

# Duty Cycle Influence on the Progenitor Properties of Millisecond Pulsars

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## Abstract

*Using a stellar evolutionary code, we carried out the investigation on the progenitor properties of millisecond pulsars with a varying duty cycle parameter. Our simulation results show that duty cycle play an important role on the final spin-period. The final spin-period for  $d = 0.001$  and  $d = 0.1$  is 4.75 ms, 1.49 ms respectively. For the same initial donor star mass and initial orbital period, the mass transfer evolutionary track indeed exists difference although the curve is quite similar. For other values of initial donor mass and initial orbital period, we find that the whole varying trend is consistent with each other.*

**Keywords:** millisecond, pulsar, duty cycle, evolution

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## 1. Introduction

Pulsar is well-known for its spin and radio pulse, namely, the lighthouse effect. There exist two kinds of pulsars. When a pulsar have a spin-period of  $P \sim 1$  s and a surface magnetic field of  $B \sim 10^{12}$  G. We call it a normal pulsar while a millisecond pulsar (denoted as MSP) should be defined as long as its spin-period is among the order of a few milliseconds. However, MSPs show some distinct observed properties such as short spin-period, low spin-down rate, old characteristic age and weak surface magnetic fields [1]. About 75% MSPs are in binary system (called binary millisecond pulsars, BMSPs), whereas the percent is only  $\sim 1\%$  for normal pulsars.

For now, two scenarios are widely accepted to account for the formation of MSPs. The first one is called recycling model, in which MSPs are proposed to be the evolutionary product of neutron star (NS) low-mass X-ray binaries (LMXBs) or intermediate-mass X-ray binaries (IMXBs) [2]. The pulsar crossed the so-called deathline accretes the mass and angular momentum from the donor star that overflows its Roche lobe, and can be subsequently spun up to a millisecond spin-period [3]. During accretion, the surface magnetic field of the NS decrease to be  $B \sim 10^8 - 10^9$  G due to accretion-induced field decay [4]. When the mass transfer ceases, a BMSP consisting of a recycling NS and a low-mass ( $\leq 0.4 M_{\text{sun}}$ ) helium white dwarf is produced. In another evolutionary process, MSPs may be formed by accretion-induced collapse (AIC) of ONeMg white dwarfs [5]. When the mass of an ONeMg white dwarf reaches the Chandrasekhar mass limit by accreting from its donor star, the electron-capture process leads to a gravitational collapse rather than a Type Ia explosion, and causes the formation an NS. Reseaches from other astrophysicists show that AIC may play an crucial role in forming MSPs [6,7]. Recently, using the recycling scenario, Liu & Chen have investigated the properties of initial parameter space of LMXBs which could evolve into BMSPs [8]. Many aspects such as thermal and viscous instability of an accretion disk, the propeller effect, spin evolution of the NSs and orbital angular momentum loss mechanism induced by magnetic braking are taken into account. Perhaps, the computing method and simulation process based on other fields of natural science could be referred to in the future exploration. [9-10].

However, the real process of the mass-transfer process maybe quite complicated and a key parameter which is still poor known maybe underestimated. This key parameter is called duty cycle (denoted as  $d$ ) which represents the ratio of the outburst time-scale to recurrence time during accretion process. In the paper above, we only take  $d = 0.01$  for an average calculation while the real influence maybe rather different. For instance, King proposes that the

range of duty cycle maybe from 0.001 to 0.1 [11]. Obviously, there exists discrepancy to be corrected and need further explanation.

In this paper, we take extension to investigate systematically the influence of different values of duty cycle on the properties of progenitors by recycling scenario and attempt to examine whether more submillisecond pulsars could be found by this detailed simulation. The structure of this paper is as follows. We describe the research method which is necessary in the evolution calculation of LMXBs in section 2. The calculated results are presented in section 3. Finally, a brief discussion and summary is given in section 4.

## 2. Research Method

Using a stellar evolution code developed by Eggleton [12-14], which has been updated with the latest input physics over the past three decades [15, 16], we calculated the evolution of binaries consisting of a NS (of mass  $M_{\text{NS}}$ ) and a main-sequence donor star (of mass  $M_d$ ), and test if they can evolve into MSPs. The stellar OPAL opacities was taken from the description for a low temperature [17]. In our calculation, the ratio of mixing length to local pressure scale height was set to be 2.0, and the overshooting parameter of the donor star (with a solar chemical composition  $X = 0.70$ ,  $Y = 0.28$ , and  $Z = 0.02$ ) was taken to be 0 [18]. However, the following description on the model is quite similar with the expressions in the paper [8].

### 2.1. Accretion Disk Instability

With nuclear evolution, the donor star begin to fill its Roche lobe, and transfer hydrogen-rich material onto the NS. Due to the high angular momentum, the accreting material forms a disk surrounding the NS. If the effective temperature in the accretion disk is below  $\sim 6500$  K (the hydrogen ionization temperature), the disk accretion is supposed to be thermally and viscous unstable [19]. Meanwhile, the accreting NS will be a transient X-ray source, which appears as short-lived outbursts phase and long-term quiescence phase.

When the mass transfer rate  $-\dot{M}_d$  is lower than the critical mass-transfer rate [20], namely,

$$\dot{M}_{\text{cr}} \approx 3.2 \times 10^{-9} \left( \frac{M_{\text{NS}}}{1.4M_{\text{sun}}} \right)^{0.5} \left( \frac{M_d}{1.0M_{\text{sun}}} \right)^{-0.2} \left( \frac{P_{\text{orb}}}{1.0\text{d}} \right)^{1.4} M_{\text{sun}} \text{yr}^{-1} \quad (1)$$

where,  $P_{\text{orb}}$  is the orbital period of the binary, the NS accretes only during outbursts. Defining a duty cycle (denoted as  $d$ ) which is a quite important parameter in this simulation to be the ratio of the outburst time-scale to the recurrence time. The accretion rate of the NS  $\dot{M}_{\text{ac}} = -\dot{M}_d/d$ . Otherwise for a high mass transfer rate  $-\dot{M}_d > \dot{M}_{\text{cr}}$ , we assume  $\dot{M}_{\text{ac}} = -\dot{M}_d$ . Certainly, the mass growth rate of the NS should suffer the limitation of the Eddington accretion rate ( $\dot{M}_{\text{Edd}} \approx 1.5 \times 10^{-8} M_{\text{sun}}$ ). The excess material is assumed to be expelled from the vicinity of the NS by radiation pressure, and carries away the specific orbital angular momentum of the NS.

### 2.2. Magnetic Braking

Low-mass donor star would be braked to spin down by the coupling between the magnetic field and the stellar winds [21]. However, the tidal interaction between the donor star and the NS would continuously spin the star back up co-rotation with the orbital rotation [22]. Therefore, magnetic braking mechanism indirectly carries away the orbital angular momentum of binaries.

In calculation, we adopt an induced magnetic braking description given by [23], in which the angular momentum loss rate is as the following formula (2), where,  $K = 2.7 \times 10^{47} \text{gcm}^2$  [24],  $\omega_{\text{crit}}$  is the critical angular velocity at which the angular momentum loss rate reaches a saturated state,  $\omega = 2\pi/P_{\text{orb}}$  and  $R_d$  are the angular velocity and the radius of the donor star, respectively.

$$J_{\text{mb}} = \begin{cases} -K\omega^3 \left( \frac{R_d}{R_{\text{sun}}} \frac{M_{\text{sun}}}{M_d} \right)^{1/2}, & \omega \leq \omega_{\text{crit}} \\ -K\omega\omega_{\text{crit}}^2 \left( \frac{R_d}{R_{\text{sun}}} \frac{M_{\text{sun}}}{M_d} \right)^{1/2}, & \omega > \omega_{\text{crit}} \end{cases} \quad (2)$$

Kim & Demarque [25] proposed that  $\omega_{\text{crit}}$  is inversely proportional to the convective turbulent timescale of the star when its age is 200 Myr, i. e.

$$\omega_{\text{crit}} = \omega_{\text{crit,sun}} \frac{\tau_{\text{sun}}}{\tau}, \quad (3)$$

where  $\omega_{\text{crit,sun}} = 2.9 \times 10^{-5}$  Hz,  $\tau_{\text{sun}}$  and  $\tau$  are the convective turbulent time-scales of the Sun and the donor star, respectively.

### 2.3. Spin Evolution of the NS

In stellar evolution code, we also consider the spin evolution of pulsars as follows. With the spin-up of the NS, the accreting material would interact with the magnetosphere of the NS. We simply define the magnetosphere radius as the position that the ram pressure of the infalling material is balanced by the magnetic pressure of the NS [26]. Under assumption of spherical accretion [27, 28], the magnetosphere radius is

$$r_m = 1.6 \times 10^8 \left( \frac{B_s}{10^{12} \text{G}} \right)^{4/7} \left( \frac{|\dot{M}_d|}{10^{18} \text{gs}^{-1}} \right)^{-2/7} \text{cm}, \quad (4)$$

where  $B_s$  is the surface magnetic field of the NS. Some observations and analysis argued that the mass accretion of the NS can lead to its magnetic field decay [29]. Here we adopt an empirical model given by [30], i. e.

$$B_s = \frac{B_i}{1 + \Delta M_{\text{acc}} / m_B}, \quad (5)$$

where  $B_i$  is the initial magnetic field of the NS,  $\Delta M_{\text{acc}}$  is the accreted mass of the NS, and  $m_B \sim 10^{-4} M_{\text{sun}}$ .

When the NS rotation is too fast, the gravitational force of the accreting material at  $r_m$  is less than its centrifugal force. The centrifugal barrier would eject the accreting material, and exerting a propeller spin-down torque on the NS [31]. For instance, if the magnetosphere radius is greater than the co-rotation radius

$$r_c = 1.5 \times 10^8 \left( \frac{M_{\text{NS}}}{M_{\text{sun}}} \right)^{1/3} P_s^{2/3} \text{cm}, \quad (6)$$

where  $P_s$  is the spin-period of the NS in units of second, the propeller effect occurs. The spin angular momentum loss rate via the propeller effect can be written as

$$J_p = 2\dot{M}_m r_m^2 [\Omega_K(r_m) - \Omega], \quad (7)$$

where  $\Omega_K(r_m)$  is the Keplerian angular velocity at  $r_m$ . When  $r_m < r_{\text{co}}$ , the accreting material is bound in the magnetic field lines to co-rotate with the NS, and is accreted onto its surface.

Assuming rigid body rotation and the momentum of inertia  $I = 10^{45} \text{gcm}^2$ , the spin-up torque of the accreting material exerting on the NS is given by

$$J_{\text{ac}} = \dot{M}_{\text{ac}} \sqrt{GM_{\text{NS}} R}, \quad (8)$$

where  $G$  is the gravitational constant,  $R$  is the radius of the NS.

In addition, if  $r_m$  is greater than the light cylinder radius

$$r_{\text{lc}} = \frac{c}{\Omega} = \frac{cP_s}{2\pi}, \quad (9)$$

the NS appears as a radio pulsar. As a result of magnetic dipole radiation, the spin angular momentum loss rate is

$$J_m = -\frac{2B_s^2 R^6 \Omega^3}{3c^3}, \quad (10)$$

### 3. Results and Analysis

In our calculation, with a solar chemical composition ( $X = 0.70$ ,  $Y = 0.28$ , and  $Z = 0.02$ ), the initial mass of the donor star and the NS is set to be  $1.5 M_{\text{sun}}$  and  $1.4 M_{\text{sun}}$  respectively. We take 5.0 day as the initial orbital period. Like the paper mentioned above, we adopt an initial spin-period of  $P_{s,i} = 10$  s and initial magnetic field of  $B = 10^{12}$  G for the accreting NS. Our major aim is to investigate the influence from different duty cycle value on the final evolutionary result, therefore, we take the value of duty cycle from 0.001 to 0.1 which is quite different from the paper above in which the duty cycle is a constant ( $d = 0.01$ ). If the donor star evolves into a He white dwarf and the Roche lobe overflow ends, we stop the calculation.

In Figure 1, we present two relation curves between duty cycle value and the final system parameters. We can find that with the increasing of duty cycle value, the final companion mass get a decrease from  $0.3273 M_{\text{sun}}$  to  $0.3247 M_{\text{sun}}$  whereas the final NS mass changes from  $1.47 M_{\text{sun}}$  to  $1.61 M_{\text{sun}}$ . The relationship between duty cycle and the final spin-period is denoted in Figure 2, it is clear that when the value of duty cycle changes from 0.001 to 0.1, the final spin-period obtain an obvious decrease, for example, by accreting the mass and angular momentum, the final spin-period reduces to 4.75 ms for  $d = 0.001$ , whereas the value is only 1.49 ms for  $d = 0.1$ .

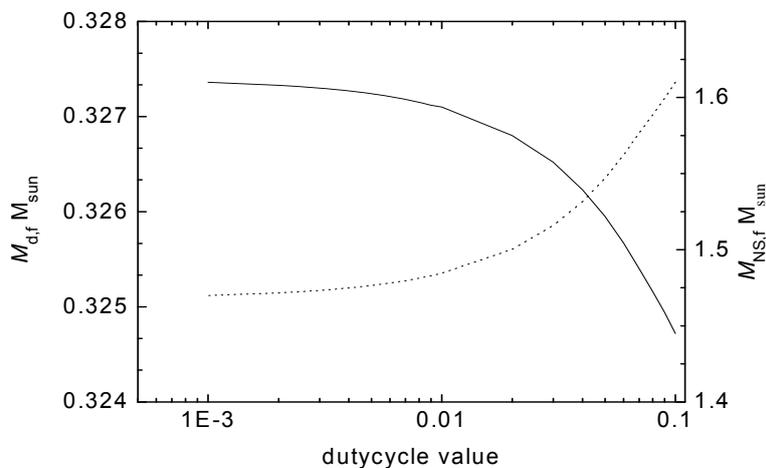


Figure 1. Relation between duty cycle value and the final system parameters. The solid line and the dot line represent the final donor star mass and the final NS mass, respectively.

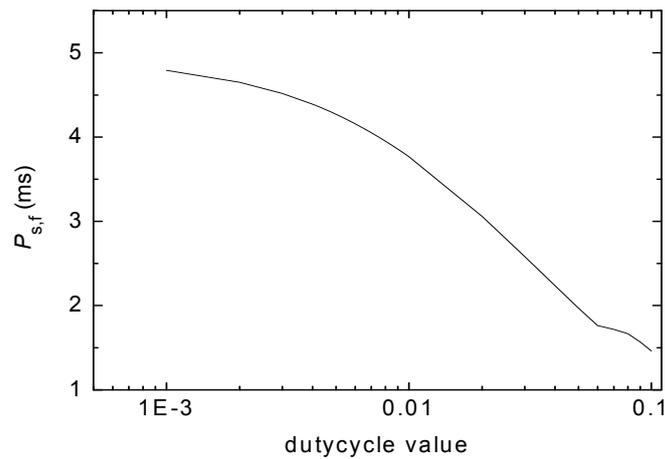


Figure 2. Relation between duty cycle value and the final spin-period  $P_{s,f}$

In Figure 3, we show a mass transfer rate evolutionary track of an LMXB with  $M_{d,i} = 1.5 M_{\text{sun}}$  and  $P_{\text{orb},i} = 5.0$  day for  $d = 0.1$ . We can find that it takes about 0.12 billion year for the Roche lobe overflow to complete except that there is a epoch during which no mass transfer take place (two points across with the abscissa). The mass transfer rate is among  $10^{-9} M_{\text{sun}} \text{ yr}^{-1} \sim 10^{-7} M_{\text{sun}} \text{ yr}^{-1}$ , the average value of mass transfer rate is about  $10^{-8} M_{\text{sun}} \text{ yr}^{-1}$ . The physical process of Figure 4 is the same with Figure 3 except for  $d = 0.01$ .

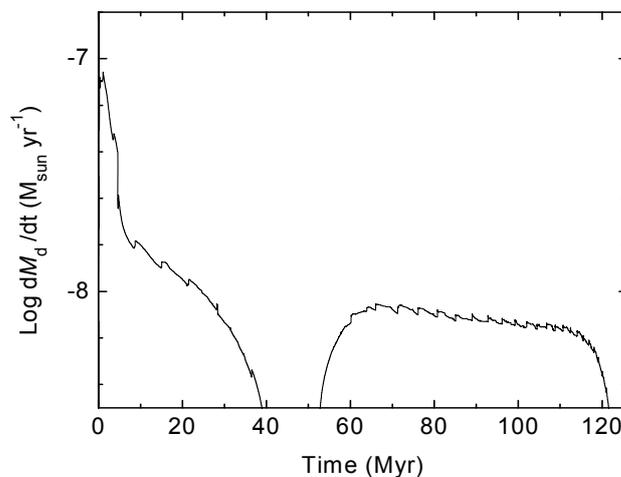


Figure 3. Mass transfer rate evolutionary track of an LMXB with  $M_{d,i} = 1.5 M_{\text{sun}}$ , and  $P_{\text{orb},i} = 5.0$  day for  $d = 0.1$

#### 4. Discussion and Conclusion

With Eggleton's stellar evolution code, in this paper, we have investigated the influence from duty cycle on the progenitors properties which could evolve into BMSPs by the classical recycling evolutionary channel. In the simulation, we take into account the disk and the propeller effect on the mass transfer process as well as spin evolution of neutron stars. An orbital angular momentum loss mechanism by magnetic winds is also considered. The main results of the paper are summarized as follows:

1. Although the key parameter duty cycle is changing from 0.001 to 0.1, all LMXBs in our scenario have a chance to evolve into a BMSP with a final spin-period of  $P_{s,f} \leq 5.0$  ms.

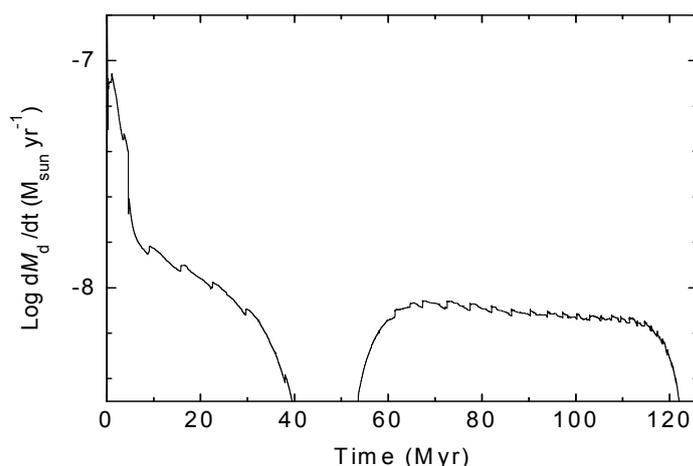


Figure 4. Similar to Figure 3, but for  $d = 0.01$

2. From Figure 2, we can find that duty cycle play an important influence on the evolutionary results. For instance, the final spin-period value for  $d = 0.001$  is two times than that for  $d = 0.1$ . Obviously, duty cycle with larger value have more chances to accrete mass and angular momentum from the donor star and so lead to a smaller spin-period.

3. We did not give the mass transfer rate evolutionary track for  $d = 0.001$  since it is almost consistent with that for  $d = 0.01$ . However, according to Figure 3 and Figure 4, there really exists distinction such as the average of mass transfer rate for  $d = 0.1$  is somewhat larger than that for  $d = 0.01$  and that the cross points for two situation is different.

Questions may arise on our choosing for the initial donor star mass and the initial orbital period. For example, what are the results if different donor star with larger or smaller mass is adopted. However, the results from our much calculation find that different donor star mass indeed exist a little distinction, while the whole varying trend is consistent with each other. For instance, if we take the same initial donor star with a smaller value of initial orbital period, it will get a smaller value for  $P_{s,f}$ . At the same time, when we take the same initial orbital period with a smaller initial companion mass, a smaller value could also be obtained for  $P_{s,f}$ . And it seems that the influence of a varying initial period is somewhat more obvious than that for a varying initial donor mass. This is the reason why we choose  $1.5 M_{\text{sun}}$  as the initial companion mass of the low Mass X-ray Binaries.

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