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Some advantages of non-binary Galois fields for digital signal processing

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ABSTRACT

It is shown that the convenient processing facilities of digital signals that varying in a finite range of amplitudes are non-binary Galois fields, the numbers of which elements are equal to prime numbers. Within choosing a sampling interval which corresponding to such a Galois field, it becomes possible to construct a Galois field Fourier transform, a distinctive feature of which is the exact correspondence with the ranges of variation of the amplitudes of the original signal and its digital spectrum. This favorably distinguishes the Galois Field Fourier Transform of the proposed type from the spectra, which calculated using, for example, the Walsh basis. It is also shown, that Galois Field Fourier Transforms of the proposed type have the same properties as the Fourier transform associated with the expansion in terms of the basis of harmonic functions. In particular, an analogue of the classical correlation, which connected the signal spectrum and its derivative, was obtained. On this basis proved, that the using of the proposed type of Galois fields makes it possible to develop a complete analogue of the transfer function apparatus, but only for signals presented in digital form.

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1. INTRODUCTION

The theory of Galois fields (finite commutative bodies) is one of the most important tools of modern information theory [1], [2], in particular, the theory of noiseless coding [3], [4]. In particular, Galois fields make it possible to construct an analog of the Fourier transform, which applicable to discrete functions and signals (Galois Field Fourier Transform), which currently also are widely used, including for the development of algorithms for encoding and decoding signals [5]-[8].

In the literature mainly using the binary Galois fields, more precisely, the most widely used fields are $GF(2^m)$ where m is an integer. This seems quite natural, since in the overwhelming majority of cases using binary codes and binary logic, and the number of elements in the $GF(2^m)$, field exactly corresponds to the number of code binary combinations of length m.

Recently, however, there has been a renewed interest [9]-[11] in multivalued logics that go back to the logic of Lukasiewicz [12], which he considered as an alternative to Aristotle's logic, since the law of the excluded middle did not take place in it. This interest connected, among other things, with the fact that multivalued logics are of significant interest to the development of artificial intelligence [13], [14]. In the literature there are numerous works proving the advantages of using ternary logic [15]-[17] in information theory, and here appears a natural analogue of the widely used binary unit of information measurement bits, called "trit". It is also pertinent to note that non-binary Galois fields also find application for the implementation of encoding and decoding procedures [18]-[21].

However, the possibilities of using multivalued logics have one more aspect, which has not yet received sufficient attention in the literature. Namely, multi-valued, in particular, ternary logic is a promising means of slowly changing signals [22] analyzing (more broadly, signals with a limited derivative). This line of research seems to be quite important, firstly, because many signals, for example, registered by monitoring systems [23], [24], really change rather slowly to the point that they described by functions with ε–coverage (the difference between signals at the next and previous unit of time by the module does not exceed one). Second, as shown in [20], consideration of slowly varying signals makes it possible to reveal the relationship between the rate of signal change and the spectral sub-bands in which the signal carries the corresponding information. Specifically, in the cited work, it is shown that the spectral range, in which concentrated the spectrum of a signal which is possessed of ε-coverage, is naturally divided into three sub-bands, two of which carry information about signal variations with an amplitude reduced to unity. This representation of signals turns out to be closely related to ternary logic [22], which serves as one more argument in favor of its use. This question can be posed even more broadly. Specifically, in this work shown that non-binary Galois fields also have very significant advantages for digital processing of discrete signals of finite amplitude (which, strictly speaking, include all signals actually used in practice).

2. THE USAGE OF GALOIS FIELDS TO DESCRIBE DISCRETE SIGNALS WITH A FINITE RANGE OF AMPLITUDES VARIATION

Any discrete signals which using in practice, vary over a finite range of amplitudes. For example, the number of signal levels (which are handled by standard microprocessors that carry out analog-to-digital conversion) is equal to 256 levels, which corresponds to the standard analog-to-digital converter (ADC) capacity of 8, although in some cases using ADC of large capacity [25]. Accordingly, in practice using digital signals that correspond to a finite set of levels.

Each of these levels can be associated with an element of some Galois field, on conditions that the number of signal levels corresponds to the number of elements of this field. It is important to emphasize that the transition character from a continuous signal to a discrete signal is a matter of agreement, i.e., the scale of levels and their number, strictly speaking, can be selected based on considerations of convenience.

In particular, as will be clear from what follows, there is a certain scope of functions, for which it is expedient to choose the number of nonzero levels equal to some prime number p. In this case, it is permissible to establish a correspondence between the set of discrete signal levels and the Galois field $GF(p)$. We emphasize that a homomorphism of the ring of integers into residue-class rings generates a field (specifically, a Galois field) only if p is a prime number. In this case, a discrete signal can be considered as a function of time $g(t)$, taking values in the $GF(p)$ field or, when dividing the signal into unite of time, as an ordered g_i, each part of which is an element of the Galois field, and i is the unit of time number.

Establishing the specified correspondence between the signal levels and the elements of the Galois field is sufficient to construct the Galois Field Fourier Transform. Indeed, for any element ζ of an arbitrary Galois field containing $n+1$ elements, we have (1).

$$\zeta^{n+1} - \zeta = 0 \text{ and } \zeta^{n} - 1 = 0 \tag{1}$$

Further, one has a general theorem to the sum of primal element degree.

$$1 + \zeta + \zeta^2 + \cdots + \zeta^{n-1} = \begin{cases} \text{"n"}, \zeta = 1 \\ 0, \zeta \neq 1 \end{cases} \tag{2}$$

where n is the number of nonzero elements in the given Galois field.

This theorem is applicable to any element from any Galois field, since for $\zeta \neq 1$ one has the relation, which follows from the formula for the geometric progression.

$$1 + \zeta + \zeta^2 + \cdots + \zeta^{n-1} = \frac{1-\zeta^n}{1-\zeta} \tag{3}$$

We emphasize that in (3), the number n appears only formally, since the summation should be performed precisely in the sense of addition in this particular field, and n is far from necessarily its element.

The number n in (3), accordingly, is nothing more than a symbol implying the summation of n units. We construct the following sequences, starting from some primitive element θ, the degrees of which inclusive to $(n-1)$-th give all nonzero elements of the considered Galois field.

$$w_1 = (1, \theta, \theta^2, \theta^3, \dots, \theta^{n-1})$$
$$w_2 = \left(1, \theta^2, \theta^{2\cdot 2}, \theta^{2\cdot 3}, \dots, \theta^{2\cdot(n-1)}\right) \quad (4)$$
$$w_{n-1} = \left(1, \theta^{(n-1)}, \theta^{(n-1)\cdot 2}, \theta^{(n-1)\cdot 3}, \dots, \theta^{(n-1)\cdot(n-1)}\right)$$

These sequences can also be viewed as functions of discrete time.

We emphasize that for the field $GF(p)$, by virtue of (1), all the degrees appearing in (12), de facto, do not exceed p. Otherwise, included in them products of integers (degrees) are calculated by $mod(p+1)$. Such sequences, as follows from (4), are precisely $n = p - 1$, where n is the number of nonzero elements in the used Galois field. Let us supplement the set of these sequences with a sequence consisting only of ones.

$$w_0 = (1,1,1,1, \dots ,1) \quad (5)$$

Each sequence (4) formed in accordance with the following rule. Hold fixed some degree θ^k of the element θ. Accordingly, the m -th term of the k -th sequence will be equal to θ^{mk}, if we assume that the first term corresponds to the value $m = 0$. Sequence (5) also meets this rule if we put $k = 0$.

Examples of the form (4) sequences for $p = 17$ shown in Figure 1. When including (5) into the set (4), obviously, the number of sequences of the considered type will be equal to p - the number of elements of the considered Galois field. For each of the sequences (4), there is one and only one sequence from this set for which the condition:

$$\sum_{j=0}^{j=n-1} w_{k_1}^{(j)} w_{k_2}^{(j)} = 1 \quad (6)$$

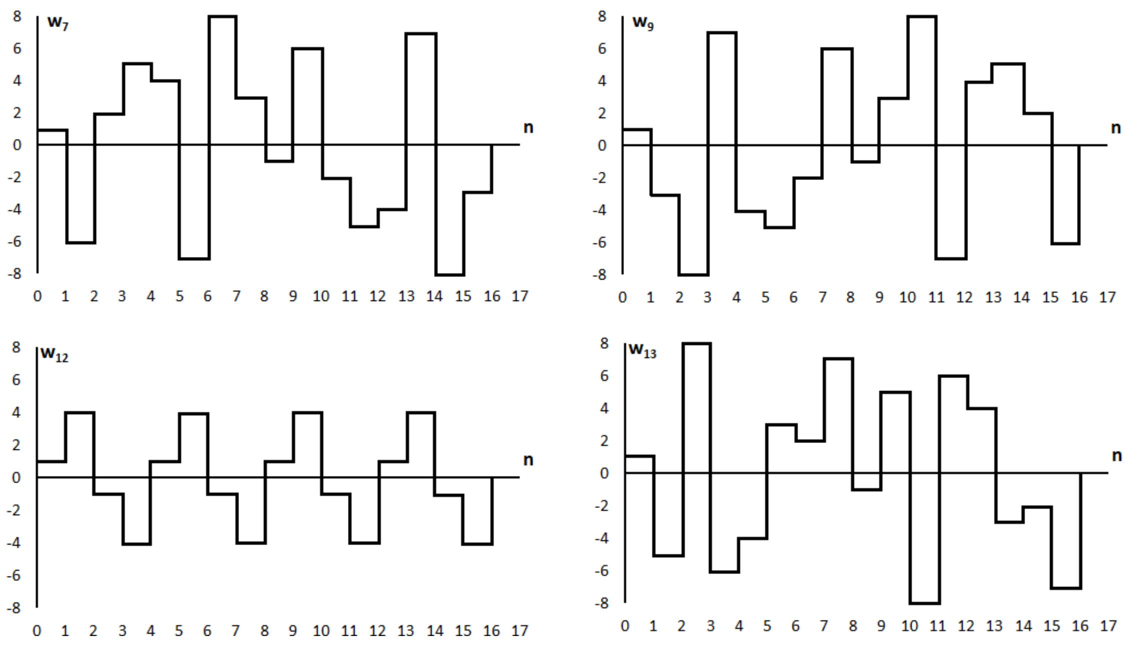

Figure 1. Examples of w_j sequence that allow the interpretation as a generalization of the Rademacher functions for different values of j (the numbers of sequence are indicated on the graphs)

Such sequences can be called conjugate; equality (6) is valid for sequences with numbers satisfying the condition:

$$k_1 \equiv k_2\big(mod(p+1)\big) \tag{7}$$

Relation (6) follows from the fact that the direct product of sequences (4) by each other has the form:

$$w_{k_1} * w_{k_2} = \left(1, \theta^{(k_1+k_2)}, \theta^{2(k_1+k_2)}, \dots, \theta^{(n-1)(k_1+k_2)}\right) \tag{8}$$

Assuming

$$\zeta = \theta^{(k_1+k_2)} \tag{9}$$

and applying (3), we obtain (10).

$$\sum_{j=0}^{j=n-1} w_{k_1}^{(j)} w_{k_2}^{(j)} = \begin{cases} 1, k_1 \equiv k_2\big(mod(p+1)\big) \\ 0, k_1 \not\equiv k_2\big(mod(p+1)\big) \end{cases} \tag{10}$$

Based on (10), one can immediately go to the spectral representation of the signal in the form

$$\vec{u} = \sum_{j=0}^{j=n-1} z_j \vec{w}_j \tag{11}$$

Multiplication of relation (11) by the vector $\vec{w}_j^T$ conjugate (in the sense of (6) with $\vec{w}_j$, by virtue of (10), gives (12).

$$\left(\vec{u}, \vec{w}_j^T\right) = \sum_{i=0}^{i=n-1} z_i \left(\vec{w}_i, \vec{w}_j^T\right) = z_j \tag{12}$$

It follows from relation (12) that the obtained sequences can be interpreted as generalized Rademacher functions [21]-[23] that make up a complete system only for the case of the minimum numbers of unit of time.

Indeed, relation (10) can be interpreted by analogy with the condition of functions orthogonality of a complex variable under condition that considered piecewise continuous functions, that are reducible to sequences containing p unit of time, which corresponds to the use of the field $GF(p)$. It can be seen that such functions forming a complete basis, i.e., therethrough can be represented any function defined in p unite of time, as well as any p-periodic function.

In this case, the elements of the z_j field that calculated by (12) have the same meaning as the spectral components calculated using one or another basis of complex-valued functions. The only one difference is that when using the orthogonal basis of complex-valued functions, the amplitudes of the spectral components are also appeared complex quantities, and when using the spectral representation in non-binary Galois fields, they are elements of the same field.

It is essential that due to this, the amount of data that needed to transmit information about the spectrum turn out to be significantly less than, for example, when using the Walsh basis [24], [25]. Indeed, the amplitude of the spectral components calculated by using the Walsh basis can vary over a very wide range, and in the case under consideration, it knowingly lies in the same range as the original signal.

3. DESCRIPTION OF THE DIGITAL SPECTRUM OF THE SIGNAL DERIVATIVE IN TERMS OF NON-BINARY GALOIS FIELDS

The functions presented above, which can be interpreted as generalized Rademacher functions, make it possible to obtain a result that is a direct analogue of the well-known property of harmonic signals, which consists in the fact that the Fourier transform of a signal, which time translated, differs from the initial one only by the phase factor.

$$F[u(t-t_0)] = e^{-i\omega t_0} \int e^{-i\omega t_1} u(t) dt = e^{-i\omega t_0} F[u(t)] \tag{13}$$

where t and ω are time and frequency variables, $F[u(t)]$ is the designation of the Fourier transform of the function (t), t_0 is the shift interval along the time axis. We obtain a direct analogue of property (13) for spectra, which obtained from generalized Rademacher functions.

The analogy with the time translation for periodic signals, that satisfy to the reporting system under consideration, the amplitude values of which displayed in a certain Galois field, is as follows. There is an original sequence.

$$\vec{f}(0) = (f_0, f_1, f_2, f_3, \dots, f_{n-1}) \quad (14)$$

Cyclic permutation by one position corresponds to a time shift by one cycle.

$$\vec{f}(1) = (f_{n-1}, f_0, f_1, f_2, \dots, f_{n-2}) \quad (15)$$

$$\vec{f}(2) = (f_{n-2}, f_{n-1}, f_0, f_1, \dots, f_{n-3}) \quad (16)$$

Let us form a direct product.

$$\vec{w}_k * \vec{f}(1) = \left(1 \cdot f_{n-1}, \theta^k \cdot f_0, \theta^{2k} \cdot f_1, \dots, \theta^{(n-1)k} \cdot f_{n-2}\right) \quad (17)$$

When summing (the operation of calculating the element of the Galois field corresponding to the amplitude of an individual spectral component), one can make a permutation.

$$\left(\vec{w}_k, \vec{f}(1)\right) = f_0\theta^k + f_1\theta^{2k} + \dots + f_{n-2}\theta^{(n-1)k} + f_{n-1} \quad (18)$$

Taking the factor θ^k outside the bracket, we obtain

$$\left(\vec{w}_k, \vec{f}(1)\right) = \theta^k\left(w_k, f(0)\right) \quad (19)$$

where it is considered that $\theta^k\theta^{(n-1)k} = \theta^{nk} = 1$.

The same way,

$$\left(\vec{w}_k, \vec{f}(m)\right) = \theta^{mk}\left(\vec{w}_k, \vec{f}(0)\right) \quad (20)$$

It can be seen that the obtained (20) is a direct analogue of property (13) inherent in spectra, that calculated based on the decomposition of the signal by harmonic functions. This formula will be needed below to describe the spectra of the numerical derivatives of signals that represented in discrete form.

A special case of such an operation is the numerical search of the first derivative that reduces to calculating the difference.

$$\Delta u(t) = u(t) - u(t - t_0) \quad (21)$$

where is the duration of one cycle, during which the signal remains constant.

When passing to periodic or artificially periodic signals, that represented by sequences of the form (14), the operation (21) corresponds to calculating the difference between the two sequences.

$$\vec{u}(0) = (u_0, u_1, u_2, \dots, u_{n-1}) \quad (22)$$

$$\vec{u}(1) = (u_{n-1}, u_0, u_1, \dots, u_{n-2}) \quad (23)$$

Let us show that in the spectral representation it described through the analogue of the transfer function. Calculating the spectra of sequences (22) and (23), subtracting the obtained result from each other and using relation (20), we obtain (24).

$$\left(\vec{w}_k, \vec{u}(1) - \vec{u}(0)\right) = (\theta^k - 1)\left(\vec{w}_k, \vec{u}(0)\right) \quad (24)$$

i.e., a discrete analog of the differentiation operation in spectra terms, that constructed based on generalized Rademacher functions is indeed described through the analog of the transfer function.

This formula (25) clearly correlates with the well-known fact that, in the frequency representation the differentiation operation also reduced to the appearance of the simplest type of transfer function. Exactly,

$$F\left[\frac{du}{dx}\right] = \int e^{-i\omega x}\frac{du}{dx}dx = -\int u(x)\frac{d}{dx}e^{i\omega x}dx = i\omega\int u(x)\,e^{i\omega x}dx = i\omega F[u] \quad (25)$$

This form of the transfer function of the differentiation operation, in particular, determines the well-known form of the formula for the impedance of a capacitor with C capacitance.

$$Z_C = \frac{1}{i\omega C} \quad (26)$$

Generalized Rademacher functions are valuable because they allow us to develop a similar approach but applied to digitized signals. In particular, for $k \neq 0$, (27) is valid.

$$(\vec{w}_k, \vec{u}) = \frac{1}{(\theta^k - 1)}(\vec{w}_k, \Delta\vec{u}), k \neq 0 \quad (27)$$

The condition $k \neq 0$ means that all components can be restored, except for the constant.

The resulting relation (27) opens up a number of additional possibilities for both digital signal processing and for their transmission. In particular, the numerical derivative of a signal with an ε-covering can take only three values by definition (which corresponds to ternary logic). Obviously, that if transmit only information about the derivative then for this will require less bandwidth. However, this approach is not acceptable for practical use due to the inevitable accumulation of errors. On the contrary, the noted circumstance can be used if transmit the information about the spectra - each spectral component (more precisely, the corresponding element of the Galois field) is calculated based on information about the entire signal profile over the all p-interval, and the number p can be made large enough.

Further, the functions, interpreted as generalized Rademacher functions, make it possible to develop a digital analogue of the transfer function apparatus for any linear radio-engineering networks (more broadly, for any linear systems). This follows from the fact that any linear operations performed on functions in the digital representation (if they have the property of invariance with respect to time translation) in the spectral representation take the form.

$$(\vec{w}_k, \vec{u}) = K(\theta)(\vec{w}_k, \Delta\vec{u}) \quad (28)$$

which can be shown in the same way in which relation (27) was obtained.

The expediency of using formulas of the form (28) for radio-technical circuits of this type can obviously be disputed. However, they can certainly be useful, for example, for solving problems of finding equivalent radio circuits of various kinds of processes (for example, equivalent circuits of electrochemical cells, etc.). In this case, the task can be formulated as follows. There is a definite function of time (signal), which appears as a response to some influences on the system, which studied experimentally. It is required to establish whether there are radio-technical circuits that is equivalent to the system under consideration in terms of signal conversion. And if so, which one?

To solve problems of this kind, the transition to digital representation is obviously useful, since it significantly simplifies computational procedures, in many respects it turns out to remove the question of measurement errors, etc.

4. CONCLUSION

The using of non-binary Galois fields indeed creates quite definite advantages for processing digital signals that varying in a finite range of amplitudes. In this case, it becomes possible a certain element of the Galois field put in correspondence to each of the signal amplitude levels. This approach allows, firstly, to construct a wide variety of analogs of orthogonal bases, differing in that the operations which necessary for digital signal processing are carried out in the sense of multiplication, addition, etc. Galois fields. An illustration of the additional possibilities that arise is, in particular, the construction of a basis in which the functions that included in it, can be interpreted as generalized Rademacher functions. It is essential that such functions form complete bases for any p-interval, to which correspond to the signals that defined at p-unit of time, where p is a prime number.

Such complete bases have (in contrast to the complete Walsh basis, which is built on the basis of Rademacher functions) many properties of harmonic functions, in particular, in the spectral representation, the operations of differentiation and shift along the time axis are described through analogs of transfer functions, etc. All this in aggregate makes it possible to assert that orthogonal bases that constructed in Galois fields are a promising means of discrete description of any nature systems, the amplitude of signals in which changes in a finite range.

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