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High-Dimensional Chaotic System and its Circuit Simulation

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Abstract

The chaotic system plays an important role in electronic and electrical equipment, computer cryptography, computer communication and so on. In this paper, we established three new six-dimensional complex chaotic systems and one new ten-dimensional complex chaotic system. The regularity to generate high-dimensional chaotic system is also found by overlaying a series of low-dimensional chaotic systems with Duffing chaotic system. The circiuts of the new high-dimensional complex chaotic system are designed. The simulation experiments of the high-dimensional complex chaotic circuits are tested. The results of theoretical analysis and experiment show that new high-dimensional complex chaotic systems and their circiuts have the hyperchaotic characteristics.

Keywords: *electrical equipment, cryptography, chaotic circiut, high-dimensional chaos*

1. Introduction

The chaotic system, the quantum mechanics and the theory of relativity are three important scientific discoveries in the 20th century [1]. The chaotic system is widespread in electrical and electronic equipments [2], communication [3], astrophysics, aerospace, weather forecast, computer cryptography [4] and power network [5]. To any chaotic encryption system, the higher dimension it has, the better security it has [6]. By adding the state feedback controller on three-dimensional chaotic system, some five-dimensional chaotic systems are generated. For example: in 2009, Huaqing Li added state feedback controller on the three-dimensional Lorenz chaotic system to generate a five-dimensional Lorenz chaotic system [7]. In 2010, Feng Han added state feedback controller on the three-dimensional Lu chaotic system to generate a five-dimensional Lu chaotic system [8]. In 2011, Lu Huang added state feedbacks controller on the three-dimensional Chen chaotic system to generate a five-dimensional Chen chaotic system [9]. In this paper, we will study how to generate a six-dimensional chaotic system, a ten-dimensional chaotic system and a chaotic simulation circuit. Then, we will explore the regularity to generate high-dimensional chaotic system. The result of this study will has practical significance in electrical and electronic circuits, chaotic cryptography and communication.

2. New Duffing-Lorenz Chaotic System

2.1. The Design of Duffing-Lorenz Chaotic System

Duffing chaotic system is one of the commonly used models in signal transmission and frequency transform field [10]. The form of the Duffing chaotic system is:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -dy - x^3 + e\cos wt \end{cases} \tag{1}$$

D and e are real constants. The characteristic properties of the Lorenz system as the first discovered physical chaotic system are found in more and more fields. The form of the three-dimension Lorenz chaotic system is as the following:

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$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = bx - xz - y \\ \dot{z} = xy - cz \end{cases} \quad (2)$$

The parameters of a~c are real constants. The external excitation part of the original Duffing system is replaced by an autonomous part. By the bridge of the autonomous part, Equation (1) and Equation (2) are combined into a new six-dimensional complex chaotic system.

$$\begin{cases} \dot{x} = a(y - x) + gv \\ \dot{y} = bx - xz - y + w \\ \dot{z} = xy - cz \\ \dot{u} = v \\ \dot{v} = -dv - u^3 + e\cos(w) \\ \dot{w} = fxv \end{cases} \quad (3)$$

The parameters of a~g are real constants.

2.2. Lyapunov Exponent Analysis

When the initial conditions are a=10, b=55, c=8/3, d=0.6, e=-3, f=1, x=1, y=1, z=1, w=1 and dt=0.005, the Lyapunov exponents are 1.399, 0.852, 0.039, -0.097, -1.453 and -14.977. Since there are three positive Lyapunov exponents, the system is in the hyperchaotic state [11].

3. New Duffing-Chen Chaotic System

3.1. The Design of Duffing-Chen Chaotic System

The form of Chen chaotic system is:

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = (c - a)x - xz + cy \\ \dot{z} = xy - bz \end{cases} \quad (4)$$

Equation (1) and Equation (4) are overlaid into a new six-dimensional Duffing-Chen complex hyperchaotic system:

$$\begin{cases} \dot{x} = a(y - x) + gv \\ \dot{y} = (b - a)x - xz + hy + w \\ \dot{z} = xy - cz \\ \dot{u} = v \\ \dot{v} = -dv - u^3 + e\cos(w) \\ \dot{w} = fxv \end{cases} \quad (5)$$

The parameters of a~h are real constants.

3.2. Lyapunov Exponent Analysis

When the parameters are a=10, b=55, c=8/3, d=0.6, e=-3, f=1, g=3, h=1, x=1, y=1, z=1, u=1, v=1, w=1 and dt=0.005, the Lyapunov exponents are 0.970, 3.108, -1.468, -0.461, -3.574 and -10.829. Since there are two positive Lyapunov exponents, the system is in the hyperchaotic state.

4. The Exploration of Regularity

By the analysis of the above mentioned two kinds of the new six-dimensional complex hyperchaotic systems, the regularity to generate the high-dimensional complex hyperchaotic systems is discovered. The external excitation part of the Duffing system is replaced by an autonomous part. The positive feedback is added. Then, by the bridge of the autonomous part, the Duffing chaotic system and the lower-dimensional chaotic system are combined into a new six-dimensional complex hyperchaotic system. The new overlaying regularity of complex hyperchaotic system is shown in Figure 1.

$$\begin{aligned}&part\ 1: Lorenz\ (or\ Another\ )\ chaotic\ \ system\ +Feedback\ \ 1\\&part\ 2: Duffing\ \ chaotic\ \ system\\&part\ 3: +Feedback\ \ 2\end{aligned}$$

Figure 1. The Overlaying Regularity

The above process can be subdivided into three steps: (1) The parameters of positive feedback and the form of feedback are ascertained; (2) The autonomous part of the Duffing chaotic system is ascertained; (3) The two chaotic systems are combined into a new high-dimensional complex hyperchaotic system.

5. The Verify of Regularity and its Circuit Simulation

5.1. The Design of Duffing-Lu Complex Chaotic System

Lu chaotic system is as the following:

$$\begin{cases}\dot{x}=a(y-x)\\ \dot{y}=-xz+cy\\ \dot{z}=xy-dz\end{cases} \tag{6}$$

Be based on the regularity to generate high-dimensional complex chaotic system, the Duffing chaotic system of Equation (1) and the Lu chaotic system of Equation (6) are combined into a new six-dimensional Duffing-Lu complex hyperchaotic system.

$$\begin{cases}\dot{x}=a(y-x)+bw\\ \dot{y}=-xz+cy\\ \dot{z}=x^2-dz\\ \dot{u}=v\\ \dot{v}=-ev-u^3+f\cos(w)\\ \dot{w}=gyz\end{cases} \tag{7}$$

The parameters of a~g are real constants.

5.2. Lyapunov Exponent Analysis

When the initial conditions are a=36, b=1, c=20, d=3, e=0.6, f=3, g=1, x=1, y=1, z=1, u=1, v=1, w=1 and dt=0.005, the Lyapunov exponent are 1.22, 0.16, -0.38, -0.54, -1.17 and -

18.85. Since there are two positive Lyapunov exponents, the system of Equation (7) is in the hyperchaotic state.

5.3. The Circuit Simulation

Be based on Multisim 7, the simulation circuit of Equation (7) is shown in Figure 2.

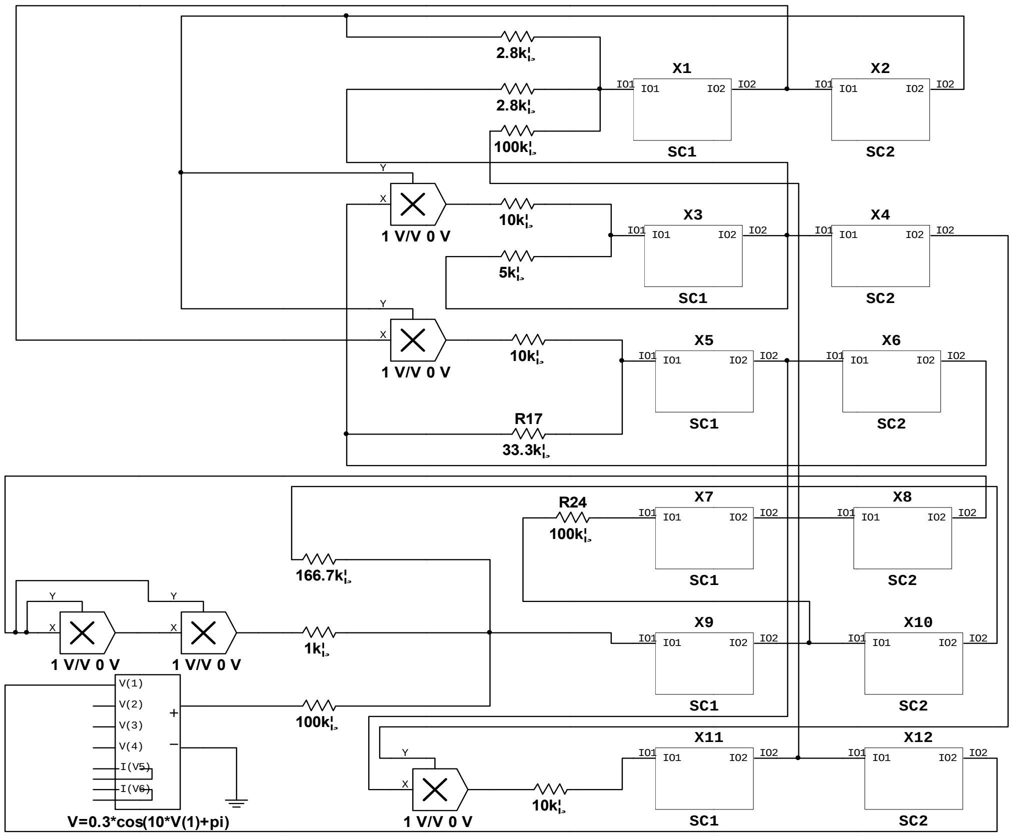
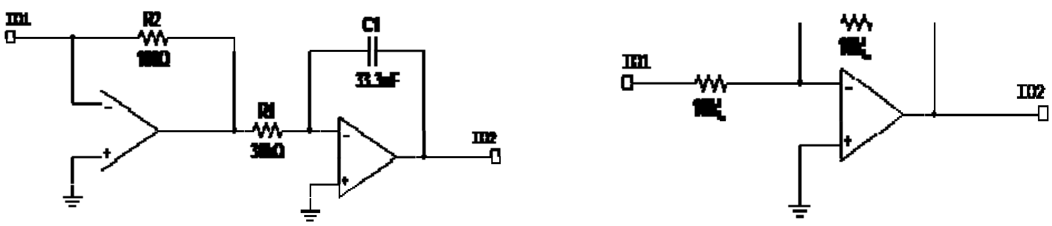


Figure 2. The Simulation Circuit

SC2 is opposition circuit. SC1 is proportion amplification circuit.The circuit pictures are:



(a) (b)

Figure 3. The Internal Connection Pictures of the Sub-circuit.(a)SC1.(b)SC2

The outputs of the simulation circuit are shown in Figure 4.

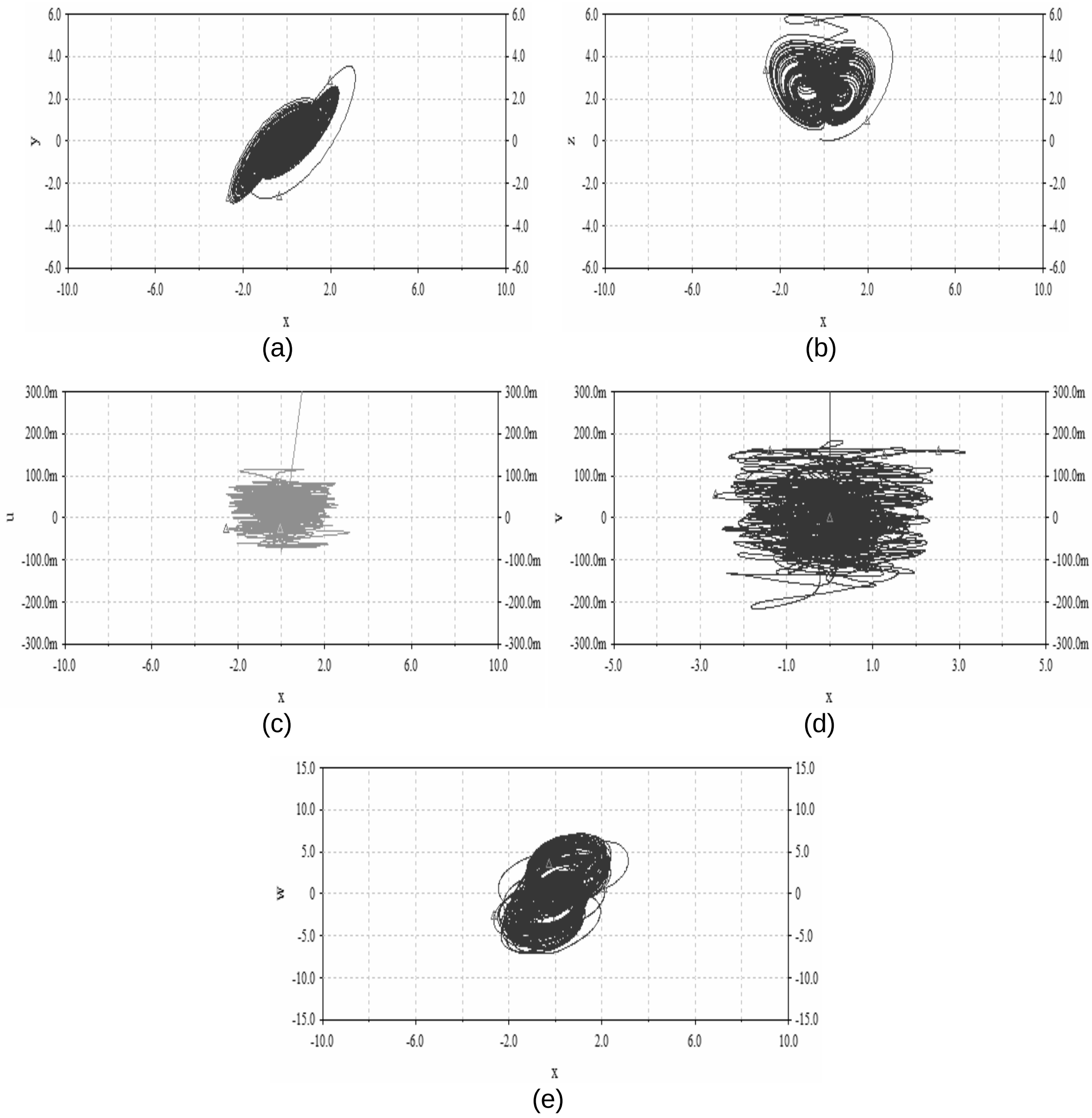


Figure 4. The Outputs of the Simulation Circuit (a)x-y, (b)x-z,(c)x-u, (d)x-v,(e)x-w

6. Another Verify of Regularity and its Circuit Simulation

6.1. The Design of Duffing-Lorenz-Sprott J Complex Chaotic System

The form of the three-dimensional Sprott J chaotic system is:

$$\begin{cases} \mathring{x} = az \\ \mathring{y} = -by + z \\ \mathring{z} = -x + y + y^2 \end{cases} \tag{8}$$

The parameters a and b are real constants.

Be based on the mentioned regularity of the high-dimensional complex chaotic system, the external excitation part of the original Duffing system is replaced by an autonomous part. The positive feedback is added. By the bridge of the autonomous part in the Duffing chaotic system, Equation (2), Equation (1) and Equation (8) are combined into a new ten-dimensional Duffing-Lorenz-Sprott J complex hyperchaotic system:

$$\begin{cases} \overset{\circ}{x} = a(y-x) + dzs - w \\ \overset{\circ}{y} = cx - xz - y \\ \overset{\circ}{z} = xy - bz \\ \overset{\circ}{u} = v \\ \overset{\circ}{v} = -ev - u^3 + f\cos(w) \\ \overset{\circ}{w} = gx \\ \overset{\circ}{p} = hr \\ \overset{\circ}{q} = -jq + r + kpr \\ \overset{\circ}{r} = -p + q + q^2 \\ \overset{\circ}{s} = -pr - s + p \end{cases} \tag{9}$$

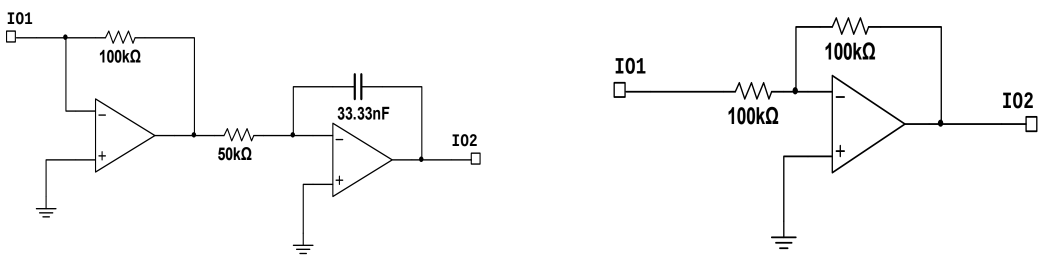
The parameters a~j are real constants.

6.2. Lyapunov Exponent Analysis

When the initial conditions are a=10, b=8/3, c=28, d=-2.5, e=0.6, f=-8, g=28, h=2, j=2, k=-2, x=1, y=1, z=1, u=1, v=1, w=1, p=1, q=2, r=1, s=1 and dt=0.005, the Lyapunov exponent are 0.622, 0.366, 0.116, 0.033, -0.364, -1.026, -0.926, -1.846, -2.727 and -11.107. Since there are four positive Lyapunov exponents, the system is in the hyperchaotic state.

6.3. The Circuit Simulation

Because of the limit of simulation experiment, the output of the simulation circuit is diminished 10 times. The circuit of SC2 is opposition circuit. The circuit of SC1 is proportion amplification circuit and integral circuit. The circuits of SC2 and SC1 are shown in Figure 5(a)-(b).



(a) (b)

Figure 5. The Internal Connection Picture of the Sub-circuit. (a)SC1,(b)SC2

W is diminished 20 times. P, q and r are enhanced 2 times. Based on the Multisim 7, the simulation circuit of Equation (9) is shown in Figure 6.

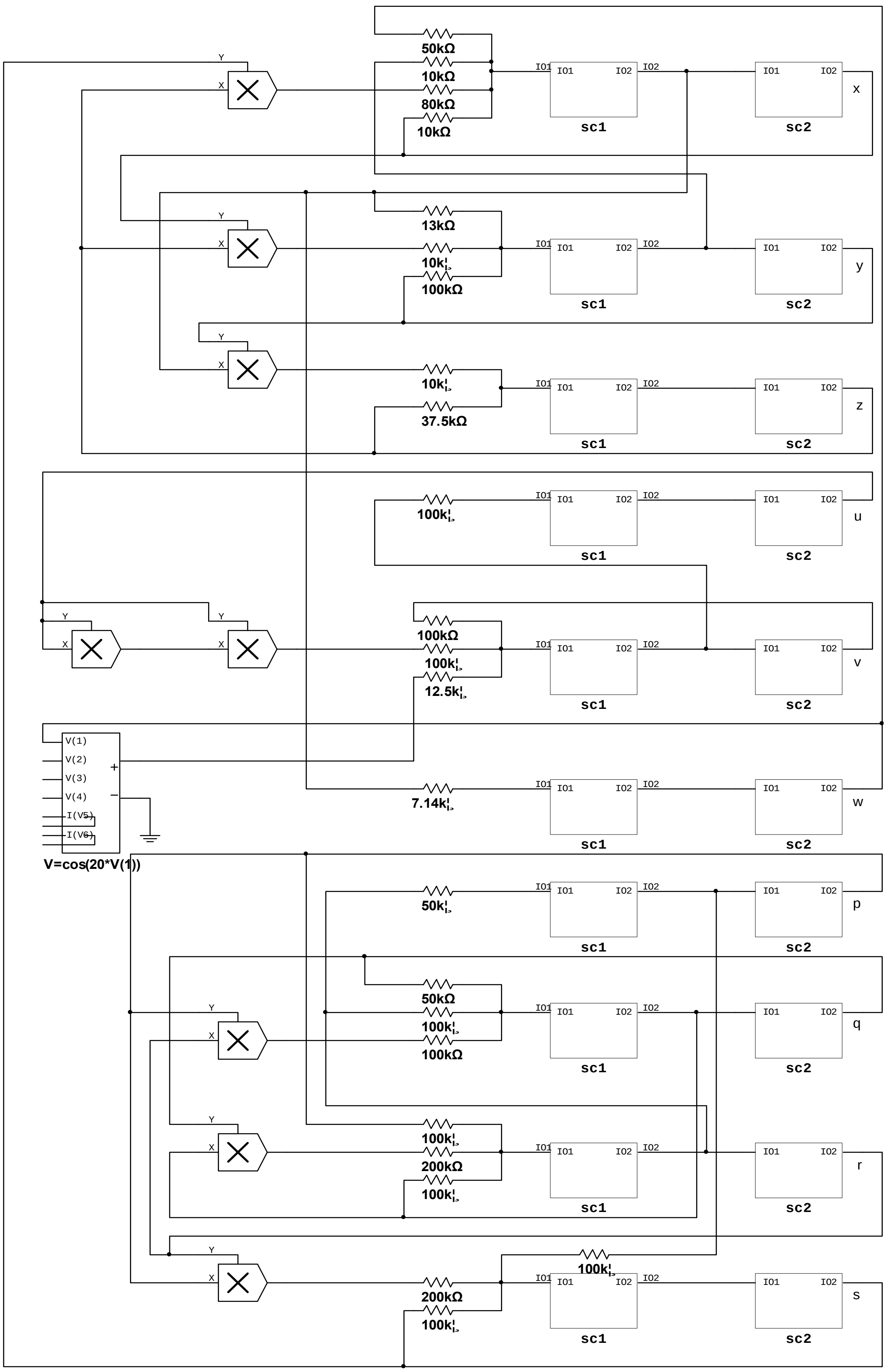


Figure 6. The Simulation Circuit

The outputs of the simulation circuit are shown in Figure 7(a)-(e).

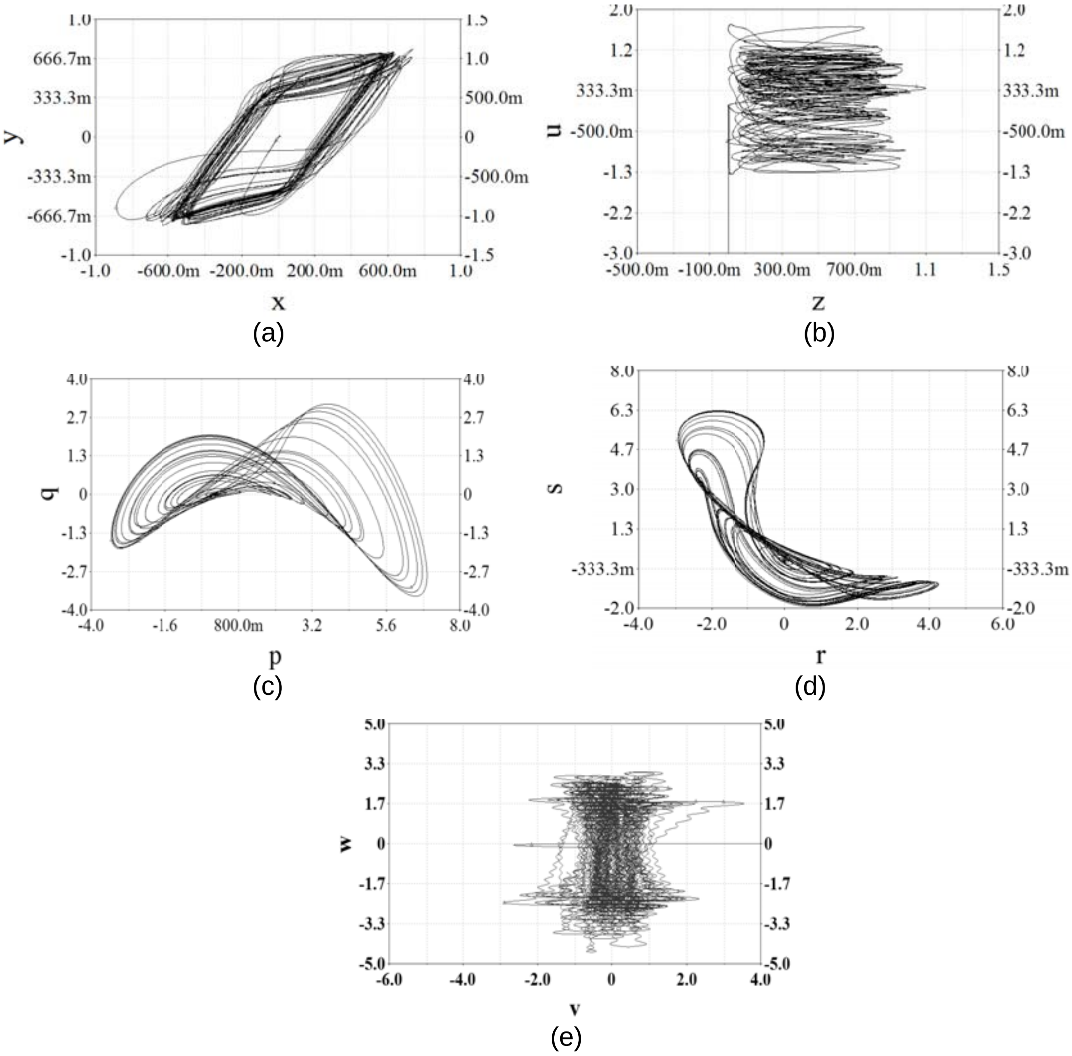


Figure 7. The Outputs of the Simulation Circuit (a) x-y,(b) z-u,(c) p-q,(d) r-s,(e) v-w

6.4. The Time-domain Test

The time-domain waveforms of the simulation circuit are shown in Figure 8(a)-(c).

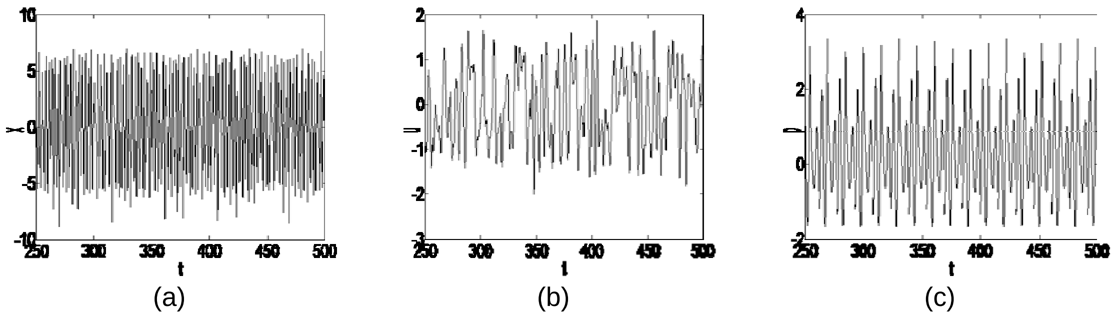


Figure 8. System Time-domains (a)t-x, (b)t-u, (c)t-p

7. Conclusion

Now, the popular researching direction of chaotic system is to design higher-dimensional hyperchaotic system. The five-dimensional complex hyperchaotic systems have been built by adding state feedback controller. In this paper, there are three new six-dimensional complex hyperchaotic systems and a new ten-dimensional complex hyperchaotic system to be found. When combining the low-dimensional chaotic system and the Duffing chaotic system to generate high-dimensional complex hyperchaotic system, the new combining regularity is found. The result of this study will lay the foundation to design a variety of new high-dimensional complex hyperchaotic systems in the future.

The experiments of the simulation circuits are designed and tested. The outputs of the simulation experiments confirmed the performance and effectiveness of the designed high-dimensional complex hyperchaotic systems. The research on high-dimensional complex hyperchaotic system and its circuit implementation will have important significance for the design of communication equipment, electronic equipment, electrical system and chaotic encryption system in the future.

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