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Research of Graph Compression in Information Storage

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Abstract

In the information storage, the image is often encountered, but the image storage and analysis are very complex, in order to more save memory space and more be used as a compressed representation of a graph, we give a new definition in the paper. We show some properties and give the lower integral sum number of some graph.

Keywords*: information processing, graph compression, graph, labeling*

1. Introduction

In the information storage, the image is often encountered. Image compression for image storage and transmission are very necessary [1, 2], but the image storage and analysis are very complex. In order to more save memory space and more be used as a compressed representation of a image, a image is mapped into a graph, we can use the labeling of graph to compress. Some relevant results about the (integral) sum number of graphs can be found in [3-6]. In this paper we give a new definition, it is more save memory space.

Let $\lfloor x \rfloor$ denote the largest integer which is not larger than the real x, Q^* denote the set of all the positive reals. The lower integral sum graph $G_+(S)$ of a nonempty finite subset $S \subset Q^*$ is the graph (S, E) with $uv \in E$ if and only if $\lfloor u+v \rfloor \in S$. A graph G is said to be a lower integral sum graph if it is isomorphic to the lower integral sum graph of some $S \subset Q^*$. We said that S is one of the lower integral sum labeling, and we consider the vertices and labeling as the same. The lower integral sum number $\sigma'(G)$ is the smallest number of isolated vertices which when added to G resulted in a lower integral sum graph.

It is obvious that $\sigma'(G) \le \sigma(G)$ for any graph G, so lower integral sum labeling not only for more saving memory space but also for more be used as a compressed representation of a graph.

A vertex w is called a working vertex, if there exists an edge uv such that $w = \lfloor u+v \rfloor$.

2. Result and Proof

The constitutive properties of lower integral sum graph have the universality, and we determined the lower integral sum number through the properties.

Theorem 2.1. If G is a sum graph, then $K_1 \vee G$ is a lower integral sum graph.

Proof. Let S is a sum labeling of graph G, we will prove that the labeling $S' = S \bigcup \{k\}$ is a lower integral sum labeling of $K_1 \vee G$, where $0 < k < 1$. For any $u, v \in S'$, if $u, v \in V(G)$, then there exsit w such that $u + v = w$, so $uv \in E(K_1 \vee G)$; If $u = V(K_1)$, then $\lfloor k + v \rfloor = v$, so $uv \in E(K_1 \vee G)$; For any $u, v \in S'$, if $uv \notin E(G)$, then there are not exsit w such that $u + v = w$, since S is a sum labeling and $0 < k < 1$, so $uv \notin E(K_1 \vee G)$, thus the labeling $S' = S \bigcup \{k\}$ is a lower integral sum labeling of $K_1 \vee G$.

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Theorem 2.2. Suppose that G is a nonempty lower integral sum graph, then:

a) There exist no vertices v_1, v_2, v_3, v_4 such that $\lfloor v_1 \rfloor = \lfloor v_4 \rfloor, \lfloor v_2 \rfloor = \lfloor v_3 \rfloor$, $v_1v_2, v_3v_4 \notin E, v_1v_3, v_2v_4 \in E$.

b) There exist no vertices v_i $(1 \le i \le 5)$, such that $0 < v_i - v_5 \le 1$ $(1 \le i \le 4)$, $\lfloor v_1 + v_5 \rfloor = \lfloor v_4 + v_5 \rfloor$, $v_5v_1, v_5v_4, v_1v_3, v_2v_4 \in E, v_1v_3, v_2v_4 \notin E.$

c) There exist no vertices $v_i (1 \le i \le 6)$ such that $\lfloor v_i \rfloor = k$ $(1 \le i \le 3)$, $v_1v_4, v_2v_4, v_1v_5, v_3v_5, v_2v_6, v_3v_6 \in E$, $v_3v_4, v_2v_5, v_1v_6 \notin E$; or $v_1v_4, v_2v_4, v_2v_5, v_3v_5, v_2v_6, v_3v_6 \in E$, $v_3v_4, v_1v_5, v_3v_5, v_1v_6 \notin E$; or v_1v_4, v_2v_4, v_2v_5, $v_1v_6 \in E$, $v_3v_4, v_1v_5, v_3v_5, v_2v_6, v_3v_6 \notin E$; or $v_3v_4, v_2v_5, v_1v_6 \in E$, $v_1v_4, v_2v_4, v_1v_5, v_3v_5, v_2v_6, v_3v_6 \notin E$.

Theorem 2.3. For any lower integral sum graph G , if G have one of these graphs $C_5, C_6, K_{2,2,2}$ and $\overline{P}_6$ as an induced subgraph, then the number of working vertices of G is more than one.

Proof. We only prove the case that C_5 as the induced subgraph of G , similar to other cases can be proved by contradiction. Suppose the number of working vertices is one. We may assume that k is the working vertex. Let :

$V(C_5) = \{v_i, 1 \le i \le 5\}, v_i = k_i + c_i (k_i = \lfloor v_i \rfloor, 1 \le i \le 5)$, then $0 \le c_i < 1$. Since:

$$\lfloor v_i + v_{i+1} \rfloor = k_i + k_{i+1} + \lfloor c_i + c_{i+1} \rfloor = k \quad (1 \le i \le 4) \tag{1}$$

$$\lfloor v_1 + v_5 \rfloor = k_1 + k_5 + \lfloor c_1 + c_5 \rfloor = k \tag{2}$$

We have:

$$|k_1 - k_3| \le 1 \tag{3}$$

$$|k_3 - k_5| \le 1 \tag{4}$$

$$|k_1 - k_4| \le 1 \tag{5}$$

We may assume without loss of generality that $k_1 \ge k_3$. By (3), we will consider two cases.

Case 1 $k_1 = k_3 = a$

By (5), we have:

$$a - 1 \le k_4 \le a - 1 \tag{6}$$

By (1), (6), we have:

$$2a - 1 \le k \le 2a + 2 \tag{7}$$

We consider two subcases.

Subcase 1 $\lfloor c_2 + c_1 \rfloor = 0$

By (1), (7), we have $a - 1 \le k_2 \le a + 2$.

a) If $k_2 = a - 1$, by (1), we have $k_5 = a - 1$ .By (1)(6), we have $k_4 = a - 1$, so $k_4 = k_5 = a - 1, k_1 = k_3 = a$, contradicting Theorem 2.2a).

b) If $k_2 = a$, by (1), we have $k = 2a, k_4 = a-1$ or a. Considering v_1, v_2, v_3, v_4, from Theorem 2.2a), we have $k_4 \neq k_1 = a$, so $k_4 = a-1$. By (1), (2) we have $k_5 = a$, thus $k_1 = k_2 = k_3 = k_5 = a$, contradicting Theorem 2.2a).

c) If $k_2 = a+1$, by (1), we have $k = 2a+1$, $k_4 = a$ or $a+1$. If $k_4 = a$, by (1), $k_5 = a$ or $a+1$. From Theorem 2.2a), considering v_1, v_3, v_4, v_5, we have $k_5 \neq k_4 = a$, but considering v_1, v_2, v_3, v_5, there have $k_5 \neq k_2 = a+1$, which is a contradiction. If $k_4 = a+1$, by (1), (2), we have $k_5 = a$. So $k_2 = k_4 = a+1$, $k_1 = k_5 = a$, contradicting Theorem 2.2a).

d) If $k_2 = a+2$, by (1)(5), we have $k = 2a+2, k_4 = a+1$, by (1), (2) we have $k_5 = a+1$. So $k_1 = k_3 = a$, $k_4 = k_5 = a+1$, contradicting Theorem 2.2a).

Subcase 2 $\lfloor c_2 + c_1 \rfloor = 1$.

By (1), (7), we have $a-2 \leq k_2 \leq a+1$. Similar to the proof of Subcase 1, we have the contradiction.

Case 2 $k_1 = a+1, k_3 = a$

By (1), we have $1 + \lfloor c_1 + c_2 \rfloor - \lfloor c_3 + c_2 \rfloor = 0$, thus $\lfloor c_2 + c_1 \rfloor = 0$, $\lfloor c_2 + c_3 \rfloor = 1$ So, we have:

$$a \leq k_4 \leq a+2 \quad (8)$$

$$2a \leq k \leq 2a+3 \quad (9)$$

By (1)(9), we have $a-1 \leq k_2 \leq a+2$. Similar to the proof of Subcase 1, we have the contradiction.

Therefore, the theorem holds.

3. The Lower Intergal Sum Number of some Graph

It is clear that star S_n is a lower integral sum graph , there are a lower integral sum labeling of $S_n : S = \{c = 1, a_i = (\frac{1}{10})^i \ (i = 1, 2, \cdots, n-1)\}$.

Theorem 3.1. $\sigma'(S_n \vee K_1) = 0 \quad (n \geq 2)$.

Proof. Let $E = E(S_n \vee K_1)$, we consider the following labeling of $S_n \vee K_1$:

$S = \{c_1 = \frac{1}{2}, c_2 = \frac{3}{4}, a_1 = 1, a_i = 2(n-1) + i \ (2 \leq i \leq n-1)\}$. It is easy to verify that the following assertions are true.

a) The vertices in S are distinct and $S \subset Q^*$.

b) For any $2 \leq i, j \leq n-1$, $a_i a_j \notin E$.

c) Since $\lfloor c_1 + c_2 \rfloor = 1 \in S$, then $c_1 c_2 \in E$.

d) For any $2 \leq i \leq n-1, 1 \leq j \leq 2$, since $\lfloor a_i + c_j \rfloor = a_i \in S$, then $a_i c_j \in E$.

Therefore, we know that the above labeling is a lower integral sum labeling, and $\sigma'(S_n \vee K_1) = 0 \quad (n \geq 2)$.

Theorem 3.2. $\sigma'(C_5 \cup K_n) = 1 \quad (n \geq 2)$

Proof. Let $E = E(C_5 \cup K_n)$, $V(C_5) = \{a_i \mid 1 \leq i \leq 5\}$, $V(K_n) = \{b_j \mid 1 \leq j \leq n\}$

$E(C_5)=\{a_i a_{i+1}\in E\,|\,1\le i\le 5\,(a_6=a_1)\}$,.

1) First we prove $\sigma'(C_5\bigcup K_n)\ne 0$. Suppose $\sigma'(C_5\bigcup K_n)=0$, according to the symmetry, let a_1 or b_1 be the largest integer, we just need to discuss the following two cases:

a) If a_1 is the largest , then $a_2, a_5<1, a_3, a_4, b_j\ge 1$. a_3 is not an integer, if not, we have $a_3a_5\in E$, which is a contradiction. Symmetrically, a_4 is not an integer. $b_j(1\le j\le n)$ are not integers, if not, we have $b_j a_2\in E$, which is a contradiction. So a_1 is only one working vertex, contradicting Theorem 2.3.

b) If b_1 is the largest integer, then $a_i\ge 1\,(1\le i\le 5), b_j<1\,(2\le j\le n)$. $a_i\ge 1\,(1\le i\le 5)$ are not integers, if not, we have $a_i b_2\in E\ (1\le i\le 5)$, which is a contradiction. So b_1 is only one working vertex, contradicting Theorem 2.3.

(2) Let $S=\{a_1=6, a_2=3.8, a_3=2.6, a_4=6.4, a_5=3.3, b_j=4.5+(\frac{1}{10})^{j+1}\ (1\le j\le n)\}$ $w=9,\ S_1=S\bigcup\{w\}$. It is easy to verify that the following assertions are true.

a) The vertices in S_1 are distinct and $S_1\subset Q^*$.

b) $a_i a_{i+1}\in E, a_5a_1\in E,\ a_i a_j\notin E(j\ne i+1), wa_i\notin E$, $wa_5\notin E\ (1\le i\le 4)$.

c) For any $1\le i<j\le n$, $b_i b_j\in E$, $wb_j\in E$, $a_k b_j\notin E\ (1\le k\le 5)$

Therefore, we know that S_1 is a lower integral sum labeling of $C_5\bigcup K_n\bigcup K_1$.

Theorem 3.3. $K_{r,s}-E(mK_2)\,(r,s\ge m)$ is a lower integral sum graph if and only if $m=1,2,3$.

Proof. Let $E=E(K_{r,s}-E(mK_2))$ and $V(K_{r,s})=(V,U)$ be the bipartition of $K_{r,s}$, $V=\{a_i\,|\,1\le i\le r\}, U=\{b_j\,|\,1\le i\le s\}$, $E(mK_2)=\{a_i b_i\,|\,1\le i\le m\}, S=V\bigcup U$.

1) We consider the following labeling of $K_{r,s}-E(K_2)$:

$$S=\{a_1=2, a_i=2-(\frac{1}{5})^{i-1}\ (2\le i\le r),\ b_1=1+(\frac{1}{6})^{r-1},\ b_j=1-(\frac{1}{4})^{j-1}\ (2\le j\le s)\}.$$

It is easy to verify that the following assertions are true.

a) The vertices in S are distinct and $S\subset Q^*$.

b) For any $2\le j\le s$, $2\le i\le r$, $a_1 b_j\in E$, $a_i b_1\in E$, $a_i b_j\in E$.

c) Since $\lfloor a_1+b_1\rfloor=3\notin S$, then $a_1b_1\notin E$.

d) For any $1\le i<j\le r$, since $\lfloor a_i+a_j\rfloor=3\notin S,$ then $a_i a_j\notin E$.

e) For any $1\le i<j\le s$, since $\lfloor b_i+b_j\rfloor=1\notin S,$ then $b_i b_j\notin E$.

Therefore, we know that the above labeling is a lower integral sum labeling, and $\sigma'(K_{r,s}-E(K_2))=0$.

2) We consider the following labeling of $K_{r,s}-E(2K_2)$:

$$S=\{a_1=5, a_2=3.9, a_3=4, a_i=4-(\frac{1}{10})^{i-2}\ (4\le i\le r),\ b_1=1.1, b_2=0.05,$$

$$b_j=1-(\frac{1}{5})^j\ (3\le j\le s)\}\cdot$$

Follow case 1, we can similar to prove that the above labeling is a lower integral sum labeling, and $\sigma'(K_{r,s}-E(2K_2))=0$.

3) We consider the following labeling of $K_{r,s}-E(3K_2)$:

$$S=\{a_1=5, a_2=1.5, a_3=2.45, a_4=2, a_i=2+(\frac{1}{10})^{i-4}\ (5\le i\le r),\ b_1=3.5, b_2=0.4,$$

$$b_3=0.55, b_j=\frac{1}{2}+(\frac{1}{10})^j\ (4\le j\le s)\}.$$

Follow case 1, we can similar to prove that the above labeling is a lower integral sum labeling, and $\sigma'(K_{r,s}-E(3K_2))=0$.

4) Suppose $\sigma'(K_{r,s}-E(mK_2))=0\ (m\ge 4)$.

According to the symmetry, the largest integer marking may suppose is a_m or $a_n\ (n>m)$. Since $m\ge 4$, then $\lfloor b_i \rfloor=0\ (1\le i\le 3)$, thus there exist $a_i, b_i\ (1\le i\le 3)$ satisfying $b_2a_1, b_3a_1, b_1a_2, b_1a_3, b_3a_2, b_2a_3\in E,\ b_1a_1, b_2a_2, b_3a_3\notin E$, but contradicting Theorem 2.2c).

From the above discussion we have that the theorem holds.

4. Conclusion

From a practical point of view, sum graph labeling can be used as a compressed representation of a graph, a data structure for representing the graph. Data compression is important not only for saving memory space but also for speeding up some graph algorithms when adapted to work with the compressed representation of the input graph. So lower integral sum labeling not only for more saving memory space but also for more be used as a compressed representation of a graph.

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