

N-tangle of the Superposition of Greenberger-Horne-Zeilinger States

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Abstract

In the paper, use the n -tangle, as a measure of entanglement to investigate the superposition of Greenberger-Horne-Zeilinger states for even n qubits. The paper show that orthogonal basis with the GHZ state can be divided into two group via SLOCC –invariant. In the same group, an expression for any two superposed state is provided, when the superposition coefficient is real, the n -tangle of superposed state is invariable. Otherwise in the different group, which can get a different expression if the superposed state come from different group.

Keywords: N -tangle invariability, superposition states, GHZ state.

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1. Introduction

Quantum entanglement is a key element for applications of quantum communications and quantum information. In particular, the entanglement can be used as a quantum resource to carry out a number of computational and information processing tasks, which include teleportation of an unknown quantum state. Quantum teleportation is a prime example of a quantum information processing task, which an unknown state can be perfectly transported from one place to another by using previously shared entanglement and classical communication between the sender and the receiver [1]. Analysis of such quantum phenomena may provide us a better understanding of the structure of the quantum mechanics framework. Therefore, it is important to find out the ways of classifying and quantifying the entanglement properties of quantum states. The idea that locally invariant quantities can be used to characterize entanglement is central to this work. Invariants under local unitary (LU) and more general transformations, such as, stochastic local operations and classical communication (SLOCC), have been extensively studied in this context [2–13].

Recently a new question has been raised concerning the entanglement of the superposed states [14–20]. Yang et al [22] have presented a way to teleport multi-qubit quantum information via the control of many agents in a network. Also reported in other areas [23, 24]. ZHANG [15] studied the entanglement properties of the superposed state of orthogonal maximally entangled states and discussed the relation between the superposed state and the mutually unbiased state. Preeti Parashar and Swapan Ranat [14] calculated the analytic expression for geometric measure of entanglement for arbitrary superposition of two N -qubit canonical orthonormal Greenberger-Horne-Zeilinger (GHZ) states and the same for two W states. Recently, Seyed Javad Akhtarshenas [21] investigated lower and upper bounds on the concurrence of the superposition of two states.

The paper will analyze expression of SLOCC –invariant and n -tangle for arbitrary superposition of two N -qubit canonical orthogonal GHZ states.

2. The SLOCC –Invariant and n -tangle of GHZ States

For even n qubits, the 2 degree SLOCC invariants are defined [4]

$$H_{12\dots n} = \langle \psi^* | \hat{\sigma}_{1y} \hat{\sigma}_{2y} \cdots \hat{\sigma}_{ny} | \psi \rangle \quad (1)$$

Alexander Wong define the n-tangle by [25]

$$\tau_{12\dots n} = |H_{12\dots n}|^2 = \left| \langle \psi^* | \hat{\sigma}_{1y} \hat{\sigma}_{2y} \cdots \hat{\sigma}_{ny} | \psi \rangle \right|^2 \quad (2)$$

For two qubits, the concurrence C is defined as [26]

$$C(\psi) = \left| \langle \psi | \hat{\sigma}_{1y} \hat{\sigma}_{2y} | \psi^* \rangle \right|^2; \quad (3)$$

$$\text{i.e. } C(\psi) = |H_{12}|^2, \quad (4)$$

it is easy to show ,

$$H_{12} = 2(a_1 a_2 - a_0 a_3), \quad (5)$$

For four qubits, it is obtained [6]

$$H_{1234} = 2(a_0 a_{15} - a_1 a_{14} - a_2 a_{13} + a_3 a_{12} - a_4 a_{11} + a_5 a_{10} + a_6 a_9 - a_7 a_8) \quad (6)$$

As ref[10], The full set of canonical orthogonal N-qubit GHZ states is given by

$$|\psi_G^\pm\rangle_{12\dots n} = \frac{1}{\sqrt{2}} \left(|0i_2 i_3 \cdots i_n\rangle \pm |0\bar{i}_2 \bar{i}_3 \cdots \bar{i}_n\rangle \right), \quad (7)$$

where $i=0,1 \forall k=2,3,\dots,n$ and a bar over a bit value indicates its logical negation.

Obviously, get that n-tangle $\tau_{12\dots n} = 1$ for all HZ states. But for 2 degree SLOCC invariants

$$H_{12\dots n} = \langle \psi^* | \hat{\sigma}_{1y} \hat{\sigma}_{2y} \cdots \hat{\sigma}_{ny} | \psi \rangle = \pm 1. \quad (8)$$

Therefore, according to 2 degree SLOCC invariants, all GHZ states can be divided into two sets, one is $H_{12\dots n} = 1$, the others is $H_{12\dots n} = -1$.

For two qubits, there are four orthogonal maximally entangled states, i.e., four Bell states,

$$\begin{aligned} |\psi\rangle^\pm &= \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle), \\ |\varphi\rangle^\pm &= \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle). \end{aligned} \quad (9)$$

For convenience, let us express

$$\begin{aligned} |\varphi^1\rangle &= \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle), & |\varphi^2\rangle &= \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle), \\ |\psi^1\rangle &= \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle), & |\psi^2\rangle &= \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) \end{aligned} \quad (10)$$

The two sets $\{|\varphi^1\rangle, |\varphi^2\rangle\}, \{|\psi^1\rangle, |\psi^2\rangle\}$,

For the four-qubit system., There are sixty orthogonal maximally entangled states, i.e., GHZ states,

$$\begin{aligned}
 |\varphi^1\rangle &= \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle), \\
 |\varphi^2\rangle &= \frac{1}{\sqrt{2}}(|0001\rangle - |1110\rangle), \\
 |\varphi^3\rangle &= \frac{1}{\sqrt{2}}(|0010\rangle - |1101\rangle), \\
 |\varphi^4\rangle &= \frac{1}{\sqrt{2}}(|0011\rangle + |1100\rangle), \\
 |\varphi^5\rangle &= \frac{1}{\sqrt{2}}(|0100\rangle - |1011\rangle), \\
 |\varphi^6\rangle &= \frac{1}{\sqrt{2}}(|0101\rangle + |1010\rangle), \\
 |\varphi^7\rangle &= \frac{1}{\sqrt{2}}(|0101\rangle + |1010\rangle), \\
 |\varphi^8\rangle &= \frac{1}{\sqrt{2}}(|0111\rangle - |1000\rangle), \\
 |\psi^1\rangle &= \frac{1}{\sqrt{2}}(|0000\rangle - |1111\rangle), \\
 |\psi^2\rangle &= \frac{1}{\sqrt{2}}(|0001\rangle + |1110\rangle), \\
 |\psi^3\rangle &= \frac{1}{\sqrt{2}}(|0010\rangle + |1101\rangle), \\
 |\psi^4\rangle &= \frac{1}{\sqrt{2}}(|0011\rangle - |1100\rangle), \\
 |\psi^5\rangle &= \frac{1}{\sqrt{2}}(|0100\rangle + |1011\rangle), \\
 |\psi^6\rangle &= \frac{1}{\sqrt{2}}(|0101\rangle - |1010\rangle), \\
 |\psi^7\rangle &= \frac{1}{\sqrt{2}}(|0101\rangle - |1010\rangle), \\
 |\psi^8\rangle &= \frac{1}{\sqrt{2}}(|0111\rangle + |1000\rangle).
 \end{aligned} \tag{11}$$

The two sets $\{|\varphi^1\rangle, |\varphi^2\rangle, \dots, |\varphi^8\rangle\}, \{|\psi^1\rangle, |\psi^2\rangle, \dots, |\psi^8\rangle\}$.similarly, for even n qubits, there are two sets $\{|\varphi^1\rangle, |\varphi^2\rangle, \dots\}, \{|\psi^1\rangle, |\psi^2\rangle, \dots\}$ GHZ states.

3. The Superposition of GHZ States

The superposition of two GHZ states in same sets is

$$|\psi\rangle_{AB} = c_1|\varphi^1\rangle + c_2|\varphi^2\rangle = a_1e^{i\theta_1}|\varphi^1\rangle + a_2e^{i\theta_2}|\varphi^2\rangle \quad (12)$$

Then the the n-tangle

$$\tau_{12\dots n} = \left| a_1^2 e^{2i\theta_1} + a_2^2 e^{2i\theta_2} \right|^2 \quad (13)$$

If $\Delta\theta = \theta_1 - \theta_2 = k\pi$, $\tau_{12\dots n} = 1$, The state superposed of the two GHZ states's n-tangle is also equal to 1. Obviously, if $\theta_1 = \theta_2$, The n-tangle of two GHZ superposed state is also equal to 1. Especially if $\theta_1 = \theta_2 = 0$, i.e., the superposition coefficient is real, the n-tangle is not changed for two GHZ superposition states in same sets.

On the other hand, The superposition of two different set GHZ states is

$$|\psi\rangle_{AB} = c_1|\varphi^1\rangle + c_2|\psi^1\rangle = a_1e^{i\theta_1}|\varphi^1\rangle + a_2e^{i\theta_2}|\psi^1\rangle \quad (14)$$

Then the concurrence

$$\tau_{12\dots n} = \left| a_1^2 e^{2i\theta_1} - a_2^2 e^{2i\theta_2} \right|^2 \quad (15)$$

$$\text{If } \Delta\theta = \theta_1 - \theta_2 = \left(k + \frac{1}{2} \right) \pi, \tau_{12\dots n} = 1,$$

Obviously, if $\theta_1 = \theta_2$, $\tau_{12\dots n} = \left| a_1^2 - a_2^2 \right|^2$. If $\theta_1 = \theta_2 = 0$, and $a_1 = a_2$, i.e., the superposition coefficient is real and equal, The state superposed of the GHZ states is a separable state.

4. Conclusion

In summary, the paper has discussed the n-tangle of superposition of Greenberger-Horne-Zeilinger states via the SLOCC invariant for even n qubits and find that orthogonal basis with the GHZ state can be into two group via SLOCC -invariant. The paper has shown that there exists a relation between the n-tangle and the SLOCC invariant for superposition of Greenberger-Horne-Zeilinger states. It could be affirmed that if the 2 degree SLOCC invariant of Greenberger-Horne-Zeilinger states is same, the n-tangle of superposition state is determined by the coefficient of phase difference. Especially, if the superposition coefficient is real, the n-tangle is not change in same sets; but in different sets, the n-tangle of superposed state is determined not only by phase difference but also by amplitude.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No.10864007), the Key Discipline of Theoretical Physics of Xinjiang (Grant No.LLWLY201111), Xinjiang Normal University Graduate Student Innovation Fund Projects of Xinjiang, China (20121215) and the scientific research foundation for the excellent young teachers of Xinjiang Normal University, Project number:XJNU0921.

References

- [1] Zhang T, Tu XL, Wang XM, Zha XW. Using less Quantum Resource for Probabilistic Controlled Teleportation of a Triplet W State. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2012; 10(6): 1227-1234.
- [2] Dür W, Vidal G, Cirac J. Three Qubits Can Be Entangled in Two nonequivalent Ways. *Phys. Rev. A*. 2000; 62(6): 062314.
- [3] Verstraete F, Dehaene J, Moor BD, Verschelde H. Four Qubits Can Be Entangled in Nine Different Ways. *Phys. Rev. A*. 2002; 65(5): 052112.
- [4] Miyake A. Classification of Multipartite Entangled States by Multidimensional Determinants. *Phys. Rev. A*. 2003; 67(1): 012108.
- [5] Osterloh A, Siewert J. Constructing N-Qubit Entanglement Monotones from Antilinear Operators. *Phys. Rev. A*. 2005; 72(1): 012337.
- [6] Osterloh A, Siewert J. Entanglement Monotones and Maximally Entangled States in Multipartite Qubit Systems. *Int. J. Quantum Inf.* 2006; 4(3): 531.
- [7] Luque JG, Thibon JY. Polynomial Invariants of Four Qubits. *Phys. Rev. A*. 2003; 67(4): 042303.
- [8] Luque JG, Thibon JY. Algebraic Invariants of Five Qubits. *J. Phys. A: Math. Gen.* 2006; 39(2): 371.
- [9] Li DF, Li XR, Huang HT, Li XX. Classification of Four-Qubit States by Means of a Stochastic Local Operation and the Classical Communication Invariant and Semi-Invariants. *Phys. Rev. A*. 2007; 76(5): 052311.
- [10] Ren XJ, Jiang W, Zhou XX, Zhou ZW, Guo GC. Permutation-Invariant Monotones for Multipartite Entanglement Characterization. *Phys. Rev. A*. 2008; 78(1): 012343.
- [11] Zha XW, Ren KF. General Relation between the Transformation Operator and an Invariant under Stochastic Local Operations and Classical Communication in Quantum Teleportation. *Phys. Rev. A*. 2008; 77(1): 014306.
- [12] Zha XW, Song HY, Ren KF. The Relation between Local Unitary Transformation Invariant and Perfect Quantum Teleportation. *IJQI*. 2010; 8(8): 1251-1256.
- [13] Zha XW, Ma GL. Classification of Four-Qubit States by Means of a Stochastic Local Operation and the Classical Communication Invariant. *Chin. Phys. Lett.* 2011; 28(2): 20301.
- [14] Parashar P, Rana S. Entanglement and Discord of the Superposition of Greenberger-Horne-Zeilinger States. *Phys. Rev. A*. 2011; 83(3): 032301.
- [15] Zhang DH, Zhou DL, Fan H. Entanglement of Superpositions of Orthogonal Maximally Entangled States. *Chin. Phys. Lett.* 2010; 27(9): 090306.
- [16] Linden N, Popescu S, Smolin JA. Entanglement of Superpositions. *Phys. Rev. Lett.* 2006; 97(10): 100502.
- [17] Yu CS, Yi XX, Song HS. Concurrence of Superpositions. *Phys. Rev. A*. 2007; 75(2): 022332.
- [18] Ou YC, Fan H. Bounds on negativity of superpositions. *Phys. Rev. A*. 2007; 76(2): 022320.
- [19] Song W, Liu NL, Chen ZB. Bounds on the Multipartite Entanglement of Superpositions. *Phys. Rev. A*. 2007; 76(5): 054303.
- [20] Gour G, Roy A. Entanglement of Subspaces in Terms of Entanglement of Superpositions. *Phys. Rev. A*. 2008; 77(1): 012336.
- [21] Akhtarshenas SJ. Concurrence of superpositions of many states. *Phys. Rev. A*. 2011; 83(4): 042306.
- [22] Yang CP, Chu SI, Han SY. Efficient Many-Party Controlled Teleportation of Multiqubit Quantum Information via Entanglement. *Phys. Rev. A*. 2004; 70(2): 022329.
- [23] Paulus I Santosa. Cost and Benefit of Information Search Using Two Different Strategies. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2010; 8(3): 195-206.
- [24] Sarwosri, Darlis H, et al. Klasifikasi Secara Efisien pada Database Multi Relasi dengan Algoritma Crossmine. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2008; 6(1): 7-14.
- [25] Wong A, Christensen N. Potential multiparticle entanglement measure. *Phys. Rev. A*. 2001; 63(4): 044301.
- [26] Wootters WK. Entanglement of Formation of an Arbitrary State of Two Qubits. *Phys. Rev. Lett.* 1998; 80(10): 2245.