum_{k\in\mathbf{Z}} \lambda_k f(\cdot + k/\Omega) \sin c\left(\Omega t - k\right)$$

Similarly, we define another truncation error:

$$\left| f(t) - \sum_{k=-N}^{N} \lambda_k f\left(\cdot + \frac{k}{\Omega}\right) \sin c\left(\Omega t - k\right) \right|$$

Li and Ye have established uniform bound for:

$$\left| f(t) - \sum_{k=-N}^{N} f\left(\frac{k}{\Omega}\right) \sin c\left(\Omega t - k\right) \right|$$

And

$$\left| f(t) - \sum_{k=-N}^{N} \lambda_k f\left(\square + \frac{k}{\Omega}\right) \sin c\left(\Omega t - k\right) \right|$$

When $f \in B_{\Omega}^{p}(R)$ with decay condition.

Theorem A. [5] *Let* $f \in B_{\Omega}^{2}(R)$ satisfy the inequality:

$$\left| f(t) \right| \le L / \left| t \right|^{\delta}, \; t \ne 0$$

Where $L > 0$ and $\delta > 1$, then for $N \ge 8$,

$$\left| f(t) - \sum_{k=-N}^{N} f\left(\frac{k}{\Omega}\right) \sin c\left(\Omega t - k\right) \right|$$
$$\le L\left(2\Omega\right)^{\delta} \square \frac{2\sqrt{2}e}{\pi} \square \frac{(\pi + \ln N)}{N^{\delta}}.$$

Theorem B. [11] *Let* $1 \le p \le \infty$, $f \in B_{\Omega}^{p}(R)$, and satisfy the inequality.

$$\left| f(t) \right| \le L / \left| t \right|^{\delta}, \; t \ne 0$$

Where $L > 0$ and $\delta > 0$, then for $e^{2/\delta} \le N \le N_0$, $N_0 = \left[\Delta^{-1/\delta}\Omega\right]$, we have:

$$\left| f(t) - \sum_{k=-N}^{N} \lambda_k f\left(\square + k/\Omega\right) \sin c\left(\Omega t - k\right) \right|$$
$$\le A \square \left(\Omega / N\right)^{\delta} \square \ln N,$$

Where $[x]$ denotes the integral part of x.

In this section, we will truncate the series on the right hand side of (1) in two different ways. This method is based on localized sampling, which is motivated by the result of [6]. That is, if we want to estimate $f(t)$ we only sum over values of f on a part of $\mathbf{Z}/\Omega$ near t. Thus for any N we consider the finite sum.

$$\left(S_{\Omega,N} f\right)(t) := \sum_{\Omega t - k \in (-N,N]} f\left(\frac{k}{\Omega}\right) \sin c\left(\Omega t - k\right)$$

Or $$\left(S_{\Omega,N}^{\lambda} f\right)(t) := \sum_{\Omega t - k \in (-N,N]} \lambda_k f\left(\square + k/\Omega\right) \sin c\left(\Omega t - k\right)$$

as a approximation to f(t).

Then, we can define the associated truncation errors:

$$\left(E_{\Omega,N} f\right)(t) := \left| f(t) - (S_{\Omega,N} f)(t) \right|$$

$$\left(E_{\Omega,N}^{\lambda} f\right)(t) := \left| f(t) - (S_{\Omega,N}^{\lambda} f)(t) \right|$$

Our aims are to give the uniform bounds for $\left(E_{\Omega,N} f\right)(t)$ and $\left(E_{\Omega,N}^{\lambda} f\right)(t)$ when $f \in B_{\Omega,\delta}^{p}(R)$. Now we state our results.

Theorem 2.1. Let $f \in B_{\Omega,\delta}^{p}(R)$, $1 \le p \le \infty$. Then we have

$$\left(E_{\Omega,N} f\right)(t) \le A\left(\Omega / N\right)^{\delta + \frac{1}{p}} \left\| f(\square)\left(1 + \left|\square\right|^{\delta}\right) \right\|_{p}.$$

Theorem 2.2. Let $f \in B^p_{\Omega,\delta}(R)$, $1 \le p \le \infty$, and $\Omega > 1$, then for $e \le N \le N_0 = \left[\Delta^{-\frac{p}{p\delta+1}} \Omega \right]$ we have:

$$\left(E^{\lambda}_{\Omega,N} f \right)(t) \le A \left(\Omega / N \right)^{\delta + \frac{1}{p}} \ln N$$

Where $[x]$ denotes the integral part of x.

Remark 2.1. To compare Theorem A and 2.1, we will see that when the decay condition is replaced by a weaker one, a weighted Bernstein class, the truncation error magnitude is improved from $O\left(N^{-\delta} \ln N\right)$ to $O\left(N^{-\delta-1/p}\right)$. For Theorem B and 2.2, the truncation error magnitude is improved from $O\left(N^{-\delta} \ln N\right)$ to $O\left(N^{-\delta-1/p} \ln N\right)$.

We first establish some lemmas which will be used in the following proofs.

Lemma 2.1. [12] Let $f \in B^p_{\Omega}(R)$, $1 \le p \le \infty$ and $\{t_n\}$ be real separated sequence, i.e., $\inf_{i \ne j} \left| t_i - t_j \right| \ge \varepsilon > 0$, $i, j \in \mathbf{Z}$.

Then: $\sum_{n \in \mathbf{Z}} | f(t_n) |^p \le B \| f \|_p^p$ Where $B = \dfrac{8(e^{p\varepsilon\Omega/2} - 1)}{p\pi\Omega\varepsilon^2}$

Lemma 2.2. [13] For $t \in R$, $q \ge 1$, and $\Omega > 1$, we have:

$$\sum_{k \in \mathbf{Z}} \left| \sin c\left(\Omega t - k\right) \right|^q \le \frac{q}{q-1}.$$

Proof of Theorem 2.1. It is easy to see that $| f(t) | \le \left| f(t) \left(1 + |t|^{\delta}\right) \right|$, then by the definition of entire functions of exponential type Ω, we have the representation (1) of f. Then applying the Hölder's inequality with exponent p on $\left(E_{\Omega,N} f\right)(t)$, we have:

$$\left(E_{\Omega,N} f\right)(t)$$

$$= \left| \sum_{\Omega t - k \notin (-N,N]} f\left(\frac{k}{\Omega}\right)\left(1 + \left|\frac{k}{\Omega}\right|^{\delta}\right) \sin c\left(\Omega t - k\right)\left(1 + \left|\frac{k}{\Omega}\right|^{\delta}\right)^{-1} \right| \le \left(\sum_{\Omega t - k \notin (-N,N]} \left| f\left(\frac{k}{\Omega}\right)\left(1 + \left|\frac{k}{\Omega}\right|^{\delta}\right) \right|^p \right)^{\frac{1}{p}} \times \left(\sum_{\Omega t - k \notin (-N,N]} \left| \sin c\left(\Omega t - k\right)\left(1 + \left|\frac{k}{\Omega}\right|^{\delta}\right)^{-1} \right|^q \right)^{\frac{1}{q}}$$

Where $1/p + 1/q = 1$. Noticing that $f \in B^p_{\Omega,\delta}(R)$, then by Lemma 2.1 we have:

$$\left(\sum_{\Omega t - k \notin (-N,N]} \left| f\left(\frac{k}{\Omega}\right)\left(1 + \left|\frac{k}{\Omega}\right|^{\delta}\right) \right|^p \right)^{\frac{1}{p}} \le A\Omega^{\frac{1}{p}} \left\| f(\cdot)\left(1 + |\cdot|^{\delta}\right) \right\|_p$$

Set
$$h(t) = \left(\sum_{\Omega t - k \notin (-N,N]} \left| \sin c\left(\Omega t - k\right)\left(1 + \left|\frac{k}{\Omega}\right|^{\delta}\right)^{-1} \right|^q \right)^{\frac{1}{q}}$$

Note that $h(t + m/\Omega) = h(t),$ for all $t \in R$ and $m \in \mathbf{Z}$. Thus, we only need to consider its upper bound for $t \in [0, 1/\Omega]$,

$$h(t) \le \left(A \sum_{k \notin (-N,N]} |k|^{-q} \left(1 + \left|\frac{k}{\Omega}\right|^{\delta}\right)^{-q} \right)^{\frac{1}{q}}$$

$$\le \left(A \sum_{k \notin (-N,N]} |k|^{-q} \left|\frac{k}{\Omega}\right|^{-\delta q} \right)^{\frac{1}{q}}$$

$$\le A\Omega^{\delta} N^{-\delta-1/p}.$$

Collecting these facts, we have obtained:

$$\left(E_{\Omega,N} f\right)(t) \le A\left(\frac{\Omega}{N}\right)^{\delta+\frac{1}{p}} \left\| f(\cdot)\left(1+|\cdot|^{\delta}\right)\right\|_p .$$

Proof of Theorem 2.2. By triangle inequality, we get:

$$\begin{aligned}&\left(E_{\Omega,N}^{\lambda} f\right)(t)\\ &\le \sum_{\Omega t-k\in(-N,N]} \left|\left(f\left(\frac{k}{\Omega}\right)-\lambda_k f\left(\cdot+\frac{k}{\Omega}\right)\right)\sin c(\Omega t-k)\right|\\ &+\sum_{\Omega t-k\notin(-N,N]} \left| f\left(\frac{k}{\Omega}\right)\sin c(\Omega t-k)\right|\end{aligned}$$

We first estimate the first item of right-side. Applying the Hölder's inequality with exponent p_0 and by Lemma 2.2.

$$\begin{aligned}&\sum_{\Omega t-k\in(-N,N]} \left|\left(f\left(\frac{k}{\Omega}\right)-\lambda_k f\left(\cdot+\frac{k}{\Omega}\right)\right)\sin c(\Omega t-k)\right|\\ &\le (2N+1)^{1/p_0}\,\Delta p_0 .\end{aligned}$$

Where $1/p_0+1/q_0=1$. Let $p_0=\ln N$. It is obvious that $p_0\ge 1$ and $N=e^{p_0}$, then $\left(2N+1\right)^{1/p_0}\le Ae$ $\ln N\ge 1$.
Hence:

$$\begin{aligned}&\sum_{\Omega t-k\in(-N,N]} \left|\left(f\left(\frac{k}{\Omega}\right)-\lambda_k f\left(\cdot+\frac{k}{\Omega}\right)\right)\sin c(\Omega t-k)\right|\\ &\le A\Delta\ln N.\end{aligned}$$

The assumption $N\le N_0$ implies $\Delta\le\left(\Omega/N\right)^{\delta+1/p}$, and by Theorem 2.1, we have desired result.

3. Some Applications

In this section, we apply Theorems 2.2 to some practical examples. The first one is that the measured sampled values are given by local average. Let $\lambda=\{\lambda_k\}$ be a sequence of continuous linear functionals. The sampled values are given by the local averages of a function may be formulated by the following integral representation.

$$\lambda_k(f):=\int f(v)u_k(v)dv , \qquad (2)$$

Where u_k for each $k\in\mathbf{Z}$ is a weight function characterizing the inertia of measuring apparatus. Particularly, in the ideal case, the function u_k is given by Dirac δ-function, $u_k=\delta(\cdot-t_k), t_k=k/\Omega,$ then $\lambda_k(f)=f(t_k)$ is the exact value of t_k. Gröchenig first studied the reconstruction of signal from local averages in 1992 [14]. After that some authors studied the approximation error when local averages are used as sampled values [15–17]. Now we assume that the functionals λ_k are given by (2) in terms of the weight functions u_k, and u_k satisfy the following properties:

i). $\operatorname{supp} u_k\subset\left[\frac{k}{\Omega}-\sigma_k^{'},\frac{k}{\Omega}+\sigma_k^{''}\right],\quad \frac{\sigma}{4}\le\sigma_k^{'},\sigma_k^{''}\le\frac{\sigma}{2},\quad \sigma$ is a positive constant;

ii). $u_k(t)\ge 0$, $\int u_k(t)dt=1$; (3)

iii). $\alpha = \inf_{k\in\mathbf{Z}}\{\alpha_k\}$, where $\alpha_k := \int_{k/\Omega-\sigma/4}^{k/\Omega+\sigma/4} u_k(t)dt$.

Note that now:

$$\lambda_k(f) = \int_{k/\Omega-\sigma_k^{'}}^{k/\Omega+\sigma_k^{''}} u_k(t) f(t)dt$$

To estimate $\left(E_{\Omega,N}^{\lambda} f\right)(t)$, we need the concept of the modulus of continuity. For $f \in C_0(R)$, we define the modulus of continuity by:

$$\omega(f,\sigma) = \sup_{|h|\le\sigma} \left\| f(\cdot+h) - f(\cdot)\right\|_{\infty}$$

Where σ is any positive number. Note that the functions in $C_0(R)$ are uniformly continuous on R. Therefore, we have $\lim_{\sigma\to 0}\omega(f,\sigma) = 0$.

Theorem 3.1. Let $f \in B_{\Omega,\delta}^{p}(R)$, $1 \le p \le \infty$, $\delta > 0$, $\Delta_\Omega(f,\lambda) \le \Omega^{-1}$ and $\Omega > e$. Suppose $\lambda_k(f)$ is obtained by the rule (2) and (3). If $\omega(f,\sigma/4) \le \min\left\{\left(((2-\alpha)e)^{-1}, ((2-\alpha)\Omega)^{-1}\right)\right\}$, then for $N > N_0$:

$$\left(E_{\Omega,N}^{\lambda} f\right)(t) \le A(\alpha)\omega(f,\sigma/4)\ln\frac{1}{\omega(f,\sigma/4)}$$

Proof. As in the proof of Theorem 2.2 and by the assumption $\Delta \le \Omega^{-1}$, we have:

$$\left(E_{\Omega,N_0}^{\lambda} f\right)(t) \le A\Delta_\Omega(f,\lambda)\ln\frac{1}{\Delta_\Omega(f,\lambda)}.$$

Then by triangle inequality,

$$\left(E_{\Omega,N}^{\lambda} f\right)(t) \le \left(E_{\Omega,N_0}^{\lambda} f\right)(t) + \left|\left(S_{\Omega,N_0}^{\lambda} f\right)(t) - \left(S_{\Omega,N}^{\lambda} f\right)(t)\right|$$

Now we are to compute $\left|\left(S_{\Omega,N_0}^{\lambda} f\right)(t) - \left(S_{\Omega,N}^{\lambda} f\right)(t)\right|$. As in the proof of Lemma 2.6 in [18], let *N* be a positive integer such that $N \ge \sigma\Omega$. If $|k| \ge N + \lceil \Omega t\rceil$, $t \in \left[-\frac{\sigma}{2},\frac{\sigma}{2}\right]$, then $\left|t+\frac{k}{\Omega}\right| \ge A\frac{|k|}{\Omega}$.

Hence,

$$\begin{aligned}
&\int_{k/\Omega-\sigma_k^{'}}^{k/\Omega+\sigma_k^{''}} u_k(t) f(t)dt \\
&\le \left(\int_{-\sigma_k^{'}}^{\sigma_k^{''}} \left|\frac{u_k(t+k/\Omega)}{1+|t+k/\Omega|^{\delta}}\right|^q dt\right)^{\frac{1}{q}} \left\| f(\cdot)\left(1+|\cdot|^{\delta}\right)\right\|_p \\
&\le A(\alpha)\left(\Omega/k\right)^{\delta}.
\end{aligned}$$

Note that:

$$\begin{aligned}
&\left|\left(S_{\Omega,N_0}^{\lambda} f\right)(t) - \left(S_{\Omega,N}^{\lambda} f\right)(t)\right| \\
&\le \sum_{\Omega t-k\in D}\left|\int_{k/\Omega-\sigma_k^{'}}^{k/\Omega+\sigma_k^{''}} u_k(t) f(t)dt \sin c(\Omega t-k)\right|,
\end{aligned}$$

Where $D = [-N,N]/[-N_0,N_0]$. Applying the Hölder's inequality with exponent $p_0 = \ln N_0$, and by the assumption $\Delta \le \Omega^{-1}$, we get:

$$\left|\left(S^{\lambda}_{\Omega,N_0} f\right)(t)-\left(S^{\lambda}_{\Omega,N} f\right)(t)\right| \le A(\alpha)\Delta_{\Omega}(f,\lambda)\ln\frac{1}{\Delta_{\Omega}(f,\lambda)}$$

Hence,

$$\left(E^{\lambda}_{\Omega,N} f\right)(t) \le A(\alpha)\Delta_{\Omega}(f,\lambda)\ln\frac{1}{\Delta_{\Omega}(f,\lambda)}.$$

It is clear that:

$$\begin{aligned}&\Delta_{\Omega}(f,\lambda)\\ &=\sup_{k\in\mathbf{Z}}\left|\int_{-\sigma_k}^{\sigma_k} u_k(t+k/\Omega)f(t+k/\Omega)-f(k/\Omega)dt\right|\\ &\le(2-\alpha)\omega(f,\sigma/4).\end{aligned}$$

By the assumption, we have $\Delta_{\Omega}(f,\lambda)\le e^{-1}$, and the function $x\to x\ln(1/x)$ is monotonely increasing for $x\in(0,e^{-1})$, thus we get the desired result.

Next, consider four usual types of errors existed in sampling series: the amplitude error, the time-jitter error, the truncation errors and the aliasing errors. We assume that the amplitude error is resulted from quantization, which means the functional value $f(t)$ of a function f at moment *t* is replaced by the nearest discrete value or machine number $\overline{f}(t)$. The quantization size is often known before hand or can be chosen arbitrarily. We may assume that the local error at any moment *t* is bounded by a constant $\varepsilon>0$, i.e., $\left|\overline{f}(t)-f(t)\right|\le\varepsilon$. The time -jitter error arises if the sampled instances are not met correctly but might differ from the exact ones by $\sigma_k, k\in\mathbf{Z}$, we assume $\sigma_k\le\sigma$ for all *k*. Now consider the combination error,

$$\left(E_N f\right)(t):=\left|f(t)-\sum_{\Omega t-k\in(-N,N]}\overline{f}(k/\Omega+\sigma_k)\operatorname{sinc}\left(\Omega t-k\right)\right|$$

Theorem 3.2. Let $f\in B^{p}_{\Omega,\delta}(R)$, $1\le p\le\infty$, $\delta>0$, $\Omega>e$. If $\left|\overline{f}(t)-f(t)\right|\le c_1\Omega^{-1}$, $\omega(f,\sigma/2)\le c_2\Omega^{-1}$, where c_1,c_2 are positive constants, $c_1+c_2\le 1$, then:

$$\left(E_N f\right)(t)\le A\Omega^{-1}\ln\Omega.$$

Proof. We define:

$$\lambda_k=\frac{\overline{f}(k/\Omega+\sigma_k)}{f(k/\Omega+\sigma_k)}\delta(\cdot-\sigma_k)$$

Where δ is the Dirac distribution. Then $\lambda=\{\lambda_k\}$ is a sequence of linear functional on $C_0(R)$. It is clear that $\lambda_k f(\cdot+k/\Omega)=\overline{f}(k/\Omega+\sigma_k)$, and:

$$\begin{aligned}&\left|\lambda_k f(\cdot+k/\Omega)-f(k/\Omega)\right|\\ &\le\left|\overline{f}(k/\Omega+\sigma_k)-f(k/\Omega+\sigma_k)\right|+\left|f(k/\Omega+\sigma_k)-f(k/\Omega)\right|\\ &\le\Omega^{-1}\end{aligned}$$

The follows from Theorem 2.2 and the monotonicity of the the function $x\to x\ln(1/x)$ for $x\in(0,e^{-1})$.

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