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A method of friction disturbance compensation based on velocity feedback

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Abstract

The friction disturbance influences servo system in follow-up performance directly, active compensation for the friction disturbance can improve the servo performance. The current compensation algorithms have some disadvantages, such as the complication of algorithms and the high requirement on hardware. In this paper, we propose a new method based on velocity observer. By this method, analytic solution of compensation voltage can be obtained aim to friction disturbance. The method is validated by comparing the results between simulation and experiment. This method has advantages of low requirement on hardware and simple algorithm. It is easy to be applied in practice and provides reference to other servo system for compensation of disturbance.

Keywords: *velocity observer, disturbance compensation, system identification*

1. Introduction

The friction disturbance influences servo system in follow-up performance directly, active compensation for the friction disturbance can improve the servo performance. There are two compensation methods in current practice [1-2]: compensation based on friction force model and based on dynamics model.

The former is from process, so the relationship between system parameter and friction is needed to be obtained, the model is complicated and with obvious error; the latter is from results, by observing system output signal and its dynamics model, the friction force can be calculated. The latter has less error than the former and needn’t to add hardware, so it widely applied to practice, i.e. those compensation method based on all kind of observer [3-5].

In the compensation research by observer, current method is based on displacement and torque feedback mainly, but the former algorithm is complicated, so it needs high performance hardware [6]; the latter needs to add current sensor.

In this paper, taking a system driven by current servo-system as subject, a compensation method was provided based on velocity observer. It is easy to be applied to controller design in practice, and provided a reference to other system about the friction compensation.

2. The Friction Compensation Algorithm Based on a Velocity Observer

2.1. Principle of Velocity Oberserver

Principal of the disturbance observer is as follow in Figure 1 [7].

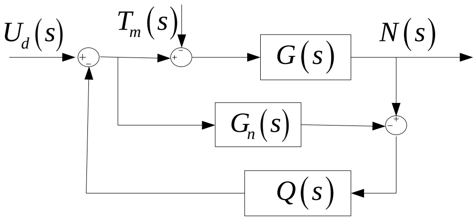

Figure 1. Principle of velocity oberserver

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where, *G*(s) is the real system, G_n(s) is the inverse identified model, *Q*(s) is a low band filter, U_d(s) is the control signal, *N*(s) is the output signal, T_m(s) is the disturbance signal.

Let the output of a real system with control and disturbance signal subtract the output of its identification model with the sole control signal, add the result to control signal. Set a low band filter in the feedback loop, that can restrain the influence of noise to velocity measurement. By the method, the friction disturbance can be identified The transfer function between the disturbance and the output signal can be obtained by Figure 1, it is as follow:

$$G_{Tn}(s)=\frac{n(s)}{T_m(s)}=-\frac{G(s)\left[1-Q(s)\right]}{Q(s)\left[G(s)G_n(s)-1\right]+1} \tag{1}$$

The higher the order of low band filter, the faster the system response will be, but if the order is too high, the system attenuation characteristic will be too bad[8]. Therefore, Bong Keun Kim[9] provides two kind of the low band filter model:

A low band filter model for first-order system

$$Q(s)=\frac{1}{\tau s+1} \tag{2}$$

A low band filter model for second-order system

$$Q(s)=\frac{3(\tau s)+1}{(\tau s)^3+3(\tau s)^2+3(\tau s)+1}$$

where, τ is the time constant of the low band filter system.

2.2. Deduction of the Compensation Algorithm

In a system driven by motor, inertia, damping constant and friction torque of all moving parts can be regarded a equivalent parameter of motor respectively, so the system model can be simplified to Figure 2.

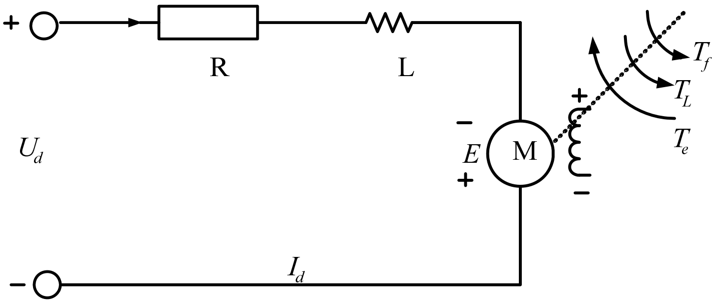

Figure 2. The simplified model of system driven by a DC motor

The mathematic model of such system driven by servo-motor can be obtained from Figure 2, it is shown in equation (3)

$$\begin{cases} U_d = RI_d + L\dfrac{dI_d}{dt} + E \\ T_e = J_e\ddot{\theta}_m + B_e\dot{\theta}_m + T_L + T_f \\ E = 2\pi K_e\dot{\theta}_m \\ T_e = K_m I_d \end{cases} \tag{3}$$

where U_d is the rotor voltage; I_d is the rotor current; R is the rotor resistance; L is the rotor inductance; E is the induced voltage; T_e is the output torque of motor; J_e is the equivalent system inertia; B_e is the equivalent damping constant; T_L is the equivalent load torque; T_f is the equivalent friction torque; K_e is the induced voltage constant; K_m is the torque constant; θ_m is the rotor angle. Among them, J_e, B_e and T_L need to be obtained by identification and other parameter can be known from motor manual.

After simplifying equation (3) the relationship between rotor voltage and its acceleration, velocity can be expressed as follow

$$U_d - \frac{R(T_L + T_f)}{K_m} = \frac{LJ_e}{K_m}\dddot{\theta}_{m1} + \left(\frac{RJ_e}{K_m} + \frac{LB_e}{K_m}\right)\ddot{\theta}_{m1} + \left(\frac{RB_e}{K_m} + 2\pi K_e\right)\dot{\theta}_{m1} \quad (4)$$

If the system works without friction, i.e. Tf=0, the equation (4) can be simplified to

$$U_d = \frac{LJ_e}{K_m}\dddot{\theta}_{m2} + \left(\frac{RJ_e}{K_m} + \frac{LB_e}{K_m}\right)\ddot{\theta}_{m2} + \left(\frac{RB_e}{K_m} + 2\pi K_e\right)\dot{\theta}_{m2} + \frac{RT_L}{K_m} \quad (5)$$

let the equation (5) subtract equation (4), the analytic expression of friction torque can be obtained

$$T_f = LJ_e\left(\dddot{\theta}_{m2} - \dddot{\theta}_{m1}\right)/R + \left(J_e + LB_e/R\right)\left(\ddot{\theta}_{m2} - \ddot{\theta}_{m1}\right) + \left(B_e + 2\pi K_e K_m/R\right)\left(\dot{\theta}_{m2} - \dot{\theta}_{m1}\right) + \Delta T_L \quad (6)$$

where, ΔT_L is variation of the load torqre. The compensation voltage for the friction disturbance is that:

$$\Delta U = \frac{RT_f}{K_m} = \frac{LJ_e}{K_m}\left(\dddot{\theta}_{m2} - \dddot{\theta}_{m1}\right) + \left(\frac{RJ_e}{K_m} + \frac{LB_e}{K_m}\right)\left(\ddot{\theta}_{m2} - \ddot{\theta}_{m1}\right) + \left(\frac{RB_e}{K_m} + 2\pi K_e\right)\left(\dot{\theta}_{m2} - \dot{\theta}_{m1}\right) + \frac{R\Delta T_L}{K_m} \quad (7)$$

The value of rotor inductance is usually very small with standard unit, i.e. $LJ_e / K_m \approx 0$, so equation (7) can be simplified to

$$\Delta U = \left(\frac{RJ_e}{K_m} + \frac{LB_e}{K_m}\right)\left(\ddot{\theta}_{m2} - \ddot{\theta}_{m1}\right) + \left(\frac{RB_e}{K_m} + 2\pi K_e\right)\left(\dot{\theta}_{m2} - \dot{\theta}_{m1}\right) + \frac{R\Delta T_L}{K_m} \quad (8)$$

where, $\ddot{\theta}_{m1}$ and $\dot{\theta}_{m1}$ are the real acceleration and velocity respectively; $\ddot{\theta}_{m2}$ and $\dot{\theta}_{m2}$ are the unreal acceleration and velocity without friction respectively.

The real velocity and acceleration can be know by a velocity sensor, but the unreal velocity and acceleration can be obtained only by calculation. The equation (8) is a first order differential equation, so the unreal velocity can be expressed as follow by relative calculation

$$\dot{\theta}_{m2} = Ce^{-\frac{RB_e + 2\pi K_e K_m}{RJ_e + LB_e}t} + \frac{U_d K_m - RT_L}{RB_e + 2\pi K_e K_m} \quad (9)$$

where, the constant C can be calculated with condition the initial velocity of motor is zero.

$$C = -\frac{U_d K_m - RT_L}{RB_e + 2\pi K_e K_m} \quad (10)$$

The acceleration can be obtained by differentiating equation (9)

$$\ddot{\theta}_{m2} = \frac{U_d K_m - RT_L}{RJ_e + LB_e} e^{-\frac{RB_e + 2\pi K_e K_m}{RJ_e + LB_e}t} \quad (11)$$

taking equation (9),(11) into equation(8), the compensation voltage can be calculated

$$\Delta U=\left(\frac{RJ_e}{K_m}+\frac{LB_e}{K_m}\right)\ddot{\theta}_{m1}+\left(\frac{RB_e}{K_m}+2\pi K_e\right)\dot{\theta}_{m1}-U_d+\frac{RT_L}{K_m} \tag{12}$$

where, J_e and B_e can't be know from the motor manual, in this paper, they will be calculated by an identification method as follow.

3. The Method of Integral Experiment Identification

The integral experiment identification [10] is carried out by sensor information (displacement, velocity, and torque). With this information, the system inertia, friction and damping paramenter can be calculated by a least square method. Besides, the experiment identification is corresponding to the real system. Therefore it is more effective and applied more widely. The principle of parameter identification is as follows: let J_e, B_e and T_L be the unknown variables, constructing equations by detected voltage and angle from experiment. The mechanical parameter will be then calculated by the least square theory.

3.1. Identification of B_e and T_f

During stable situation, the driving torque and resistance torque are equal, and the rotor current is almost constant, namely $\mathrm{d}I_d/\mathrm{d}t\approx 0$,so equation(3) can be simplified as

$$\frac{U_d-E}{R}=\frac{1}{K_m}\left(J_e\ddot{\theta}_m+B_e\dot{\theta}_m+T_L+T_f\right) \tag{13}$$

On the other hand, during stable situation, the angular acceleration $\ddot{\theta}_m\approx 0$, so equation (13) can be simplified further as

$$\frac{U_d-E}{R}=\frac{1}{K_m}\left(B_e\dot{\theta}_m+T_L\right) \tag{14}$$

The stable rotor velocity with different voltage can be detected by multiple experiments. Then multiple equations can be obtained through equation (14). However equation (14) cannot be balanced strictly with friction disturbance and these equations should be written including error as below

$$\frac{U_i-\pi K_e\dot{\theta}_i}{R}=\frac{1}{K_m}\left(\bar{B}_e\frac{\dot{\theta}_i}{2}+T_L+T_f\right)+e_i \qquad i=1,2,\ldots n \tag{15}$$

where i is a member of experimental data ,n is the number of the data classes(it should be ensurred that $n\geq 3$, the bigger the n value, the more accurate the identification result will be. Here, we choose n=6); e_i is voltage error generated by resistance torque disturbance, $\bar{B}_e$ and $\bar{J}_e$ are discreet value of equivalent damping constant and equivalent inertia respectively. To obtain more accurate $\bar{B}_e$and $\bar{J}_e$, we should ensure the error sum of squares being the smallest[11], namely, s value in equation(16) is smallest.

$$s=\sum_{i=1}^{6}e_i^2=\sum_{i=1}^{6}\left[U_i-\pi K_e\dot{\theta}_i-\frac{R}{K_m}\left(\bar{B}_e\frac{\dot{\theta}_i}{2}+\bar{T}_f\right)\right]^2 \tag{16}$$

where s is a function with two variables $\bar{B}_e$ and $\bar{T}_f$. To obtain the maximum of s, the gradient of this function should be zero, namely

$$\begin{cases} \dfrac{\partial s}{\partial \bar{B}_e} = -\sum_{i=1}^{6}\left[U_i - \pi K_e\dot{\theta}_i - \dfrac{R}{K_m}\left(\bar{B}_e\dfrac{\dot{\theta}_i}{2} + \bar{T}_f\right)\right]\dfrac{R\dot{\theta}_i}{K_m} = 0 \\ \dfrac{\partial s}{\partial \bar{T}_f} = -2\sum_{i=1}^{6}\left[U_i - \pi K_e\dot{\theta}_i - \dfrac{R}{K_m}\left(\bar{B}_e\dfrac{\dot{\theta}_i}{2} + \bar{T}_f\right)\right]\dfrac{R}{K_m} = 0 \end{cases} \quad (17)$$

$\bar{B}_e$ and $\bar{T}_f$ can be calculated by solving equation (17).

3.2. Identification of J_e

The data to indentify the rotation inertia should be collected during acceleration stage. To avoid the disturbance from start and stop of movement, collecting the data in time with half maximum velocity, equation (18) can be obtained in the same way.

$$\frac{U_i - \pi K_e\dot{\theta}_i}{R} = \frac{1}{K_m}\left(\bar{J}_e\ddot{\theta}_i + \bar{B}_e\frac{\dot{\theta}_i}{2} + T_L + T_f\right) + e_i \qquad i = 1, 2, \ldots n \quad (18)$$

where, meaning of e_i is the same to equation (15), $\bar{J}_e$ is discreet value of equivalent inertia. To obtain more accurate $\bar{J}_e$, we should ensure the error sum of squares being smallest, namely, s value in equation(19) being smallest

$$s = \sum_{i=1}^{6}\left[U_i - \pi K_e\dot{\theta}_i - \frac{R}{K_m}\left(\bar{J}_e\ddot{\theta}_i + \bar{B}_e\frac{\dot{\theta}_i}{2} + T_L + \bar{T}_f\right)\right]^2 \quad (19)$$

Let

$$\frac{\partial s}{\partial \bar{J}_e} = -\frac{R}{K_m}\sum_{i=1}^{6}\ddot{\theta}_i\left[U_i - \pi K_e\dot{\theta}_i - \frac{R}{K_m}\left(\bar{J}_e\ddot{\theta}_i + \bar{B}_e\frac{\dot{\theta}_i}{2} + T_L + \bar{T}_f\right)\right] = 0 \quad (20)$$

$\bar{J}_e$ can be caulculated by solving equation (20).

4. Verification of the Compensation Method

Taking the mechanism of developed Chemiluminescence Immunoassay Analyzer as an experimental system, it is shown in Figure 3.

Figure 3. Scheme for the system's transfer motion

A DC servo motor with encoder drives belt through reducer and belt wheel. Then the belt drives the block gliding along two guiding axes. The two axes can't parallel strictly, so the friction on block is variable, and the system servo performance will suffer with that.

4.1. The Motor's Parameter

The relative parameters of the used DC servo motor are shown in Table 1. The equivalent inertia and dumping coefficient will be obtained by above identification method.

Table 1. The parameters of the used servo motor

Nominal voltage U_e(V)	Rotor resistance R(Ω)	Rotor inductance L(mH)	Torque constant T/I(mNm/A)	induced voltage constant U/n(V/rpm)
24	0.61	0.119	25.9	0.00043

4.2. Parameter Identification

Firstly, providing some different voltages and collecting motor velocity in stable situation by encoder many times, then constructing equations by the least square, finally the equivalent inertia and dumping coefficient can be calculated as J_e=0.2757 gm^2, B_e=0.0275 mNm/rad/s.

4.3. Construction of the Simulation Model

Performing Laplace transformation on equation (3), the The block diagram of this servo system can be obtained as Figure 4.

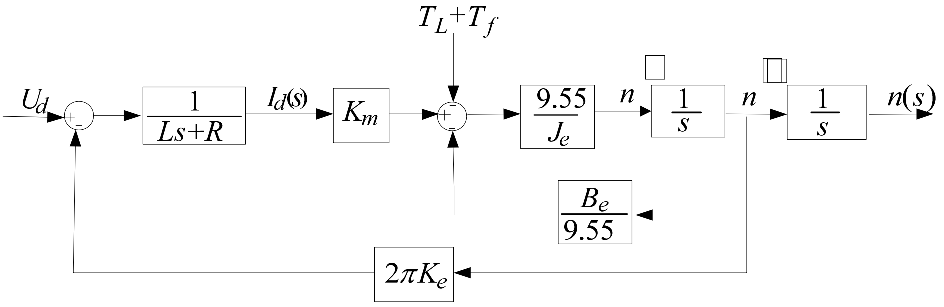

Figure 4. The block diagram of this servo system

Performing laplace transformation on equation (12) as

$$\Delta U(s)=\left(\frac{RJ_e}{K_m}+\frac{LB_e}{K_m}\right)\dot{\theta}_{m1}(s)s+\left(\frac{RB_e}{K_m}+2\pi K_e\right)\dot{\theta}_{m1}(s)-U_d(s) \tag{21}$$

Substituting the identified J_e, B_e value into diagram showed in Figure 4, taking equation (2) as the lower band filter and choosing its time constant as τ =0.001s, the simulation model with this compensation method can be obtained, as shown in Figure 5.

4.4. Analysis of the Identification Experiment and the Compensation Accuracy

To evaluate effectiveness of this identification method, providing experimental velocity and the output velocity based on this simulation model, the comparison between the two velocity-time curves is shown in Figure 6. In case of the voltage is 2.4v, 2.64v,2.88v, the velocity-time curve error between simulation and experiment is small, therefore, error of the identification result is also small, so it can be used to this compensation method.

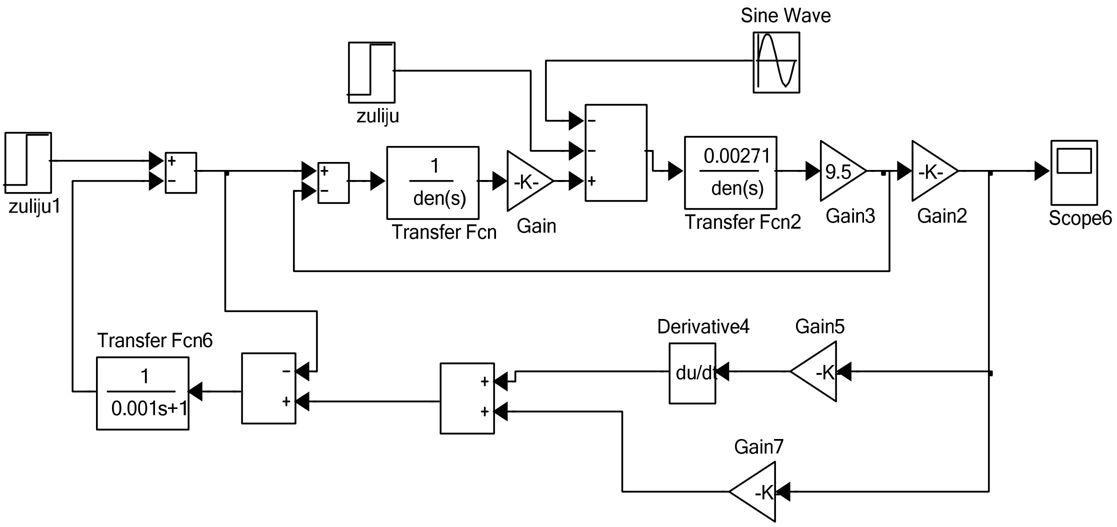

Figure 5. The simulation model with this compensation method

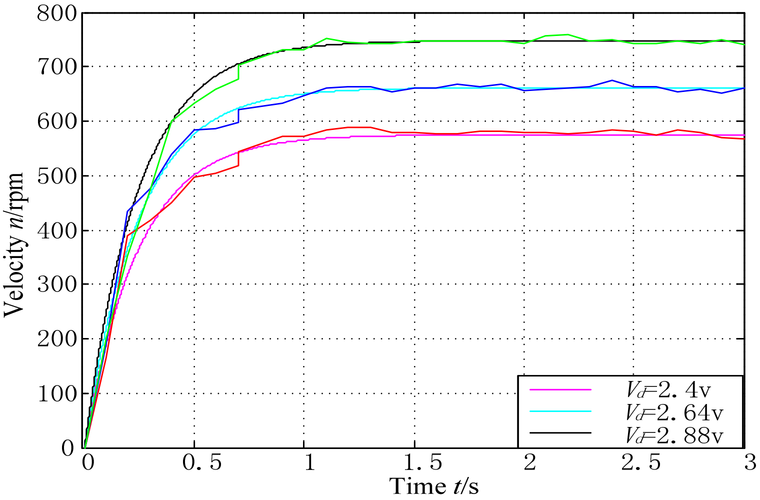

Figure 6. A group of comparison between the simulation and experiment velocity-time curves

To validate this compensation method, changing the belt preload and the grease in the guiding axis, so the friction and damping coefficient would be changed also. The velocity-time curves with and without compensation were shown below. There, the input voltage is 2.4V.

It can be know from Figure 7, the response speed with compensation is faster than without compensation. The steady-state value with compensation is near to the ideal value without friction (Error between them is in Table 2.), that indicates the compensation method has high precision.

Table 2. The friction error of the compensation method by comparison of velocity

The ideal steady-state velocity without friction *n*(rpm)	The experimental steady-value with the compensation *n*(rpm)	Error %
863.93	846.5	2.017

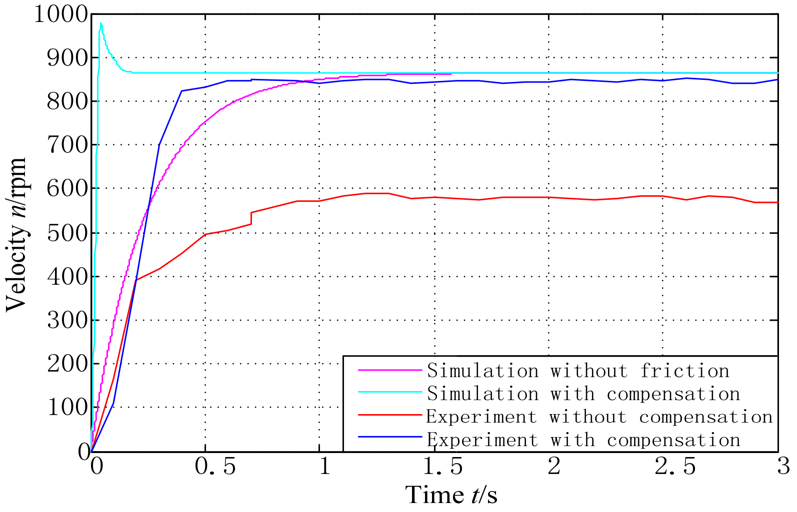

Figure 7. The velocity-time curves with and without compensation

5. Conclusion

Based on the system model, the analytic expression of friction force was deduced, farther, the needed compensation voltage was obtained. Its relative coefficients were obtained by an integral identification method. Comparison between simulation and experiment results validates the compensation method. This method has low requirement on hardware and is easy to be programmed with simple algorithm. The system response speed and tracking accuracy is better than without compensation obviously. This method can be applied to the real-time compensation to friction or other resistance torque.

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