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Outage Performance Based on Optimal Relay Location in Multi-node Relay Network

Yaolian Song*[1], Yubin Shao[1], Hua Long[1], Xuanhui Xi[2]
[1]Faculty of Information Engineering and Automation, Kunming University of Science and Technology, Kunming, 650504, China
[2]501 Station of the SARFT, Kunming 650302 China
*Corresponding author, e-mail: songyaolian@kmust.edu.cn

Abstract

In this paper, we investigate the outage performance of multi-node relay network under equal power allocation (EPA) scheme. Due to the mobility of relay nodes, we proposed to design a method that considers random relay location based on random model. We conduct deep experiments which demonstrate that there exists optimal relay location to minimize the outage probability of multi-node relay networks. Finally, simulation results show that the outage performance of the *multi-node relay network* is closely related to the relay location.

Keywords: *cooperative relay, equal power allocation, relay location*

1. Introduction

In wireless communications, the transmitted signal suffers severe fading from such as noise, attenuation, multi-path propagation and interference. Signal fading arising from multi-path propagation is particularly severe. However, due to the space constraint of the mobile destination, the use of multiple antennas is limited, which leads to increased interest in cooperative diversity technique. Cooperative diversity can effectively mitigate the degradation through exploiting spatially dispersed relays. Diversity means to send more than one copy of the transmitted signal to the destination. Due to the mobility of relay nodes, the performance of wireless system is related to the location of relay.

In higher signal to noise ratio region, the formula of approximate outage probability for multi-node relay networks was provided in [1]. The methods of relay selection were presented in [2-3] under total power constraint. Optimal power allocation method and best relay selection scheme were addressed in [4] to minimize the system symbol error rate as well as outage probability. Joint relay selection and power allocation scheme was proposed in [5] to maximize system throughput with limited interference to primary used in cognitive radio system. The optimal power allocation (OPA) technique was addressed in [6] and [7]. The outage performance of single relay networks was researched in [8]. The impact of the relay location on system capacity and outage probability for cooperative relay transmission system based on equal power allocation (EPA) was investigated in [9], which demonstrated that the optimal value of the capacity and the outage probability can be achieved when the relay sits half-way between the source and the destination in AF mode. However, the previous documents didn’t consider the mobility of relays.

To solve above problems, the mobility of the relay nodes is taken into account. Different relay location can lead to different performance of *multi-node relay network*. In this paper, we proposed a new method that the relay moves along random relay model. The outage performance of multi-node relay system can be affected by different relay location for EPA scheme. Analysis and simulation results show that there exists optimal relay location to minimize the outage probability.

The paper is organized as follows, in section 1, we review the previous work relay selection of cooperative relay transmission networks. Section 2 depicts the multi-node relay system model. Outage performance is presented in section 3. Numerical and simulation results are addressed in section 4, and the paper is concluded in section 5.

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2. Research Method

In this section, a multi-node relay model is described as shown in Figure 1. The whole cooperative communication process can be divided into two phases: (1) Source broadcast information to N relays and the destination,(2) Relays retransmit received signals in amplify-and-forward (AF) mode to the destination, while the source keep silent in the second phase. We assumed that users can transmit their information in orthogonal mode in order to prevent mutual interference in both phases. In addition, we think perfect synchronization can be achieved between cooperative nodes.

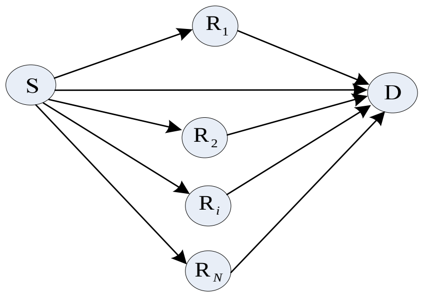

Figure1. Multi-node Relay Model

In the first phase, the received signal at the destination and the ith relay can be expressed as follows, respectively

$$y_{sd} = \sqrt{P_s} h_{sd} x_1 + u_{sd} \quad (1)$$

$$y_{sr_i} = \sqrt{P_s} h_{sr_i} x_1 + u_{sr_i} \quad (2)$$

Where P_s denotes transmit power. h_{sd} and h_{sr_i} are the fading channel coefficients between the source and the destination and the ith relay. u_{sd} and u_{sr_i} are additive white Gaussian noise (AWGN) with zero mean and σ_n^2 variance at the destination and the ith relay, respectively.

In the second phase, the destination received the signal from the ith relay, which can be written as follows:

$$y_{r_i d} = \beta_i h_{r_i d} y_{sr_i} + u_{r_i d} \quad (3)$$

Due to power constraint of each relay, the amplification factor β_i can be given as $\beta_i \le \sqrt{\dfrac{P_i}{P_s \left|h_{sr_i}\right|^2 + \sigma_n^2}}$. P_i is transmit power of the ith relay. $h_{r_i d}$ denotes channel coefficient between the ith relay and the destination. In this paper, the channel coefficients h_{sd} , h_{sr_i} and $h_{r_i d}$ are modeled as zero mean complex Gaussian random variables with variances δ_{sd}^2 , $\delta_{sr_i}^2$ and $\delta_{r_i d}^2$, respectively. Where $\delta_{sd}^2 \propto d_{sd}^{-\alpha}$, $\delta_{sr_i}^2 \propto d_{sr_i}^{-\alpha}$ and $\delta_{r_i d}^2 \propto d_{r_i d}^{-\alpha}$, d stands for the distance between two nodes, α is the path loss exponent. We assume that the channel coefficients can be available at the received nodes but not at the transmit node. The noise terms take zero mean and σ_n^2 variance Gaussian distribution. h_{sd} , h_{sr_i} and $h_{r_i d}$ can be expressed as:

$$h_{sd} = \sqrt{d_{sd}^{-\alpha}}\, h(t),\; h_{sr_i} = \sqrt{d_{sr_i}^{-\alpha}}\, h_{ir}(t),\; h_{r_i d} = \sqrt{d_{r_i d}^{-\alpha}}\, h_{id}(t) \tag{4}$$

Where $h(t)$, $h_{ir}(t)$ and $h_{id}(t)$ are modeled as Rayleigh distributed random variable with zero mean and unit variances. We assumed that the source and the destination locate at fixed position separated by unit distance, and $d_{sd} = 1$, as shown in Figure 2. The source locates at base point (0,0), while the destination lies in point (1,0), and relays locate at random position (l, h). The distance between two nodes can be expressed as follows.

$$\begin{cases} d_{sd} = 1 \\ d_{sr_i} = \sqrt{l^2 + h^2} \\ d_{r_i d} = \sqrt{(1-l)^2 + h^2} \end{cases} \tag{5}$$

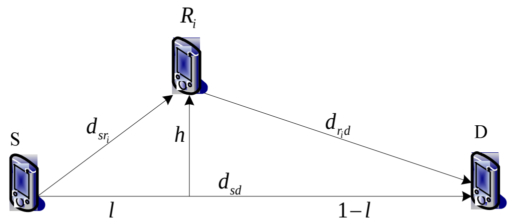

Figure 2. Relay Random Location Model

The formula (4) can be rewritten as:

$$\begin{cases} h_{sd} = h(t) \\ h_{sr_i} = \left(l^2 + h^2\right)^{-\frac{\alpha}{4}} h_{ir}(t) \\ h_{r_i d} = \left((1-l)^2 + h^2\right)^{-\frac{\alpha}{4}} h_{id}(t) \end{cases} \tag{6}$$

$$\begin{aligned} \gamma &= \frac{P_s \left|h_{sd}\right|^2}{\sigma_n^2} + \sum_{i=1}^{N} \frac{\frac{P_s}{\sigma_n^2}\left|h_{sr_i}\right|^2 \frac{P_i}{\sigma_n^2}\left|h_{r_i d}\right|^2}{\frac{P_s}{\sigma_n^2}\left|h_{sr_i}\right|^2 + \frac{P_i}{\sigma_n^2}\left|h_{r_i d}\right|^2 + 1} \\ &= \frac{P_s d_{sd}^{-a} \left|h(t)\right|^2}{\sigma_n^2} + \sum_{i=1}^{N} \frac{\frac{P_s}{\sigma_n^2} d_{sr_i}^{-a}\left|h_{ir}(t)\right|^2 \frac{P_i}{\sigma_n^2} d_{r_i d}^{-a}\left|h_{id}(t)\right|^2}{\frac{P_s}{\sigma_n^2} d_{sr_i}^{-a}\left|h_{ir}(t)\right|^2 + \frac{P_i}{\sigma_n^2} d_{r_i d}^{-a}\left|h_{id}(t)\right|^2 + 1} \\ &= \frac{P\left|h(t)\right|^2}{(N+1)\sigma_n^2} + \sum_{i=1}^{N} \frac{\frac{P}{(N+1)\sigma_n^2}\left(l^2+h^2\right)^{-\frac{\alpha}{2}}\left|h_{ir}(t)\right|^2 \frac{P}{(N+1)\sigma_n^2}\left((1-l)^2+h^2\right)^{-\frac{\alpha}{2}}\left|h_{id}(t)\right|^2}{\frac{P}{(N+1)\sigma_n^2}\left(l^2+h^2\right)^{-\frac{\alpha}{2}}\left|h_{ir}(t)\right|^2 + \frac{P}{(N+1)\sigma_n^2}\left((1-l)^2+h^2\right)^{-\frac{\alpha}{2}}\left|h_{id}(t)\right|^2 + 1} \\ &= \frac{P\left|h(t)\right|^2}{(N+1)\sigma_n^2} + \frac{1}{N+1}\sum_{i=1}^{N} \frac{\frac{P^2}{\sigma_n^4}\left|h_{ir}(t)\right|^2\left|h_{id}(t)\right|^2\left(l^2+h^2\right)^{-\frac{\alpha}{2}}\left((1-l)^2+h^2\right)^{-\frac{\alpha}{2}}}{\frac{P}{\sigma_n^2}\left|h_{ir}(t)\right|^2\left(l^2+h^2\right)^{-\frac{\alpha}{2}} + \frac{P}{\sigma_n^2}\left|h_{id}(t)\right|^2\left((1-l)^2+h^2\right)^{-\frac{\alpha}{2}} + (N+1)} \end{aligned} \tag{7}$$

Let $A = \frac{P}{\sigma_n^2}$, formula (7) can be expressed as formula (8).

The source and relays are allocated equal power $P_s = P_i = \frac{P}{N+1}$. Maximal ratio combiner is used at the destination, then, the received signal to noise ratio of the destination can be formulated as formula (7).

$$\max\gamma = \frac{A\left|h(t)\right|^2}{N+1} + \frac{1}{(N+1)}\sum_{i=1}^{N}\frac{A^2\left|h_{ir}(t)\right|^2\left|h_{id}(t)\right|^2\left(l^2+h^2\right)^{-\frac{\alpha}{2}}\left((1-l)^2+h^2\right)^{-\frac{\alpha}{2}}}{A\left(\left|h_{ir}(t)\right|^2\left(l^2+h^2\right)^{-\frac{\alpha}{2}}+\left|h_{id}(t)\right|^2\left((1-l)^2+h^2\right)^{-\frac{\alpha}{2}}\right)+(N+1)} \tag{8}$$

From formula (8), we can know that the maximum of signal to noise ratio is irrelevant to the first term $\frac{A\left|h(t)\right|^2}{N+1}$. The maximum of signal to noise ratio of the destination is decided by the maximum of the second term of formula (8). Let path loss exponent $\alpha = 4$, formula (8) can be simplified as follows:

$$\begin{aligned}\max\gamma &\sim \max\sum_{i=1}^{N}\frac{A^2\left|h_{ir}(t)\right|^2\left|h_{id}(t)\right|^2\left(l^2+h^2\right)^{-2}\left((1-l)^2+h^2\right)^{-2}}{A\left(\left|h_{ir}(t)\right|^2\left(l^2+h^2\right)^{-2}+\left|h_{id}(t)\right|^2\left((1-l)^2+h^2\right)^{-2}\right)+(N+1)}\\ &= \max\sum_{i=1}^{N}\frac{A^2\left|h_{ir}(t)\right|^2\left|h_{id}(t)\right|^2}{A\left(\left|h_{ir}(t)\right|^2\left((1-l)^2+h^2\right)^2+\left|h_{id}(t)\right|^2\left(l^2+h^2\right)^2\right)+(N+1)\left(l^2+h^2\right)^2\left((1-l)^2+h^2\right)^2}\end{aligned} \tag{9}$$

If we can achieve the minimum of the denominator of the formula (9), the maximum of formula (9) can be obtained.

$$\min\left(A\left(\left|h_{ir}(t)\right|^2\left((1-l)^2+h^2\right)^2+\left|h_{id}(t)\right|^2\left(l^2+h^2\right)^2\right)+(N+1)\left(l^2+h^2\right)^2\left((1-l)^2+h^2\right)^2\right) \tag{10}$$

$$\text{s.t}\begin{cases}0 \le l \le 1\\ -\frac{\sqrt{2}}{2} \le h \le \frac{\sqrt{2}}{2}\end{cases} \tag{11}$$

We can get the optimal value $l = 0.5, h = 0$ by solving formula (10) and (11), which can make signal to noise ratio of the destination maximized. When relays locate at point (0.5,0), the capacity of multi-node relay networks can be maximized. The maximal capacity of multi-node relay network can be expressed as formula (12).

$$\begin{aligned}I_{AF}^{\max} &= \frac{1}{N+1}\log_2\left(1+\max\gamma\right)\\ &= \frac{1}{N+1}\log_2\left(1+\frac{A\left|h(t)\right|^2}{N+1}+\frac{16A^2}{(N+1)}\sum_{i=1}^{N}\frac{\left|h_{ir}(t)\right|^2\left|h_{id}(t)\right|^2}{A\left(\left|h_{ir}(t)\right|^2+\left|h_{id}(t)\right|^2\right)+(N+1)4^{-2}}\right)\end{aligned} \tag{12}$$

3. Outage Performance

An outage event will occur when the capacity I falls under required spectral efficiency R, with outage probability $P_{AF}^{out} = \Pr\left[I \prec R\right]$. It is extremely difficult to directly compute the exact outage probability. Therefore, we conduct deep experiments of formula (13).

$$P_{AF}^{out} = \Pr\left[I \prec R\right] = \Pr\left[\frac{1}{N+1}\log_2\left(1+\gamma\right) \prec R\right] = \Pr\left[\gamma \prec 2^{(N+1)R} - 1\right] \quad (13)$$

4. Simulation Results

In this section we report enough simulation results to deeply analyze the outage performance of multi-node relay networks. Throughout this paper, we let $d_{sd} = 1$, required spectrum efficiency $R = 1$ bit/s and path loss exponent $\alpha = 4$. Figure 3 shows the signal to noise ratio of the destination versus l and h when there is only one relay to help the source with transmitting signal to the destination. We can know that when the relay locates at $l = 0.5$ and $h = 0$, the signal to noise ratio of the destination can be maximized. The value of signal to noise ratio is closely related to the distance between random relay location and optimal relay location (0.5,0).

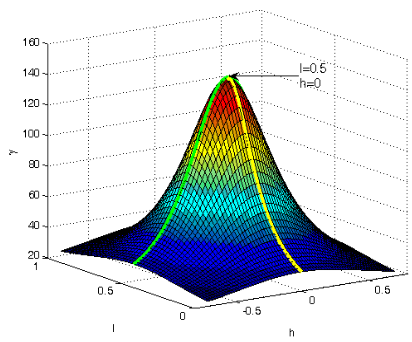

Figure 3. γ versus l and h

Figure 4 demonstrates the outage probability of different relay position for one relay network under equal power allocation scheme. It is easy to see that the outage probability of relay location (0.5,0) is optimal, while the outage performance of random location A is worst. From Figure 5 we can know that the distance between A and (0.5,0) is biggest, random location C is nearest to the optimal location (0.5,0). Therefore, the outage performance of random location C outperforms that of other random location.

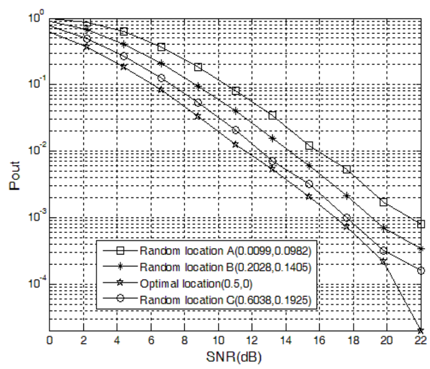

Figure 4. Outage Probability of Different Relay Location for One Relay Network

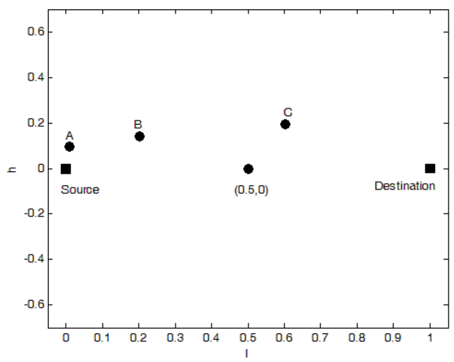

Figure 5. Random Relay Location

We compare the outage probability of equal power allocation scheme with that of optimal power allocation (OPA) scheme, such as shown in Figure 6. For random location A and B, the outage performance of EPA scheme is superior to that of OPA scheme. For random location C, the outage probability of OPA scheme is better than that of EPA scheme. When the distance between source and relay is greater than that of relay and destination, the outage performance of OPA scheme outperforms that of EPA scheme. Otherwise, the outage performance of EPA scheme is superior to that of OPA scheme.

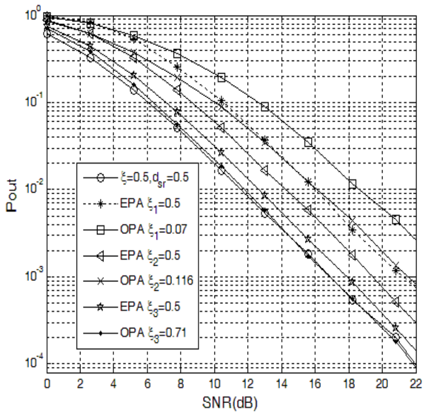

Figure 6. Outage Probability of Different Schemes for One Relay Network

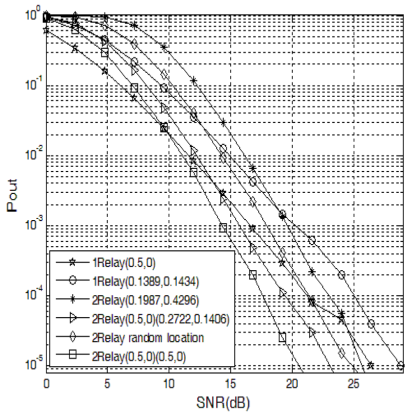

Figure 7. Outage Performance Comparison for One and Two Relay Network

Figure 7 illustrates simulation results of outage performance for one and two relay network. In lower SNR region, the outage performance of optimal relay location for one relay network is optimal. However, when relay locates at point (0.1389,0.1434), the outage performance of one relay networks is declined. For two relays networks, the outage performance of optimal relay position is optimal, while the outage performance of random position (0.1987,0.4296) is worst. When two relays locate at (0.2722,0.1406) and (0.1389,0.1434) respectively, the outage performance of random relay location is superior to that of same random location (0.1987,0.4296). This is because that the distance between random relay location (0.2722,0.1406) or (0.1389,0.1434) and (0.5,0) is 0.2677 or 0.3886 respectively, while the distance between (0.1987,0.4295) and (0.5,0) is 0.5225. Therefore, when there exist more than one relay to help transmitting data, we should choose the relay which is close to the optimal relay location.

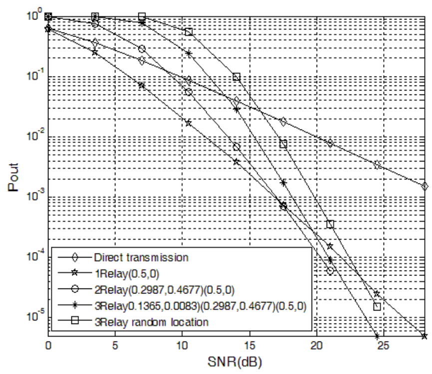

Figure 8. Outage Performance for Different Relay Network

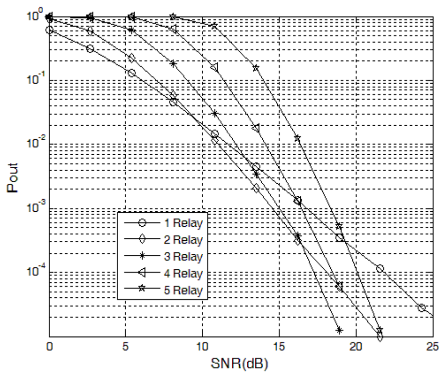

Figure 9. Outage Performance for Optimal Relay Location

Figure 8 shows outage performance comparison for different relay network. In higher SNR region, when the number of relay is increasing, the outage performance of multi-node relay network is improved. Figure 9 gives simulation results of optimal relay location under EPA scheme; we can draw the same conclusion as well as Figure 8.

5. Conclusion

It is well known that cooperative relay transmission can effectively mitigate the deterioration caused by fading and multi-path propagation, and obviously improve the outage performance of cooperative networks. The performance of cooperative relay networks can be affected by relay location. In this paper, we investigate the relationship between outage probability and relay location in AF mode. We can draw the conclusion as follows. (1) There exists optimal relay location to optimize the capacity and the outage probability of multi-node relay networks. When the relay is close to the optimal relay location (0.5,0), the outage performance can be improved. (2) When the relay is close to the destination, the outage performance of the OPA scheme is superior to that of the EPA scheme. (3) In lower SNR region, the outage performance of multi-node relay networks is decreasing when the number of relay participated in data transmission is increasing under the constraint of total transmit power. (4) In higher SNR region, the outage performance can be enhanced with the creasing number of relay.

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