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Research on Coordinate Transformation of the Three-phase Circuit

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Abstract

The different three-phase circuit coordinate transformation matrixes were given in relevant literatures, which might cause some difficulties to understand and apply the coordinate transformation. The paper presented a general expression of the coordinate transformation matrixes in three-phase circuit and pointed out that the coordinate transformation matrixes having different specific expressions are due to existing the parameters which can be selected in the general expression. On this basis, the three often used expressions of coordinate transformation matrixes rotating at arbitrary speed were obtained through selecting these different parameters, which are Park transformation, orthogonal transformation and the coordinate transformation in the theory of instantaneous power. The paper further pointed out that, through particularly choosing coordinate rotation speed, it can get commonly used special coordinate transformation matrixes between the three-phase stationary coordinate systems and between the stationary coordinate system and synchronous rotating coordinate system by matrix general expression. The work of this paper showed that the three-phase circuit coordinate transformation matrixes are essentially uniformed, though their forms and practical applications are different, which was helpful to further understanding and correctly using the coordinate transformation theory of three-phase circuit.

Keywords: *three-phase circuit, coordinate transformation matrix, transformation matrix parameters*

1. Introduction

Coordinate transformation of the three-phase circuit is the basis of AC motor vector control and instantaneous power theory. Many literatures have elaborated coordinate transformation and its application in detail [1-8]. Specifically, The method of physical concepts, space analytic geometry, vector algebra and orthogonal similarity transformation deducing AC motor coordinate transformation matrix rotating at arbitrary speed have been applied to literature [9], and the identical conclusion is drawn. Literature [10] looks on coordinate transformation as circuit equations linear transformation and reveals the essence of the coordinate transformation in the more basic theoretical level. Coordinate transformation theory is deepened and enriched by the work of literature [9-10], which provide a useful reference and inspiration for understanding and developing the AC motor coordinate transformation theory and its control strategy from a multidisciplinary point of view, and also supply a guideline to propose a new type of circuit equations linear transformation.

However, the coordinate transformation matrixes given in these literatures are not the same. Generally speaking, there are three common forms of expression of the coordinate transformation matrix, such as: coordinate transformation matrix used to vector control of AC motors in international literatures [1-3], in chinese literatures [4-6] and used to the instantaneous power theory in literatures [7-8]. It causes some difficulties in understanding and application of the coordinate transformation due to different expression forms of the coordinate transformation matrix. For example, it causes Chinese researchers, especially students to make mistakes in application of MATLAB simulation tool due to the different forms of coordinate transformation matrix expression for vector control of AC motors used in international and Chinese literatures.

Based on giving general expression of coordinate transformation matrix in three-phase circuit, the paper obtains three kinds of coordinate transformation matrix expressions, reveals the unity of its essence and discusses the differences in actual application through choosing different parameters.

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2. Research Method
2.1. The General Expression of the Transformation Matrix

The three-phase coordinate system abc to the coordinates dq0 transform and inverse transform relationship are as follows:

$$x_{\mathrm{dq0}} = C_{\mathrm{abc/dq0}}\, x_{\mathrm{abc}} \tag{1}$$

$$x_{\mathrm{abc}} = C^{-1}_{\mathrm{abc/dq0}}\, x_{\mathrm{dq0}} = C_{\mathrm{dq0/abc}}\, x_{\mathrm{dq0}} \tag{2}$$

Where:

$$x_{\mathrm{abc}} = \begin{bmatrix} x_{\mathrm{a}} & x_{\mathrm{b}} & x_{\mathrm{c}} \end{bmatrix}^{\mathrm{T}} \tag{3}$$

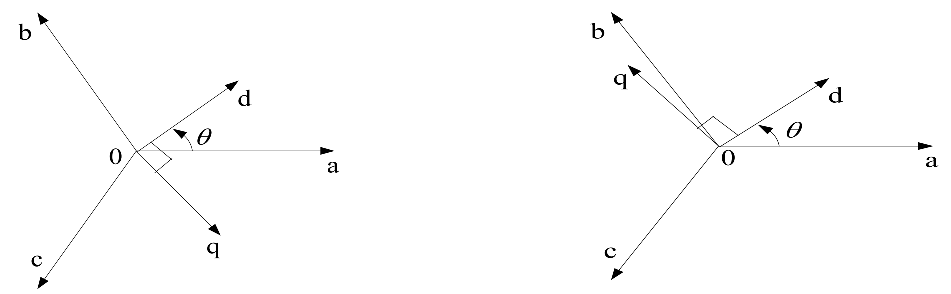
$$x_{\mathrm{dq0}} = \begin{bmatrix} x_{\mathrm{d}} & x_{\mathrm{q}} & x_{0} \end{bmatrix}^{\mathrm{T}} \tag{4}$$

Both of them can represent physical quantities such as voltage, current, magnetic field in the three-phase coordinate system of abc and dq0 respectively.
According to the literature [9-10], the transformation matrix in formula (1), (2) respectively is:

$$C_{\mathrm{abc/dq0}} = k_1 \begin{bmatrix} \cos\theta & \cos(\theta - \frac{3\pi}{2}) & \cos(\theta + \frac{3\pi}{2}) \\ \pm\sin\theta & \pm\sin(\theta - \frac{3\pi}{2}) & \pm\sin(\theta + \frac{3\pi}{2}) \\ k_2 & k_2 & k_2 \end{bmatrix} \tag{5}$$

$$C_{\mathrm{dq0/abc}} = C^{-1}_{\mathrm{abc/dq0}} = \frac{2}{3k_1} \begin{bmatrix} \cos\theta & \pm\sin\theta & \frac{1}{2k_2} \\ \cos(\theta - \frac{2\pi}{3}) & \pm\sin(\theta - \frac{2\pi}{3}) & \frac{1}{2k_2} \\ \cos(\theta + \frac{2\pi}{3}) & \pm\sin(\theta + \frac{2\pi}{3}) & \frac{1}{2k_2} \end{bmatrix} \tag{6}$$

Where, k_1, k_2 can value arbitrary non-zero real numbers, θ is d-axis counterclockwise angle ahead of the a-axis and can value arbitrary real number, "+" corresponds to the q-axis counterclockwise backward d-axis $\frac{\pi}{2}$, "-" corresponds to the q-axis counterclockwise ahead d-axis $\frac{\pi}{2}$, these can be shown as Figure 1(a), (b).



(a) q-axis counterclockwise backward d-axis $\frac{\pi}{2}$ (b) q-axis counterclockwise ahead d-axis $\frac{\pi}{2}$

Figure 1. The Relationship of Three-phase Coordinate System abc and dq0

Formula (5), (6) are the general expression of transformation matrix between the physical quantity of the three-phase coordinate system abc and the coordinate dq0, they are reversible, and they're orthogonal transformation $C^{-1}_{\mathrm{abc/dq0}} = C^{\mathrm{T}}_{\mathrm{abc/dq0}}$ when $k_1 = \sqrt{\frac{2}{3}}, k_2 = \frac{\sqrt{2}}{2}$. Coordinate transformation matrix presences different specific forms of expression due to the optional of k_1, k_2, θ.

2.2. The value of k_1, k_2, θ

Although k_1, k_2 can value arbitrary non-zero real number, θ can value arbitrary real number in the formula (5), (6), several values are as follows

(1) $k_1 = \frac{2}{3}, k_2 = \frac{1}{2}, \theta = \omega t - \frac{\pi}{2}$ (ω is the counterclockwise speed of coordinate system $\mathrm{dq0}$ relative to abc, the same as below), q-axis is ahead of the d-axis $\frac{\pi}{2}$, Figure 2 can be obtained by Figure 1(b).

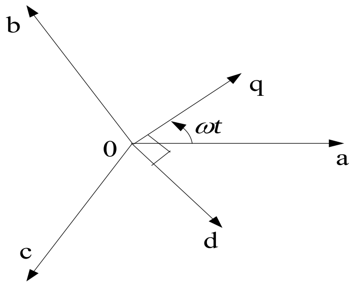


Figure 2. $\theta = \omega t - \frac{\pi}{2}$, the transform schematic diagram of q-axis ahead of the d-axis $\frac{\pi}{2}$

By the formula (5) can be obtained:

$$C_{\mathrm{abc/dq0}} = \frac{2}{3}\begin{bmatrix} \sin \omega t & \sin(\omega t - \frac{2\pi}{3}) & \sin(\omega t + \frac{2\pi}{3}) \\ \cos \omega t & \cos(\omega t - \frac{2\pi}{3}) & \cos(\omega t + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{7}$$

By the formula (6) can be obtained:

$$C_{\mathrm{dq0/abc}} = \begin{bmatrix} \sin \omega t & \cos \omega t & 1 \\ \sin(\omega t - \frac{2\pi}{3}) & \cos(\omega t - \frac{2\pi}{3}) & 1 \\ \sin(\omega t + \frac{2\pi}{3}) & \cos(\omega t + \frac{2\pi}{3}) & 1 \end{bmatrix} \tag{8}$$

Formula (7), (8) transformation as shown is the Park transformation, and is the coordinate transformation matrix of AC motor rotating at arbitrary speed commonly used in the international literature [1-3]. Because of $C^{-1}_{\mathrm{abc/dq0}} \neq C^{\mathrm{T}}_{\mathrm{abc/dq0}}$, power is not conserved before and after transformation.

(2) $k_1 = \sqrt{\frac{2}{3}}$, $k_2 = \frac{\sqrt{2}}{2}$, $\theta = \omega t$,q-axis is ahead of d-axis $\frac{\pi}{2}$, Figure 3 can be obtained by Figure 1(b).

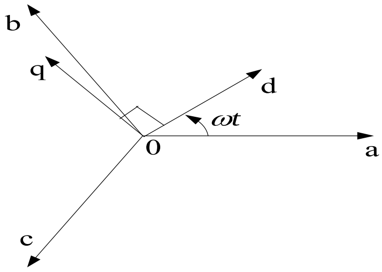


Figure 3. $\theta = \omega t$, the transformation schematic diagram of q-axis ahead of d-axis $\frac{\pi}{2}$

By the Equation (5) can be obtained:

$$C_{\mathrm{abc/dq0}} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\omega t & \cos(\omega t - \frac{2\pi}{3}) & \cos(\omega t + \frac{2\pi}{3}) \\ -\sin\omega t & -\sin(\omega t - \frac{2\pi}{3}) & -\sin(\omega t + \frac{2\pi}{3}) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{9}$$

By Equation (6) can be obtained:

$$C_{\mathrm{dq0/abc}} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\omega t & -\sin\omega t & \frac{\sqrt{2}}{2} \\ \cos(\omega t - \frac{2\pi}{3}) & -\sin(\omega t - \frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \\ \cos(\omega t + \frac{2\pi}{3}) & -\sin(\omega t + \frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{10}$$

Formula(9), (10) transformation as shown is the orthogonal transform, that $C^{-1}_{\mathrm{abc/dq0}} = C^{\mathrm{T}}_{\mathrm{abc/dq0}}$, power is conserved before and after transformation, this transformation matrix on coordinate transformation of AC motor rotating at arbitrary speed is commonly used in the chinese literature [4-6].

(3) $k_1 = \sqrt{\frac{2}{3}}$, $k_2 = \frac{\sqrt{2}}{2}$, $\theta = \omega t - \frac{\pi}{2}$,q-axis backward d-axis $\frac{\pi}{2}$, Figure 4 can be obtained by Figure 1(a).

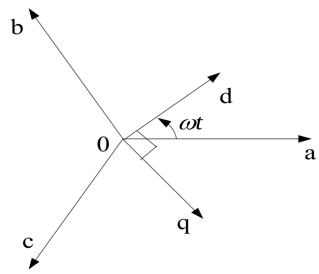


Figure 4. $\theta = \omega t - \frac{\pi}{2}$, the transformation schematic diagram of q-axis backward d-axis $\frac{\pi}{2}$

By the Equation (5) can be obtained:

$$C_{\text{abc/dq0}} = \sqrt{\frac{2}{3}}\begin{bmatrix} \sin\omega t & \sin(\omega t - \frac{2\pi}{3}) & \sin(\omega t + \frac{2\pi}{3}) \\ -\cos\omega t & -\cos(\omega t - \frac{2\pi}{3}) & -\cos(\omega t + \frac{2\pi}{3}) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{11}$$

By Equation (6) can be obtained:

$$C_{\text{dq0/abc}} = \sqrt{\frac{2}{3}}\begin{bmatrix} \sin\omega t & -\cos\omega t & \frac{\sqrt{2}}{2} \\ \sin(\omega t - \frac{2\pi}{3}) & -\cos(\omega t - \frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \\ \sin(\omega t + \frac{2\pi}{3}) & -\cos(\omega t + \frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{12}$$

Formula(11), (12) transformation as shown is the orthogonal transform, that $C^{-1}_{\text{abc/dq0}} = C^{\text{T}}_{\text{abc/dq0}}$, power is conserved before and after transformation, which is the transformation matrix that is used as rotating coordinate transformation at arbitrary speed in instantaneous power theory [7-8].

3. Discussion

Due to $C_{\text{abc/dq0}}$ is reversible, and $C^{-1}_{\text{abc/dq0}} = C_{\text{dq0/abc}}$, the values of k_1, k_2, θ only affect the concrete expression of the transformation matrix, and do not affect the feasibility and accuracy of the transformation.

However, whether the coordinate transformation is orthogonal transformation is decided by the value of k_1, k_2,which will affect the power (torque) expression. For example, in the coordinate transformation of the AC motor, if the zero-axis component is ignored, the electromagnetic torque expression corresponding to the transformation of formula (7) is:

$$T_{\text{e}} = \frac{2}{3} n_{\text{p}} L_{\text{m}} (i_{\text{q1}} i_{\text{d2}} - i_{\text{q2}} i_{\text{d1}}) \tag{13}$$

The electromagnetic torque expression corresponding to the transformation of formula (9) is:

$$T_{\text{e}} = n_{\text{p}} L_{\text{m}} (i_{\text{q1}} i_{\text{d2}} - i_{\text{q2}} i_{\text{d1}}) \tag{14}$$

Where, n_{p} is motor pole pairs, L_{m} is mutual inductance maximum value for the stator and rotor windings. $i_{\text{d1}}, i_{\text{q1}}$ is respectively for the stator current of the d-axis and q-axis component. i_{d2}, i_{q2} is respectively for the rotor current component of the d-axis and q-axis.

Obviously, the formula (13) and (14) have a coefficient difference.

In addition, it also considers the unity of the existing concepts in selecting k_1, k_2, θ. Hypothesis a three-phase circuit voltage is:

$$\begin{cases} u_{\text{a}} = \sqrt{2}U \sin\omega t \\ u_{\text{b}} = \sqrt{2}U \sin(\omega t - \frac{\pi}{3}) \\ u_{\text{c}} = \sqrt{2}U \sin(\omega t + \frac{2\pi}{3}) \end{cases} \tag{15}$$

Where, U is RMS voltage.

Define the vector:

$$\vec{u} = \sqrt{\frac{2}{3}}(u_{\mathrm{a}} + u_{\mathrm{b}} e^{j\frac{\pi}{3}} + u_{\mathrm{c}} e^{j\frac{2\pi}{3}}) \tag{16}$$

(15) and (16) can be expressed as follows:

$$\vec{u} = \sqrt{3}Ue^{j(\omega t - \frac{\pi}{2})} \tag{17}$$

Formula (17) shows, the voltage vector is coincidence with d-axis of coordinate transformation in Figure 4. Instantaneous active and instantaneous reactive power obtained by the formula (11) is [7-8]:

$$\begin{cases} p = |\vec{u}| i_{\mathrm{d}} \\ q = |\vec{u}| i_{\mathrm{q}} \end{cases} \tag{18}$$

Where, $|\vec{u}|$ is the modulus of the voltage vector.

As can be seen from Figure 4, the instantaneous active power p is the product of modulus of voltage vector $|\vec{u}|$ and d-axis current component i_{d} , so, i_{d} is also known as active current. The instantaneous reactive power q is the product of mold of voltage vector $|\vec{u}|$ and q-axis current component i_{q} , so, i_{q} is known as reactive current, the positive value of q represents inductive reactive, negative value indicates capacitive reactive, which is consistent with the concept of reactive power in general textbooks.

Particularly to be noted that, the value of ω is arbitrary, since it's the rotational speed of the coordinate system dq0 respect to the coordinate system abc in the coordinate transformation matrix of formula (7) to (10). Therefore, ω taking certain values can obtain corresponding specific transformation. As in the description of the coordinate transformation of the AC motor, if abc represents three-phase coordinates of the stator, when $\omega = 0$, it can obtain the transformation matrix of the stator three-phase coordinate system (usually represented with ABC) and the stationary coordinate system (usually represented with $\alpha\beta 0$). If abc represents three-phase coordinates of the rotor, when $\omega = -\omega_{\mathrm{r}}$ (ω_{r} is rotor speed), it can obtain the transformation matrix of the rotor three-phase coordinate system (usually represented with abc) and the stationary coordinate system $\alpha\beta 0$. If abc represents three-phase coordinates of the stator, when $\omega = \omega_1$ (ω_1 is synchronous speed), it can obtain the transformation matrix of the three-phase coordinate system ABC and the synchronous rotating coordinate system, (usually represented with $\mathrm{dq0}$). If abc represents three-phase coordinates of the rotor, when $\omega = \omega_{\mathrm{s}}$ (ω_{s} is rotor slip), it can obtain the transformation matrix of the rotor three-phase coordinate system abc and the synchronous rotating coordinate system $\mathrm{dq0}$.

4. Conclusion

(1) Although the forms of expression about the coordinate transformation matrix of the three-phase circuit are different in chinese and international literatures, their essence is same and they can be expressed by a general formula.

(2) The expressions of coordinate transformation matrixes are different when parameters take on different values in the general formula.

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