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Error Model and Accuracy Calibration of 5-Axis Machine Tool

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Abstract

To improve the machining precision and reduce the geometric errors for 5-axis machinetool, error model and calibration are presented in this paper. Error model is realized by the theory of multi-body system and characteristic matrixes, which can establish the relationship between the cutting tool and the workpiece in theory. The accuracy calibration was difficult to achieve, but by a laser approach-laser interferometer and laser tracker, the errors can be displayed accurately which is benefit for later compensation.

Keywords: *accuracy, 5-axis machine tool, laser interferometer, laser tracker, calibration*

1. Introduction

Recent years, five-axis CNC machine tools have been paid more and more attention, due to the special advantages in manufacturing. Compared with traditional 3-axis CNC machine tools, 5-axis machine tools have two more rotary axes, which can help cutting tools move more freely, thus the tools can machine workpiece at all different direction without changing jig or clamping again which can lessen the cost of expensive jigs and avoid errors of reclamping, hence the efficiency and quality of machining are both improved extremely; meanwhile, some complex surface can beachieved directly which has great inference on industry, expecially the defense industry where many complicated components are needed, such as wing unit of aircraft, the impeller of centrifugal compressor and propeller of vessel. Due to above merits, 5-axis machine tools can make much more profit than 3-axis ones.

Besides advantages, 5-axis machine tools also have disadvantages, compared with 3-axis ones. As machine tools are serial structure, two more rotary axes bring in new additional errors and makekinematic chains longer, which means that errors are superimposed and processing precision might be decreased. However, machining accuracy is the most important parameter to judge machine tools whether is good or not. Hence it is essential to improve the accuracy. Generally speaking, to improve machining precision, there are two ways, one is error avoidance, the other iserror compensation. Error avoidance means that machining precision is improved by careful designment, perfect manufacture and accurate assembly, however, it not only results in huge expenses, but also can hardly enhance the precision, when machine tools has reached certain level of accuracy. These shortcomings prevent this method from being widely adopted in the modern industry. While error compensation, which is different from above method, gains high precision by new additional errors to offset original errors. It doesn't need change the structure of machine tool, so it is convenient and economic. Thus, this method is widely considered by researchers.

There are 3 steps to enhance precision-error model, calibration and compensation. By modeling, the main errors of 5-axis machine tool are introduced and their relationships are analysed; calibration can gain the every error's specific valuewhich is utilized in later compensation. General speaking, the methods of calibration can be divided into two kinds-the direct one and indirect one. The direct method measureseach component error independently, such as laser interferometer and electronic level, while the indirect one achieves the errors by some type of artifacts or kinematic reference standards, for instance, double ball bar and laser

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tracker. Each method has its own advantage and disadvantage. Take the direct one for example, it can capture the errors more accurately, and the information it gets can be used directly in later compensation, but it costs quantities of time to collect all the errors. Compared with direct one, indirect method can gainthe information quickly, and most important of all, it can capture some information which the direct equipments hardly gets, for example, the relationship of two rotary axe. Therefore, in this paper, both two methods are adopted to achieve all errors of 5-axis machine tool. Nowadays, laser measurement equipments are more and more popular due to their high precision and digital display of results which can avoid reading errors. So, the two devices we used are both laser ones. There are, respectively, laser interferometer and laser tracker

2. Geometric Error Components of 5-axis Machine Tool

To 5-axis of machine tool, there're 3 translational axes and 2 rotary axes. 3 translational axes are X, Y and Z, respectively; while rotary axes are C and A. Generally speaking, each axis has one degree of freedom on ideal condition, however, in reality, each axis has 6 differenterrors which are all position dependent-half of them are linear errors, and the rest are angular errors. Take X-axis (translational axis) for example in Figure 1, the linear errors are one positioning error$\delta_{x(x)}$ and two straightness errors-verticalstraightness error$\delta_{y(x)}$ and horizontalstraightness error$\delta_{z(x)}$, while the angular errors are one roll error$\varepsilon_{x(x)}$ and two tilt errors-pitch error $\varepsilon_{y(x)}$and yaw error$\varepsilon_{z(x)}$ [1].

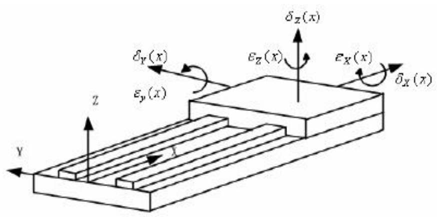

Figure 1. 6 Different Errors of X-axis

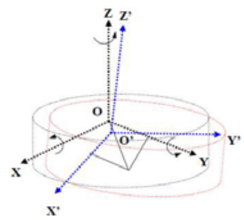

Figure 2. 6 Different Errors of Rotary Axis

Errors of rotary axis are similar with translational axis in Figure 2, take C-axis for example. Linear components are one axial error $\delta_z(C)$ and two radial error-along X direction $\delta_x(C)$ and along Y direction $\delta_y(C)$, while angular ones are one angular position error$\varepsilon_y(B)$ and two tilt error—tilt error motion around X of C $\varepsilon_x(C)$ and tilt error motion around Yof C$\varepsilon_y(C)$,whereδ is the linear error, subscript is the error direction and the positioncoordinate is inside the parenthesis, ε is the angular error, subscript is the axisof rotation and theposition coordinate is inside the parenthesis.

Besides errors of every single axis, it exists other errors between axes, the typical ones are squareness errors. As the end effector also has the influence on the machine accuracy, so the spindle should be considered, especially the relationship between the axis of the spindle and the other axe.

3. Kinematic Error Model of 5-axis Machine Tool based on the Theory of Multi-body System

The multi-body system theories are presented to study multi-body systems. The key points of the theories are number arrays of low-order body and characteristic matrixes. Machine tools, expecially CNC, are typical multi-body system, so they are fitted to be analyzed by the method [2].

The measured 5-axis machine tool is type of TTTRR, 'T' means translational axis, while 'R' stands for rotary axis, showing as Figure 3. The structural diagram is showing as Figure 4, where 0 is base, 1 is workpiece, 2 is gantry, 3 is glide board, 4 is column, 5 is swivel head, 6 is rotary head, 7 is spindle and 8 is cutting tool. It has 5 degree of freedom, so there are 5 driving axe, translational axe-X, Y and Z, rotary axe-C and A, which parallel with Z-axis and X-axis, respectively. The gantry moves along X-axis on the base, while the glide board runs along the Y-axis, meantime, the column shift along Z-axis inside the glide board. The swivel head rotates along C-axis, meanwhile the rotary head swivels along A-axis.

Figure 3. The Machine Tool

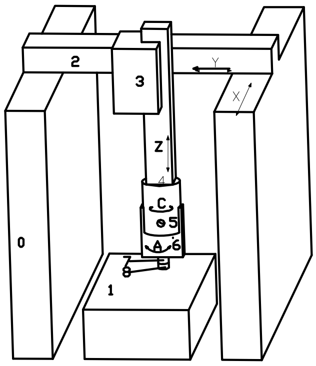

Figure 4. The Structural Diagram

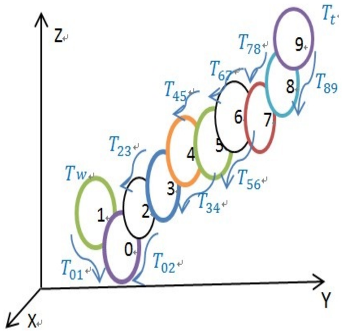

Figure 5. The Topology of Machine Tool

3.1. Number Arrays of Low-order Body

Low-order body is used to describe the topology of multi-body system. Arrays of low-order body is constructed as the following. First, chose any body i as the inertia body; second, number the body i as 0; third, number other bodies, the number of the bodies should successively increase from proximal to distal, meanwhile the bodies in a latter branch can't be numbered until the ones in the former branch are all finished [2]. According to the theory, the machine tool mentioned above can be abstracted as the topology in Figure 5. The two branches areworkpieceone and cutting tool one.

In order to aquire number arrays of low-order body ,some definition should be given first. Choose any body j as a typical body in the system, the serial number of the n-grade low-order body is defined as follow [3]:

$$\mathrm{L}^{\mathrm{n}}(\mathrm{j}) = \mathrm{i}\ (1)；\ \mathrm{L}^{\mathrm{n}}(\mathrm{j}) = \mathrm{L}(\mathrm{L}^{\mathrm{n}-1}(\mathrm{j})) \tag{2}$$

$$\mathrm{L}^{0}(\mathrm{j}) = \mathrm{j} \tag{3}$$

$$\mathrm{L}(\mathrm{j}) = \mathrm{i} \tag{4}$$

Where L is low-order operator, and the body j is called as n-grade high-order body i, it meets the Equation 2; besides, the serial number of the 0-grade low-order body j equals to j, shown in Equation 3; and when the body i and the body jare adjacent, it meets Equation 4. Hence, the low-order of the machine tool which will be measured is given in Table 1.

Table 1. Low-order Body Array of 5-axis Machine Tool

j	1	2	3	4	5	6	7	8	9	10
L^0(j)	0	1	2	3	4	5	6	7	8	9
L^1(j)	0	0	0	2	3	4	5	6	7	8
L^2(j)	0	0	0	0	2	3	4	5	6	7
L^3(j)	0	0	0	0	0	2	3	4	5	6
L^4(j)	0	0	0	0	0	0	2	3	4	5
L^5(j)	0	0	0	0	0	0	0	2	3	4
L^6(j)	0	0	0	0	0	0	0	0	2	3
L^7(j)	0	0	0	0	0	0	0	0	0	2
L^8(j)	0	0	0	0	0	0	0	0	0	0

3.2 Transformation matrix

When machining, the cutting tool should coincide with workpiece at designate point in the ideal situation, however, due to the errors, the differences always exist. In order to gain the value of the differences, characteristic matrix, which can describe the relationships of position and motion between two objects, is introduced.

In the field of low-order body, two kind characteristic matrixes are involved in, which are position characteristic matrix and posture characteristic matrix. Each of them consists of 4 matrixes, which are ideal relative steadying Tp, ideal relative motion Ts, the errors of practical relative steadying ΔT_p and the error of practical relative motion matrix ΔT_s, respectively. If the two bodies—S and V are adjacent, the following equalities 5-8 will hold:

$$T_{SV,p}=\begin{bmatrix}1&0&0&0\\0&\cos\alpha_{SV}&-\sin\alpha_{SV}&0\\0&\sin\alpha_{SV}&\cos\alpha_{SV}&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\cos\beta_{SV}&0&\sin\beta_{SV}&0\\0&1&0&0\\-\sin\beta_{SV}&0&\cos\beta_{SV}&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\cos\gamma_{SV}&-\sin\alpha_{SV}&0&0\\\sin\alpha_{SV}&\cos\gamma_{SV}&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&a_{SV}\\0&1&0&b_{SV}\\0&0&1&c_{SV}\\0&0&0&1\end{bmatrix}\quad(5)$$

$$\Delta T_{SV,p}=\begin{bmatrix}1&-\varepsilon_{z_{SV}}&\varepsilon_{y_{SV}}&\delta_{x_{SV}}\\\varepsilon_{z_{SV}}&1&-\varepsilon_{x_{SV}}&\delta_{y_{SV}}\\-\varepsilon_{y_{SV}}&\varepsilon_{x_{SV}}&1&\delta_{z_{SV}}\\0&0&0&1\end{bmatrix}\quad(6)$$

$$T_{SV,s}=\begin{bmatrix}1&0&0&0\\0&\cos\theta_{x_{SV}}&-\sin\theta_{x_{SV}}&0\\0&\sin\theta_{x_{SV}}&\cos\theta_{x_{SV}}&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\cos\theta_{y_{SV}}&0&\sin\theta_{y_{SV}}&0\\0&1&0&0\\-\sin\theta_{y_{SV}}&0&\cos\theta_{y_{SV}}&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\cos\theta_{z_{SV}}&-\sin\theta_{z_{SV}}&0&0\\\sin\theta_{z_{SV}}&\cos\theta_{z_{SV}}&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&x_{SV}\\0&1&0&y_{SV}\\0&0&1&z_{SV}\\0&0&0&1\end{bmatrix}\quad(7)$$

$$\Delta T_{SV,s}=\prod_i\begin{bmatrix}1&-\varepsilon_{z_{iSV}}&\varepsilon_{y_{iSV}}&\delta_{x_{iSV}}\\\varepsilon_{z_{iSV}}&1&-\varepsilon_{x_{iSV}}&\delta_{y_{iSV}}\\-\varepsilon_{y_{iSV}}&\varepsilon_{x_{iSV}}&1&\delta_{z_{iSV}}\\0&0&0&1\end{bmatrix},i=x,y,z,\theta_x,\theta_y,\theta_z;\quad(8)$$

Where a_{SV}, b_{SV}, c_{SV} represents position relationship between the origin of the coordinate system S and the origin the coordinate system V, when they keep static; meantime α_{SV},β_{SV}, γ_{SV} is the posture of the coordinate frame. x_{SV},y_{SV},z_{SV} is relative translational motion between the two origins, while $\theta_{x_{SV}}$,$\theta_{y_{SV}}$,$\theta_{\gamma_{SV}}$ is the angle that the coordinate system V rotates around the X, Y and Zaxis of the coordinate S

By the above equalities, total characteristic matrix between two different adjacent coordinate system can be expressed as follows [4]:

$$T_{SV}=T_{SV,p}\Delta T_{SV,p}T_{SV,s}\Delta T_{SV,s}\quad(9)$$

Take T_{01},T_{02},T_{23} for example, the matrixes are as follows:

$$T_{01,p}=\begin{bmatrix}1&0&0&x_{wd}\\0&1&0&y_{wd}\\0&0&1&z_{wd}\\0&0&0&1\end{bmatrix},\Delta T_{01,p}=\begin{bmatrix}1&-\Delta\gamma_{wd}&\Delta\beta_{wd}&\Delta x_{wd}\\\Delta\gamma_{wd}&1&-\Delta\alpha_{wd}&\Delta y_{wd}\\-\Delta\beta_{wd}&\Delta\alpha_{wd}&1&\Delta z_{wd}\\0&0&0&1\end{bmatrix},T_{01,s}=I_{4\times4},\Delta T_{01,s}=I_{4\times4};$$

$$T_{01}=T_{01,p}\times\Delta T_{01,p}\times T_{01,s}\times\Delta T_{01,s}\quad(10)$$

$$T_{02,p}=I_{4\times4},\Delta T_{02,p}=I_{4\times4},T_{02,s}=\begin{bmatrix}1&0&0&0\\0&1&0&y\\0&0&1&0\\0&0&0&1\end{bmatrix},\Delta T_{02,s}=\begin{bmatrix}1&-\Delta\gamma_y&\Delta\beta_y&\Delta x_y\\\Delta\gamma_y&1&-\Delta\alpha_y&\Delta y_y\\-\Delta\beta_y&\Delta\alpha_y&1&\Delta z_y\\0&0&0&1\end{bmatrix}$$

$$T_{02}=T_{02,p}\times\Delta T_{02,p}\times T_{02,s}\times\Delta T_{02,s}\quad(11)$$

$T_{23,p}=I_{4\times4}$, $\Delta T_{23,p}=\begin{bmatrix}1 & -\Delta\gamma_{xy} & 0 & 0\\ \Delta\gamma_{xy} & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$, $T_{23,s}=\begin{bmatrix}1 & 0 & 0 & x\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$, $\Delta T_{23,s}=\begin{bmatrix}1 & -\Delta\gamma_x & \Delta\beta_x & \Delta x_x\\ \Delta\gamma_x & 1 & -\Delta\alpha_x & \Delta y_x\\ -\Delta\beta_x & \Delta\alpha_x & 1 & \Delta z_x\\ 0 & 0 & 0 & 1\end{bmatrix}$

$$T_{23} = T_{23,p} \times \Delta T_{23,p} \times T_{23,s} \times \Delta T_{23,s} \quad (12)$$

The rest matrixes—T_{34}, T_{45}, T_{56}, T_{67}, T_{78} and T_{89} are similar with above ones—T_{01}, T_{02} and T_{23}, so they aren't elaborated here. The aim of error model is to expose the difference between the cutting tool and the workpiece, however, only in the same coordinate system can it be achieved. Seen from Figure 5, the two branch have a common node-the base, so workpiece matrix and cutting tool matrix are transformed into the base coordinate system. The cutting tool tip can be described by matrix in the coordinate system 9, then transform into the coordinate system 8 and 8 into 7,…, finally into the coordinate system 0 step by step shown in Equation 13, meantime, the workpiece can be depicted by matrix in the coordinate system 1, then transform into the coordinate system 0,shown in Equation 14.

$$^{0}T_w=T_{01}T_w \quad (13)$$

$$^{0}T_t=T_{02}T_{23}T_{34}T_{45}T_{56}T_{67}T_{78}T_{89}T_t \quad (14)$$

Where T_w is the workpiece actual position matrix in the workpiece coordinate system, which contains the information of ideal position and every single error; ^{0}Tw is the matrix in the coordinate system 0, which is also themachine tool reference coordinate system. T_t is the cutting tool tip actual position matrix in the cutting tool coordinate system, $^{0}T_t$ is the matrix in the coordinate system 0. T_{ij} is transform matrix from the coordinate system j to the coordinate system i. The error matrix which is hunted is the difference between $^{0}T_w$and$^{0}T_t$,shown in Equation 15.

$$E={}^{0}T_t - {}^{0}T_w \quad (15)$$

4. The Theory of Laser Interferometer

The initial laser interferometer dated from 1880s, named Michelson interferometer, which contains a monochromaticlight, a semitransparent lens and two reflectors in Figure 6. When light is incident upon the lens, half of it is reflected to the stationary mirror and the rest is passed to the mobile mirror [5]. The two returned light will be parallel to each other in case that the two reflectors are vertical with each other, thus the two light can go together to the detector. If the two distancesfrom the reflectors to the lens are same, there is constructive interference shown in Figure 7 and a strong signal at the detector. The constructive interference occurs not only the two distances are equal, but also the difference of the distances is a whole number (including 0) of wavelengths. Instead, if the difference is not a whole number of wavelengths but a whole number plus a half (so 0.5, 1.5, 2.5 ...), there is destructive interferenceand a weak signal in Figure 8 [6]. Hence, fix one of the reflector and move the other one, the constructive and the destructive interference may take place in turn. According to the character, the distance the reflector moving may be judged.

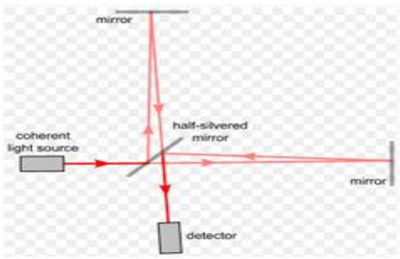

Figure 6. Michelson Interferometer

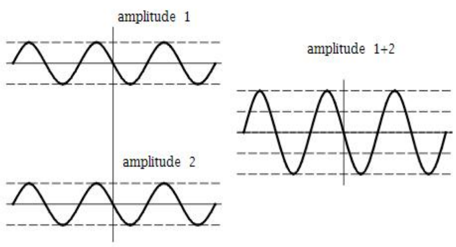

Figure 7. Constructive Interference

With 20 decades development, modern laser interferometer has a high accuracy up to 1ppm, or even much higher, but its basic principle is still similar with Michelson interferometer. Showing as Figure 9, a laser beam from the measuring head is splitted into two laser beams. One beam goes up to stationary reflector which combines with the splitter, the other runs towards the mobile reflector. After being reflected, two beams meet with each other at interferometer which makes two beams into one interfered beam. The beam is transmitted into detector. The detector can distinguish the optical path difference when the mobile reflector moves, which can calculate the distance it moves.

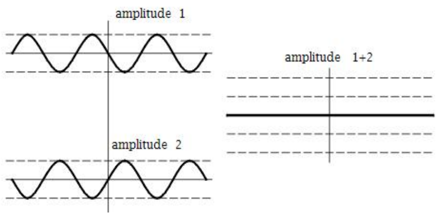

Figure 8. Destructive Interference

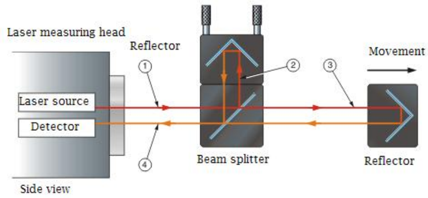

Figure 9. Laser Interferometer

5. The Theory of Laser Tracker

Laser tracker is a little similar with laser interferometer –they both have laser and reflector which is fixed on the target, but the former has an important advantage—the target do not need to move along straight line. Thus, laser tracker is more flexible. Usually, a laser tracker has following components: a laser source, a beam steering mechanism with angle encoders, interferometer block, an optical Position Sensitive Diode (PSD), beam splitting optics, a retroreflector, a control unit and software [7].

The beam from laser source is splitted into two parts, one part is used for measurement, the rest is for reference. The measurement beam travels from the splitter to the target, then turn back. Through the splitter, the back beam recognizes the reference beam and interferes with it. The interference will be used to calculate the distance of the target moves which labeled as ρ. In order to follow the target, the measurement beam could rotate along two direction-elevation and azimuth. When it tracks the target, the angular encoder will record the angle informationof elevation and azimuth, which marked as $\alpha$and β, respectively. Hence, the position of the target (x,y,z) can be gained in Equation 16, shown in Figure 10.

$$\begin{cases} x=\rho * \cos(\alpha) * \cos(\beta) \\ y=\rho * \cos(\alpha) * \sin(\beta) \\ z=\rho * \sin(\alpha) \end{cases} \tag{16}$$

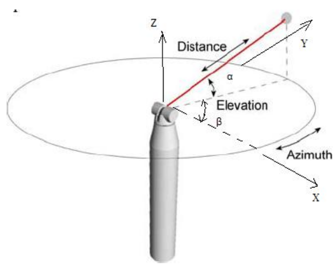

Figure 10. Laser Tracker

6. Experiment

The work volume of measured 5-axis machine tool is 7500mm×3800mm×2500mm, the travel range of A-axis and C-axis are$\pm 110°$and $\pm 300°$,respectively. The resultsof positioning errors and straightness errors of translational axe are provided by laser interferometer, while the relationship of the axe and the positioning of rotary axe are given by laser tracker. As the equipments need space to set up, therefor, the actual metrology range is smaller than the work volume, which is 6000mm×2000mm×2000mm.

Figure 11-13 shows the X-axis,Y-axis and Z-axis positioning error, separately. The curves in Figure 11 are analysed to get the value of theX-axis' backlash, positioning accuracy and repeat positioning accuracy, which are 0.004950mm, 0.069909mm and 0.009122mm, respectively.

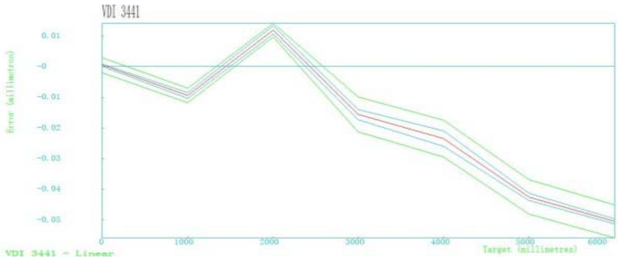

Figure11. X-axis positioning error

The backlash, positioning accuracy and repeat positioning accuracy of Y-axis also can be obtained from Figure 12, which are 0.007050mm, 0.053431mm and 0.008061mm, separately.

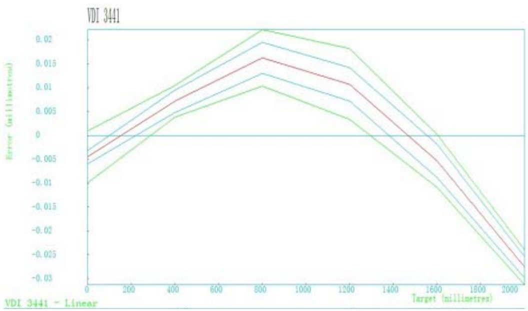

Figure 12. Y-axis Positioning Error

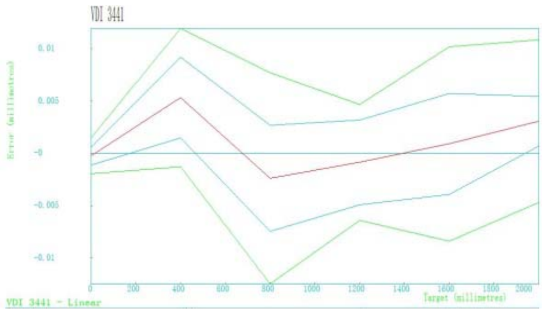

Figure 13. Z-axis Positioning Error

The results of Z-axis is similar with Y-axis and X-axis, which are 0.010150, 0.024393 and 0.010819. Figure 14-17 describes the straightness errors. Least square method is adopted to fit curves. The straightness error of X-axis in XY plan is 0.00552mm, while the one in XZ plan is 0.00779mm. Similarly, the straightness error of Y-axis in XY plan is 0.00571mm, meanwhile the one in YZ plan is 0.00211mm

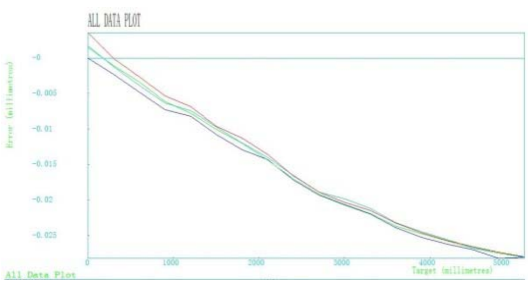

Figure 14. X-axis Straightness Error in XY Plan

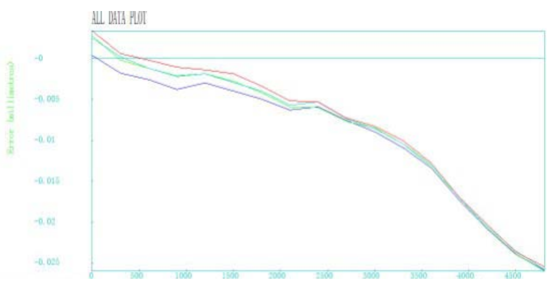

Figure 15. X-axis Straightness Error in XZ Plan

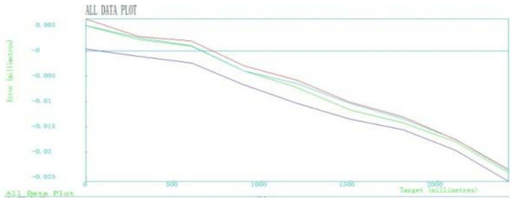

Figure 16. Y-axis Straightness Error in XY Plan

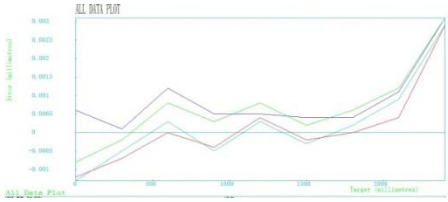

Figure 17. Y-axis Straightness Error in YZ Plan

The angular positioning errors of rotary axes can be achieved in Figure 18-19

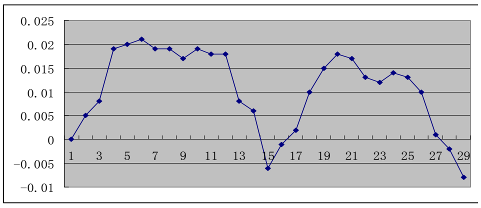

Figure 18. Angular Positioning Error of A-axis

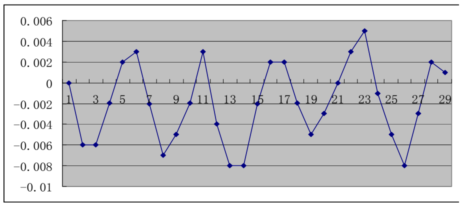

Figure19. Angular Positioning Error of C-axis

The squareness between axes are also given in Figure 20.The one between A-axis and C-axis is 89.996°, while the one between X-axis and Y-axis is 89.998°. Similarly, the perpendicularityof X-Z and Y-Z are 89.998° and 89.997. The verticality between C-axis and XY plan is 89.999, meanwhile, the one between spindle and XY plan is same with the former.

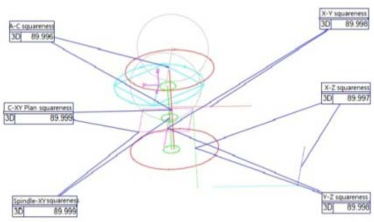

Figure 20. The Squareness

From the above, all measured errors are given in Table 2.

Table 2. All Measured Data

Axis	Item	Data	Axis	Item	Data
X-axis	Positioning error	0.07/6000	Y-axis	Positioning error	0.05/2000
	Repeat positioning error	00.1/6000		Repeat positioning error	0.008/2000
Z-axis	Positioning error	0.024/2000	C-axis	Positioning error	0.0372/360
	Repeat positioning error	0.011/2000	A-axis	Positioning error	0.016/360
Y-axis	Straightness error in XY plan	0.025/2500	X-axis	Straightness error in XY plan	0.035/5000
	Straightness error in YZ plan	0.005/2500		Straightness error in XZ plan	0.035/5000
Y-Z axis	Perpendicularity	0.017/500	X-Y axis	Perpendicularity	0.026/500
X-Z axis	Perpendicularity	0.026/500	C-axis XYplan	Perpendicularity	0.006/300
A-C axis	Perpendicularity	0.021/300	Spindle-A axis	Perpendicularity	0.003/80
Spindle-A axis	Coplanarity	0.019	C-A axis	Coplanarity	0.020
Spindle-C axis	Concentricity	0.014	Spindle-X,Y axis	Perpendicularity	0.006/30

7. Conclusion

In this paper, error model of 5-axis machine tool by the thoery of muli-body systemand characteristic matrixes is introduced and calibration by the laser equipment is presented. The experiment demonstrates the feasibility of the approachbased on the laser interferometer and the laser tracker. Due to the complex model, dynamic errors are not considered, which might be a research subject in the future.

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