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Multi-Machine Controller Design of Permanent Magnet Wind Generators using Hamiltonian Energy Method

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Abstract

In this paper, the nonlinear control problem of permanent magnet wind generators is investigated based on Hamiltonian energy method. A nonlinear design method is proposed for the multi-machine system, such that the closed-loop system is stable simultaneously. Moreover, in the presence of disturbances, the closed-loop is finite–gain L_2 stable under the action of the Hamiltonian controller. In order to illustrate the effectiveness of the proposed method, the simulations are performed which show that the gotten controller can improve the transient property and robustness of the system.

Keywords: *permanent magnet wind generator (PMWG), multi-machine system, Hamiltonian energy method, robust control*

1. Introduction

Recently, with the rapid development of economy, the electricity supply becomes the bottleneck of the development of society, and the demand for renewable energy increases dramatically. As the main renewable energy, wind power technology becomes the focus of research [1, 2]. The energy-based control of nonlinear systems has been used to design the controller for permanent magnet wind generators (PMWGs) [3, 4]. This method can thoroughly take advantage of the internal structural properties of systems and make the control design relatively simple [4-6]. The Hamiltonian energy approach was put forward in [7] for modeling of physical systems. Then, there have been several attempts to extend this approach in theoretical and practical aspects [6, 8].

In this article, we focus on the control problems of multi-machine system of PMWGs. First of all, the theorems on the nonlinear control and robust control of double-machine system are proposed and proved. Then, Hamiltonian energy function is constructed to obtain port-controlled Hamiltonian (PCH) system model. And the PCH form is transformed into port-controlled Hamiltonian system with dissipation (PCH-D) via the pre-feedback control. Based on the proposed theorem, it is known that the closed-loop system can achieve simultaneous stability. Next, the disturbance attenuation problem of multi-machine system of PMWGs is investigated in order to show the robustness property of this energy-based design method. The controller can guarantee the closed-loop system is finite-gain L_2 stable. Finally, the simulation results show the Hamiltonian energy control can improve the stability and enhance the ability of disturbance attenuation of the multi-machine system.

2. Hamiltonian Energy Method

Consider a nonlinear affine system modeled by equations as follows:

$$\begin{cases} \dot{x}(t) = f(x(t)) + G(x(t))u(t) \\ y(t) = h(x(t)) \end{cases} \tag{1}$$

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where $x \in R^n$ is state variable; $u \in R^m$, $y \in R^p$ are control input and output variable, respectively; $f: R^n \to R^n$ is smooth vector field, $G: R^n \to R^{n \times m}$ is smooth matrix-valued function, and $h: R^n \to R^p$ is smooth vector-valued function.

Assuming that there exists a Hamiltonian function $H(x)$, we have an alternative representation of nonlinear systems (1), called port-controlled Hamiltonian system with dissipation (PCH-D) system model:

$$\begin{cases} \dot{x} = (J - R)\nabla H + G(x)v \\ y = G^T(x)\nabla H \end{cases} \tag{2}$$

where $v \in R^m$ is control vector, $\nabla H = \partial H / \partial x$ is the gradient of $H(x)$, J is a skew-symmetric matrix and R is a positive semi-definite matrix. Now, we summarize the relevant results of the former research [3, 6] into the theorem as follows:

Theorem 1: For the system (2), there exists the following control law

$$v = -\mathrm{T}G^T(x)\nabla H \tag{3}$$

where T is a positive definite matrix, such that the system is stable.

Then, the control problem of double-machine system is researched. Consider the following two PCH-D systems, which have the same order dynamic model.

$$\Sigma_1 : \begin{cases} \dot{x} = (J_1 - R_1)\dfrac{\partial H_1}{\partial x} + G_1(x)v \\ y = G_1^T(x)\dfrac{\partial H_1}{\partial x} \end{cases} \tag{4}$$

$$\Sigma_2 : \begin{cases} \dot{\xi} = (J_2 - R_2)\dfrac{\partial H_2}{\partial \xi} + G_2(\xi)v \\ \eta = G_2^T(\xi)\dfrac{\partial H_2}{\partial \xi} \end{cases} \tag{5}$$

where $x, \xi \in R^n$ are state variables; $v \in R^m$, $y, \eta \in R^p$ are control input and output variables, respectively.

Considering two PCH-D systems which hold the same physical structure and mathematical model, Theorem 2 can be given as follows:

Theorem 2: For the systems (4) and (5) whose mathematical models are same and only the parameters are different, the output feedback controller

$$v = -\kappa(y + \eta) \tag{6}$$

can stabilize both of them simultaneously, where κ is a positive definite diagonal matrix.

Proof: Taking the controller (6) into PCH-D systems (4) and (5). It is obtained that

$$\dot{X} = \begin{bmatrix} J_1 - R_1 - G_1\kappa G_1^T & -G_1\kappa G_2^T \\ -G_2\kappa G_1^T & J_2 - R_2 - G_2\kappa G_2^T \end{bmatrix}\frac{\partial H}{\partial X} \tag{7}$$

where $X = \begin{bmatrix} x^T & \xi^T \end{bmatrix}^T$, $H(X) = H_1(x) + H_2(\xi)$ and $\dfrac{\partial H(X)}{\partial X} = \begin{bmatrix} \dfrac{\partial H_1(x)}{\partial x} & \dfrac{\partial H_2(\xi)}{\partial \xi} \end{bmatrix}^T$.

From the equation (7), we obtain

$$J(X)=\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix},\ R(X)=\begin{bmatrix} R_1+G_1\kappa G_1^T & G_1\kappa G_2^T \\ G_2\kappa G_1^T & R_2+G_2\kappa G_2^T \end{bmatrix}$$

Obviously, $J(X)$ is a skew-symmetric matrix, $R(X)$ is positive definite. Equation (7) is one expanded PCH-D system. Therefore, the controller stabilizes the system (4) and (5) simultaneously.

Furthermore, the robust control problem of double-machine system is investigated. In the presence of disturbance, the system (4) and (5) can be described by the following form:

$$\Sigma_{11}:\begin{cases} \dot{x}=(J_1-R_1)\dfrac{\partial H_1}{\partial x}+G_1(x)(v+\omega) \\ y=G_1^T(x)\dfrac{\partial H_1}{\partial x} \end{cases} \tag{8}$$

$$\Sigma_{22}:\begin{cases} \dot{\xi}=(J_2-R_2)\dfrac{\partial H_2}{\partial \xi}+G_2(\xi)(v+\omega) \\ \eta=G_2^T(\xi)\dfrac{\partial H_2}{\partial \xi} \end{cases} \tag{9}$$

where $\omega \in R^m$ is the unknown disturbance. The disturbance is same in the two systems because they have the same controller where the noise enters the system.

Theorem 3: For the systems (8) and (9), there exists the output feedback controller

$$v=-\kappa(y+\eta)-\Gamma(y+\eta) \tag{10}$$

can render both of them finite-gain L_2 stable with respect to disturbance ω, where κ is a positive definite diagonal matrix and Γ is a positive definite matrix.

Proof: Refer to the proof of Theorem 2 in the reference paper [4].

3. Hamiltonian Controller Design

3.1. Model of Permanent Magnet Wind Generator

The dynamic model of permanent magnet wind generator consists of the models of gearbox and permanent magnet synchronous generator, which can be described by the following third-order equations [9].

$$\Sigma:\begin{cases} J_t\dfrac{d\omega_r}{dt}=T_m-K_t\omega_r-n_gT_e \\ L_q\dfrac{di_q}{dt}=u_q-R_Si_q-L_dp\omega_gi_d-p\omega_g\psi \\ L_d\dfrac{di_d}{dt}=u_d-R_Si_d+L_qp\omega_gi_q \end{cases} \tag{11}$$

where $T_e=p(L_d-L_q)i_di_q+p\psi i_q$ is the electromagnetic torque applied on the generator shaft, $J_t=J_r+n_g^2J_g$, $K_t=K_r+n_g^2K_g$, $n_g=\omega_g/\omega_r$, ω_r is the turbine rotor speed, ω_g is the generator rotor speed, n_g is the gearbox ratio, T_m is the effective input mechanical torque to the wind turbine, J_r is the turbine rotor inertia, J_g is the generator rotor inertia, K_r is the turbine friction coefficient , K_g is the generator friction coefficient, ψ is magnet flux, p is the number of pole pairs, R_s is the resistance of stator, L_d and L_q are d-axis and q-axis generator inductances, u_d and u_q are d-axis and q-axis voltages, i_d and i_q are d-axis and q-axis currents, respectively.

3.2. Nonlinear Control of Double-Machine System

For the double-machine system $\sum_i$ ($i = 1, 2$), we design the nonlinear controller based on Hamiltonian energy method. Based on the system structure, Hamiltonian energy function can be chosen as

$$H_i = \frac{1}{2}(K_{ti}\omega_{ri} - T_{mi})^2 + \frac{1}{2}i_{qi}^2 + \frac{1}{2}i_{di}^2 \tag{12}$$

And $\nabla H_i = \begin{bmatrix} K_{ti}\omega_{ri} - T_{mi} & i_{qi} & i_{di} \end{bmatrix}^T$.

Under the Hamiltonian function, the system model (10) can be represented in PCH form

$$\frac{d}{dt}\begin{bmatrix}\omega_{ri}\\ i_{qi}\\ i_{di}\end{bmatrix} = \begin{bmatrix} -\frac{1}{J_{ti}} & -\frac{n_{gi}p_i\psi_i}{J_{ti}} & -\frac{n_{gi}p_i\left(L_{di}-L_{qi}\right)}{J_{ti}}i_{qi} \\ 0 & -\frac{R_{si}}{L_{qi}} & -\frac{L_{di}p_i\omega_{gi}}{L_{qi}} \\ 0 & \frac{L_{qi}p_i\omega_{gi}}{L_{di}} & -\frac{R_{si}}{L_{di}} \end{bmatrix}\nabla H_i + \begin{bmatrix} 0 \\ -\frac{p_i\omega_{gi}\psi_i}{L_{qi}} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{L_{qi}} & 0 \\ 0 & \frac{1}{L_{di}} \end{bmatrix}\begin{bmatrix}u_{qi}\\ u_{di}\end{bmatrix} \tag{13}$$

In order to complete the dissipative Hamiltonian realization, we employ the following control law:

$$u_i = \begin{bmatrix}u_{qi}\\ u_{di}\end{bmatrix} = K_i + v_i = \begin{bmatrix}u_{Kqi}\\ u_{Kdi}\end{bmatrix} + \begin{bmatrix}u_{vqi}\\ u_{vdi}\end{bmatrix} \tag{14}$$

By analyzing the form of (10), we design the pre-feedback control K_i, which makes the system satisfy PCH-D form. And v_i will be specially design in next step.

$$K_i = \begin{bmatrix}u_{Kqi}\\ u_{Kdi}\end{bmatrix} = \begin{bmatrix} p_i\omega_{gi}\psi_i + \frac{L_{qi}n_{gi}p_i\psi_i}{J_{ti}}\left(K_{ti}\omega_{ti} - T_{mi}\right) \\ \frac{L_{di}n_{gi}p_i\left(L_{di}-L_{qi}\right)i_{qi}}{J_{ti}}\left(K_{ti}\omega_{ti} - T_{mi}\right) + \frac{\left(L_{di}^2 - L_{qi}^2\right)p_i\omega_{gi}}{L_{qi}}i_{qi} \end{bmatrix} \tag{15}$$

By substituting (15) into (13), the closed-loop system is changed into the following form:

$$\Sigma_i : \frac{d}{dt}\begin{bmatrix}\omega_{ri}\\ i_{qi}\\ i_{di}\end{bmatrix} = \begin{bmatrix} -\frac{1}{J_{ti}} & -\frac{n_{gi}p_i\psi_i}{J_{ti}} & -\frac{n_{gi}p_i\left(L_{di}-L_{q_i}\right)}{J_{ti}}i_{qi} \\ \frac{n_{gi}p_i\psi_i}{J_{ti}} & -\frac{R_{si}}{L_{qi}} & -\frac{L_{di}p_i\omega_{gi}}{L_{qi}} \\ \frac{n_{gi}p_i\left(L_{di}-L_{qi}\right)}{J_{ti}}i_{qi} & \frac{L_{di}p_i\omega_{gi}}{L_{qi}} & -\frac{R_{si}}{L_{di}} \end{bmatrix}\nabla H_i + \begin{bmatrix} 0 & 0 \\ \frac{1}{L_{qi}} & 0 \\ 0 & \frac{1}{L_{di}} \end{bmatrix}\begin{bmatrix}u_{qi}\\ u_{di}\end{bmatrix} \tag{16}$$

where

$$J_i=\begin{bmatrix} 0 & -\frac{n_{gi}p_i\psi_i}{J_{ti}} & -\frac{n_{gi}p_i\left(L_{di}-L_{qi}\right)}{J_{ti}}i_{qi} \\ \frac{n_{gi}p_i\psi_i}{J_{ti}} & 0 & -\frac{L_{di}p_i\omega_{gi}}{L_{qi}} \\ \frac{n_{gi}p_i\left(L_{di}-L_{qi}\right)}{J_{ti}}i_{qi} & \frac{L_{di}p_i\omega_{gi}}{L_{qi}} & 0 \end{bmatrix},\ R_i=\begin{bmatrix} \frac{1}{J_{ti}} & 0 & 0 \\ 0 & \frac{R_{si}}{L_{qi}} & 0 \\ 0 & 0 & \frac{R_{si}}{L_{di}} \end{bmatrix}.$$

Obviously, J_i is a skew-symmetric matrix and R_i is a positive semi-define matrix. Therefore, the model (16) satisfies PCH-D form. At the same time, the output function is designed as

$$y_i=G_i^T\nabla H_i=\begin{bmatrix} \frac{i_{qi}}{L_{qi}} \\ \frac{i_{di}}{L_{di}} \end{bmatrix} \tag{17}$$

Based on Theorem 2, the nonlinear controller can be designed. Let positive definite matrix $\kappa=\begin{bmatrix} \kappa_{11} & 0 \\ 0 & \kappa_{22} \end{bmatrix}$, where $\kappa_{ii}>0\ (i=1,2)$. The control law v is taken as

$$v=\begin{bmatrix} u_{vq1} \\ u_{vd1} \end{bmatrix}=\begin{bmatrix} u_{vq2} \\ u_{vd2} \end{bmatrix}=-\kappa\left(G_1^T\nabla H_1+G_2^T\nabla H_2\right)=\begin{bmatrix} -\kappa_{11}\left(\frac{i_{q1}}{L_{q1}}+\frac{i_{q2}}{L_{q2}}\right) \\ -\kappa_{22}\left(\frac{i_{d1}}{L_{d1}}+\frac{i_{d2}}{L_{d2}}\right) \end{bmatrix} \tag{18}$$

3.3. Robust Control of Double-Machine System

In this part, we discuss the robust control problem of double-machine system in the presence of disturbance, whose PCH-D model $\sum_{ii}$ ($i=1,2$) is given as follows:

$$\Sigma_{ii}:\frac{d}{dt}\begin{bmatrix} \omega_{ri} \\ i_{qi} \\ i_{di} \end{bmatrix}=\begin{bmatrix} -\frac{1}{J_{ti}} & -\frac{n_{gi}p_i\psi_i}{J_{ti}} & -\frac{n_{gi}p_i\left(L_{di}-L_{q_i}\right)}{J_{ti}}i_{qi} \\ \frac{n_{gi}p_i\psi_i}{J_{ti}} & -\frac{R_{si}}{L_{qi}} & -\frac{L_{di}p_i\omega_{gi}}{L_{qi}} \\ \frac{n_{gi}p_i\left(L_{di}-L_{qi}\right)}{J_{ti}}i_{qi} & \frac{L_{di}p_i\omega_{gi}}{L_{qi}} & -\frac{R_{si}}{L_{di}} \end{bmatrix}\nabla H_i+\begin{bmatrix} 0 & 0 \\ \frac{1}{L_{qi}} & 0 \\ 0 & \frac{1}{L_{di}} \end{bmatrix}\begin{bmatrix} u_{qi}+\omega_1 \\ u_{di}+\omega_2 \end{bmatrix} \tag{19}$$

where $\omega=\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$ is the unknown disturbance.

According to Theorem 3, we can design the following controller.

$$v=\begin{bmatrix}u_{vq1}\\u_{vd1}\end{bmatrix}=\begin{bmatrix}u_{vq2}\\u_{vd2}\end{bmatrix}=-(\kappa+\Gamma)\left(G_1^T\nabla H_1+G_2^T\nabla H_2\right) \quad (20)$$

By combining the pre-feedback control (15) and the control law (18) according to (14), we obtain the nonlinear controller. This output feedback controller can stabilize the multi-machine system and render the closed-loop systems stable simultaneously.

Remark 1: By comparing (18) with (20), we notice that the double-machine system can be stabilized through adjusting the matrix $\kappa+\Gamma$. So the nonlinear controller improves the robustness property of the double-machine system and renders it finite-gain L_2stable in the presence of disturbances.

Remark 2: In this paper, in order to make the design idea simple and more understandable, we introduce the design process of multi-machine system by using the example of double-machine. When considering the case of multi-machine system (more than 2 machines), this design idea can be extended. The feasible design process is as follows: Firstly, the machines are divided into two groups and every group is described by one expanded higher order PCH system. Secondly, the controller is designed to make two higher order PCH systems stable simultaneously, that is, the stabilization problem of a double-machine system. Finally, the design result of multi-machine system is gotten from the controller of two higher order PCH systems. In order to show this design method don't limit to double-machine system, we adopt one system including three machines as the example in the simulations.

4. Simulations

In order to confirm the design results, the nonlinear control law is implemented in MATLAB. The simulations include two parts: the first one is the control results in the case of no disturbances and the second one is the control results in the presence of disturbances. The simulation results show that the nonlinear controller improves the stability and enhanced the robustness of the closed-loop system.

4.1. Simulation Results Without Disturbances

In the simulations, we control three machines based on the design method for multi-machine system. In the structure of systems (16), the different parameters are chosen to construct three groups of matrices.

$$J_1=\begin{bmatrix}0&-1&-2\\1&0&-1\\2&1&0\end{bmatrix},\quad R_1=\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad G_1=\begin{bmatrix}0&0\\2&0\\0&-2\end{bmatrix};$$

$$J_2=\begin{bmatrix}0&2&1\\-2&0&-1\\-1&1&0\end{bmatrix},\quad R_2=\begin{bmatrix}0&0&0\\0&3&0\\0&0&3\end{bmatrix},\quad G_1=\begin{bmatrix}0&0\\1&0\\0&-1\end{bmatrix};$$

$$J_3=\begin{bmatrix}0&-2&-2\\2&0&-2\\2&2&0\end{bmatrix},\quad R_1=\begin{bmatrix}0&0&0\\0&2&0\\0&0&2\end{bmatrix},\quad G_1=\begin{bmatrix}0&0\\-1&0\\0&1\end{bmatrix};$$

Let the parameter matrix $\kappa=\begin{bmatrix}1&0\\0&1\end{bmatrix}$. The following simulation results are obtained:

Figure 1 - Figure 3 show the response curves of three PMWGs, respectively. In every figure, the solid line is turbine rotor speed ω_r, dashed line is q-axis current i_q and point line is d-

axis current i_d. Under the control of the same Hamiltonian controller, the states of three machines are stable asymptotically, though the dynamic processes are various and the convergence times are different. So the nonlinear controller can stabilize three wind turbines simultaneously. Theorem 2 is proved.

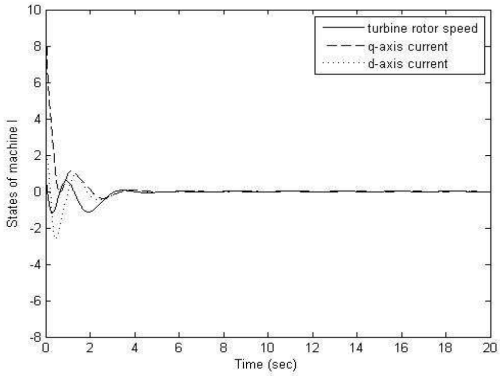

Figure 1. Dynamics of Machine 1#

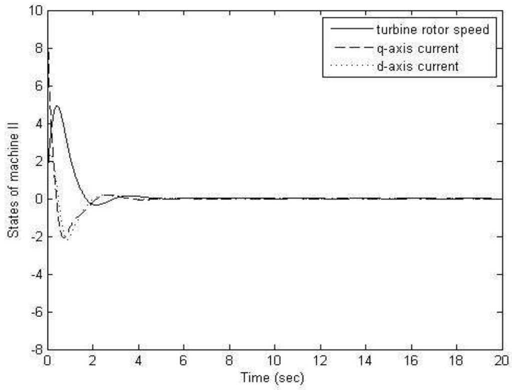

Figure 2. Dynamics of Machine 2#

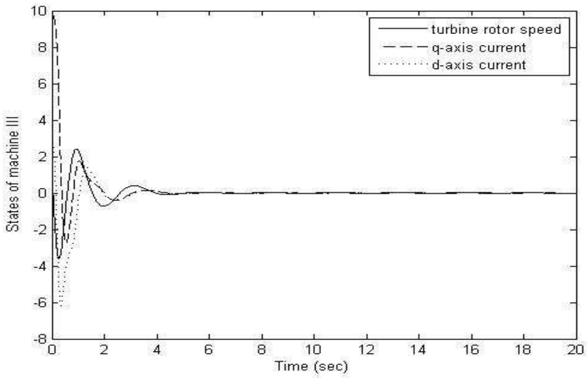

Figure 3. Dynamics of Machine 3#

4.2. Simulation Results with Disturbances

In the presence of disturbances $\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, the state response curves of three machines are shown in Figure 4 – Figure 6.

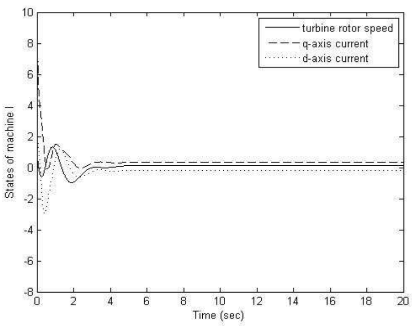

Figure 4. Dynamics of Machine 1#

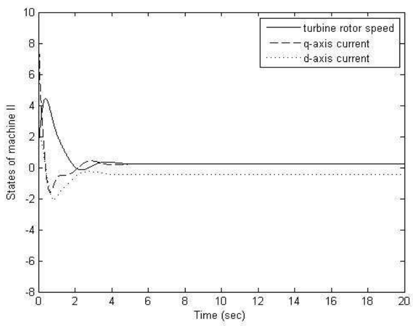

Figure 5. Dynamics of Machine 2#

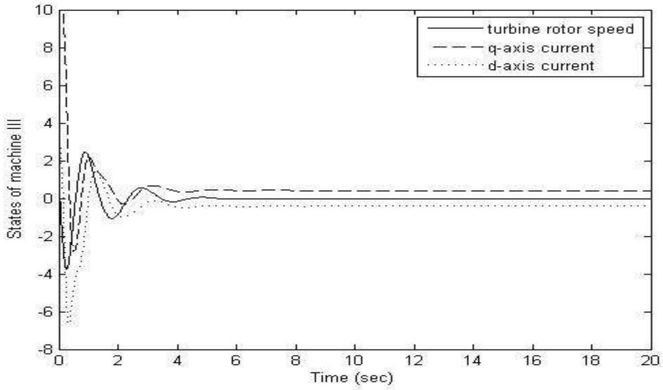

Figure 6. Dynamics of Machine 3#

Through adjusting the parameter matrix $\kappa+\Gamma=\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$, the Hamiltonian controller can still render three PMWGs stable when there exist unknown disturbances in the systems. Comparing with the simulation results of Figure 1-Figure 3, the states don't converge to the point of zero but a new equilibrium point. Therefore, the proposed controller guarantees the stability of the multi-machine system and enhances the ability of disturbance attenuation.

5. Conclusion

In this paper, the Hamiltonian energy controllers are proposed for the multi-machine system of PMWGs. Firstly, Theorem 2 is proposed and the nonlinear controller is designed for the double-machine system. Secondly, the case of double-machine system with disturbances is considered. The robust controller is constructed according to Theorem 3, which renders the closed-loop system finite-gain L_2 stable. Finally, the example of third-machine system presents in the simulation section, which extents the double-machine design to multi-machine case. The results show that the energy-based controller can stabilize the system of multi-machine wind turbines and enhance the ability of disturbance attenuation.

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