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Numerical Simulation of 1-D and 2-D Continuous Geo-electrical Magnetotelluric Forward

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Abstract

Finite element technology in one-dimensional layered media of the continuous electro-media and the two-dimensional continuous electrical is displayed in ways of the special matrix equations in the variation formula and split technical conditions. To achieve a one-dimensional layered continuous electrical finite element numerical simulation of the media, the introduction of the secondary triangulation rectangular subdivision has been used to the two-dimensional numerical simulation, and using the rectangular center to set up the virtual point method to reduce the computational workload in calculation. Then design a meaningful manner electric model, the base power model analysis of electromagnetic waves in the rock, they all provide the guidance for the actual production.

Keywords: *continuous electrical media, MT, quadratic subdivision, virtual point, theoretical model*

1. Introduction

Magnetotelluric sounding is a common form of frequency domain magnetotelluric sounding method; the basic principle is based on the electrical differences of the media to speculate through the electromagnetic response of the electromagnetic waves of different formations. One-dimensional MT forward to analytical solution for solving layered media, most of its domestic scholars are against uniform layered media and study its algorithm only need a simple iterative loop can be obtained analytical solution; anisotropic layered media [1], Li Yuguo in Germany in the 90s of the last century, made relevant discussions and studies related forward calculation, which can only be obtained numerical calculation under the conditions of the numerical solution. One-dimensional numerical simulation of a layered electrical continuous medium forward; assumed physical parameters is divided into blocks of uniform, to improve the accuracy of the calculation, made an effort to more accurately magnetotelluric inversion, the first regional rectangle split, then the use of the triangular unit the split rectangular unit, unit of electromagnetic fields and parameters, interpolation, continuous change makes a small unit block.

Deepened on study the forward technology in magnetotelluric numerical simulation of magnetotelluric sounding has been became one of the hottest issues for many years. The good inversion result based on a more accurate forward model, which is a facing task for the geophysical workers to calculate, the current inversion based on accurate forward model and the geological model in the actual technology increasingly, how to design the initial model to appropriate the forward is still our geophysical work, which is one of the difficulties in geophysics. In recent years many scholars proposed the concept of continuous electrical media and parameter design method introduces the field of numerical simulation of the electromagnetic method, a variety of numerical techniques continue to emerge. In the last part of this paper, the main conclusions can be pointed out and the deficiency and some things which are needed to be perfected in the future. Proposals were also given by the author.

2. Research Method

2.1. 1-D Response Formulization

Assumptions layered media, we tend to use the physicist who Tikhonov and Cagniard magnetotelluric model, and the basic principle is: suppose the natural source magnetotelluric is

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excited by a plane electromagnetic wave incident perpendicular to the surface, through metallic materials spread down through different media, detection effect will be different, analyze and summarize the detection results under a variety of conditions, summed up the law, to accumulate experience for the metal detector. Metal body detection material, it is assumed that the layered case, the electromagnetic field satisfies the third boundary condition, can be constructed in electromagnetic waves spread among its following function:

$$I(u)=\int_0^{\bar{z}}\left[\left(\frac{du}{dz}\right)^2 - i\text{µ«±}\,u^2\right]dz \quad (1)$$

It's left and right sides take the differential at the same time, and the variable is divided into:

$$\text{¢}I(u)=\int_0^{\bar{z}}\left(2\frac{du}{dz}\frac{d\text{¢}u}{dz}-2i\text{µ«±}\,u\text{¢}u\right)dz$$
$$=2\int_0^{\bar{z}}\left(\frac{d}{dz}\left(\frac{du}{dz}\text{¢}u\right)-\left(\frac{d^2u}{dz^2}+i\text{µ«±}\,u\right)\text{¢}u\right)dz \quad (2)$$

Final electrical continuous layered media, one-dimensional boundary value problem is equivalent to the following vibrational problem[2,10]:

$$\left.\begin{aligned}&F(u)=I(u)+au^2|_{\bar{z}}=\int_0^{\bar{z}}\left[\left(\frac{du}{dz}\right)^2-i\text{µ«±}\,u^2\right]dz+au_{\bar{z}}^2\\&\text{¢}F(u)=0\end{aligned}\right\} \quad 3$$

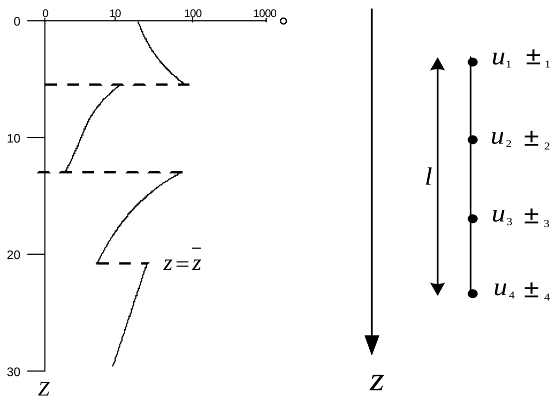

Figure 1. Layered Schematic Diagram of Continuous Electrical Medium and Single Element Nodes

2.2. 2-D Response formulization

The object of two-dimensional magnetotelluric electrical continuous medium forward is along the spatial distribution of electric medium. Since apparant resistivity is determined by conductivity, temperature, humidity, compaction degree, and so on, electromagnetic angular frequency, electric medium permittivity, conductivity can be linear or nonlinear variation. Previous numerical simulation, regard as unchangable, which reduce the simulation degree of actual situation, and along whcih the result is: inversion results are unsatisfactory as the forward model of invertion is not accurate enough. Form the forward model, it decrease the requirement of accuracy [4]. His article will base on the typical parameters u ,^{2} , ^{a} , and k with the finite element method, from a wider perspective on the problem and the precision analysis of forward, which integrates theory and practice. Ignoring the electromagnetic fields on displacement current role in rocks and ores, and basing on the TE and TM mode of Maxwell equations, the earth electromagnetic field variational problem can be summarized as follows according to Wannamaker P E[3,7]:

$$\left.\begin{aligned} &F(u)=\int_{\text{t}}[\frac{1}{2}{}^{2}(\nabla\cdot u)^{2}-\frac{1}{2}{}^{\text{a}}u^{2}]d\,\text{t}\;+\int_{CD}\frac{1}{2}{}^{2}ku^{2}d\,\text{dl}\\ &u\big|_{AB}=1,\\ &\delta F(u)=0.\end{aligned}\right\}\qquad(4)$$

In which Ω is study area (Figure 1, TE model), ∇ is two-dimensional Hamilton operator, and $\nabla=\frac{\partial}{\partial x}\mathrm{e}_x+\frac{\partial}{\partial y}\mathrm{e}_y$.

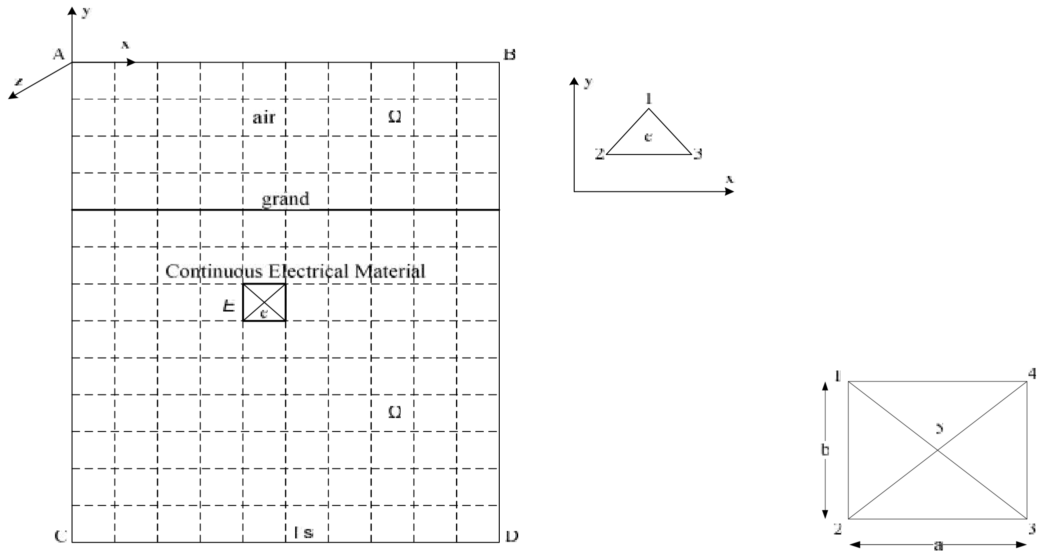

Figure 2 Diagram of Two Dimensional Continuum TE Model and Rectangular-Two Triangulation Finite Element Meshing

For the E wave: $u=E_z$, $^{2}=\frac{1}{i\omega\mu}$, $^{\text{a}}=\pm\,\text{J}\;i\omega£$, $k=\sqrt{\text{J}\,i\omega«\pm}$;

For the H wave: $u=H_z$ $\;^{2}=\frac{1}{\pm\,\text{J}\;i\omega£}$ $\;^{\text{a}}=i\omega\mu$ $\;k=\sqrt{\text{J}\,i\omega«\pm}$.

And E_z and H_z are the component of direction (Z axis) of electric field E and magnetic H field, respectively, ω is the angular frequency of electromagnetic wave, £ is dielectric constant of the medium, « is magnetic permeability, finally ± is electrical conductivity.

2.3. FEM Method

For one dimension model, the steps as follows [5]:

The first step: the entire regional area partitioning, area partitioning for solving a lot of units, each of which unit is split, and the split principle must be continuous conductivity in a typical mutation with segment continuous defined boundary, in other words, in the conductivity of the discontinuous point cannot be interpolated.

The second step: unit interpolation learned from the first chapter discusses the natural source attenuation soon, so the unit must be made smaller to teach dense, using cubic interpolation to ensure the accuracy requirements.

The third step: the unit for analysis, and Figure 2 shows a unit, the nodes at both ends is 1, 4, the intermediate node 2, 3 are equidistant. Using u_1, u_2, u_3, u_4 and $\pm_1$, $\pm_1$, $\pm_3$, $\pm_4$ field value and electrical conductivity of these nodes, in accordance with the cubic interpolation function unit, separately, these four points on the field value and conductivity function can be expressed as follows:

$$\left.\begin{aligned} \pm &= N_1\pm_1 + N_2\pm_2 + N_3\pm_3 + N_4\pm_4 = \mathbf{N}^{\mathbf{T}}\boldsymbol{\sigma} \\ u &= N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_4 = \mathbf{N}^{\mathbf{T}}\mathbf{u} \end{aligned}\right\} \quad (4)$$

The fourth step: overall each unit in synthesis. K_{1e} and K_{2e} are 4×4 matrix, firstly expanded its into $n\times n$ matrix, in Which n is the total number of nodes of the Whole. Stroke matrix equation as follows:

$$\mathbf{Ku} = \mathbf{0} \quad (5)$$

The fifth step: the boundary conditions on behalf into the 5-style, its linear equations. 5 where the matrix is a large sparse symmetric matrix, under normal circumstances, the use of the catch-up method to solve this with the typical characteristics of linear algebraic equations to solve becomes convenient, solving equations, obtained $\frac{du}{dz}$ Substituting into the following formula

$$°_a = -i\mu« \left[u \Big/ \left(\frac{du}{dz} \right) \right] \quad (6)$$

Which can be calculated apparent resistivity, and then to understand the effect of the metal surface below detection.

For two dimension model, the steps as follows:

Firstly using rectangular element E to dissect the Ω area, then quadrilateral element is divided into four triangular elements e (Figure 2). As shown in Figure2, the interface is divided into rectangular generally, secondly through the rectangular, according to its center point or a suitable locations (based on actual circumstances), for the two division, which is a method to solve the topographic fluctuation by assigning a boundary point of the region after the secondary split. Increasing the virtual point embraces the advantage of expanding adaptability without increasing the final compute nodes of the finite element meshing. Here we introduce the idea of magnetotelluric method [8, 9]. The right diagram of figure 1 is the rectangular unit secondary subdivision for triangular unit schematic diagram. As is shown, 1, 2, 3, and 4 are the four corners of the first rectangle rectangular subdivision. We can name the center point 5 (optionally), and then the conductivity and other parameters can be assigned according to the actual situation of these five points. Even though increasing the point 5, obviously, this approach has a wide range of practical adaptability resulting from the interpolation calculation of the interpolation function in the new unit. As point 5 is a visual point, we can finish the procedure in the element analysis of erasing the value of point 5 which is not included in the total points.

Secondly, We use bilinear interpolation in the electromagnetic field u and parameters ², ª, k in the unit e, i.e.

$$\mathbf{u} = \sum_{i=1}^{3} n_i u_i \quad \boldsymbol{\tau} = \sum_{i=1}^{3} n_i{}^{2}{}_i \quad \boldsymbol{\lambda} = \sum_{i=1}^{3} n_i{}^{a}{}_i \quad \mathbf{k} = \sum_{i=1}^{3} n_i k_i$$

in which $n_i = (a_i x + b_i y + c_i)/2\Delta$ is the linear function of x and y, and $\Delta = a_1 b_2 - a_2 b_1$ is the area of the triangle. $a_1 = y_2 - y_3$, $a_2 = y_3 - y_1$, $a_3 = y_1 - y_2$, $b_1 = x_3 - x_2$, $b_2 = x_1 - x_3$, $b_3 = x_2 - x_1$, $c_3 = x_2 y_3 - x_3 y_2$, (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the triangular element node coordinates. The first element integral is:

$$\int_e \frac{1}{2} 2 (\nabla u)^2 d\, \mathrm{t} = \iint \frac{1}{2} 2 [(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2] dxdy = \frac{1}{2} \boldsymbol{u}_e^T \boldsymbol{k}_{1e} \boldsymbol{u}_e \quad (7)$$

Where $\boldsymbol{k}_{1e} = (k_{1ij})$, $k_{1ij} = k_{1ji}$ $\boldsymbol{u}_e = [u_i]^T$ $i, j = 1,2,3$

$$k_{1ij} = \iint \sum_{l=1}^{3} n_l 2_l (\frac{\partial n_i}{\partial x}\frac{\partial n_j}{\partial x} + \frac{\partial n_i}{\partial y}\frac{\partial n_j}{\partial y}) dxdy = \frac{2_1 + 2_2 + 2_3}{12\Delta}(a_i a_j + b_i b_j)$$

The second element integral of (8)

$$\int_e \frac{1}{2} \mathrm{a} u^2 d\, \mathrm{t} = \iint \frac{1}{2} \mathrm{a} u^2 dxdy = \frac{1}{2} \boldsymbol{u}_e^T \boldsymbol{k}_{2e} \boldsymbol{u}_e \quad (8)$$

where $\boldsymbol{k}_{2e} = (k_{2ij})$ $k_{2ij} = k_{2ji} = \frac{\Delta}{60} ¦_{ij} \mathrm{a}$ $\mathrm{a} = (\mathrm{a}_l)^T$ $\boldsymbol{u}_e = [u_i]^T$ $i, j, l = 1,2,3$, then the matrix-vector is shown as follows:

$$\begin{bmatrix} ¦_{11} \\ ¦_{21} \\ ¦_{22} \\ ¦_{31} \\ ¦_{32} \\ ¦_{33} \end{bmatrix} = \begin{bmatrix} 6 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 6 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

Assuming integral boundary $\overline{12}$ is on the bottom boundary, and then the boundary integral is

$$\int_{\mathrm{d}_e} \frac{1}{2} 2 k u^2 d\mathrm{d} = \int_{\overline{12}} \frac{1}{2} 2 k u^2 dl = \frac{1}{2} \boldsymbol{u}_e^T \boldsymbol{k}_{3e} \boldsymbol{u}_e \quad (9)$$

where $\boldsymbol{k}_{3e} = (k_{3ij})$ $k_{3ij} = k_{3ji} = \frac{l}{60} \cdot ¢_{ij}$ $\boldsymbol{u}_e = [u_i]^T$ $i, j = 1,2$,

$¢_{11} = 12 2_1 k_1 + 3 2_1 k_2 + 3 2_2 k_1 + 2 2_2 k_2$,

$¢_{21} = 3 2_1 k_1 + 2 2_1 k_2 + 2 2_2 k_1 + 3 2_2 k_2$, $¢_{22} = 2 2_1 k_1 + 3 2_1 k_2 + 3 2_2 k_1 + 12 2_2 k_2$, After the completion of the four triangular elements integral rectangular unit, eliminate intermediate nodes then expand into the matrix and array composed by all the nodes, and finally the spreading matrix of each rectangular unit added:

$$F(u) = \sum_e F_e(u) = \sum_E F_E(u) = \frac{1}{2} \mathbf{u}^{\mathbf{T}} \mathbf{K} \mathbf{u} \quad (10)$$

If the variation of 10 is zero, then the boundary value of the line AB substituted into the equation $\mathbf{Ku} = 0$, and try to solve the equation, finally we can get $\mathbf{u}$ of each node, which represents $\mathbf{H_z}$ or $\mathbf{E_z}$ of each node. The previous scholars tend to calculate the formula, and the calculated apparent resistivity bases on Cagniard apparent resistivity [6].

3. Results and Analysis

Forward with layered media layers and layers of apparent resistivity, which they can be divided into several types using to study its response with different relationships[11]. The following examples are three and five layer model to validate the results.

3.1. Three Layer Model

In the three layer model, the five electrical parameters are the same as the two layer model. As the receptivity's of the three layers are different, the electromagnetic results are also different. Based on the differences of the three layers, we can get 4 kinds of curves (H, K, A and Q) from $N = 2^{n-1}$, which can be defined as follows:

H: $°_1 > °_2 < °_3$, K: $°_1 < °_2 > °_3$, A: $°_1 < °_2 < °_3$, Q: $°_1 > °_2 > °_3$.

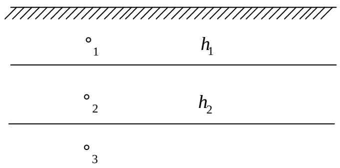

Figure 3. Diagram of Three Layer Model

K $°_1 < °_2 > °_3$, element nodes of the first layer may define as $° = [10,1000]\Omega \cdot m$, then the second $° = [1000,100]\Omega \cdot m$, and the last is $° = 100\Omega \cdot m$. We also can set $h = [0,100]m$, $h = [100,1000]m$, and the results are shown below:

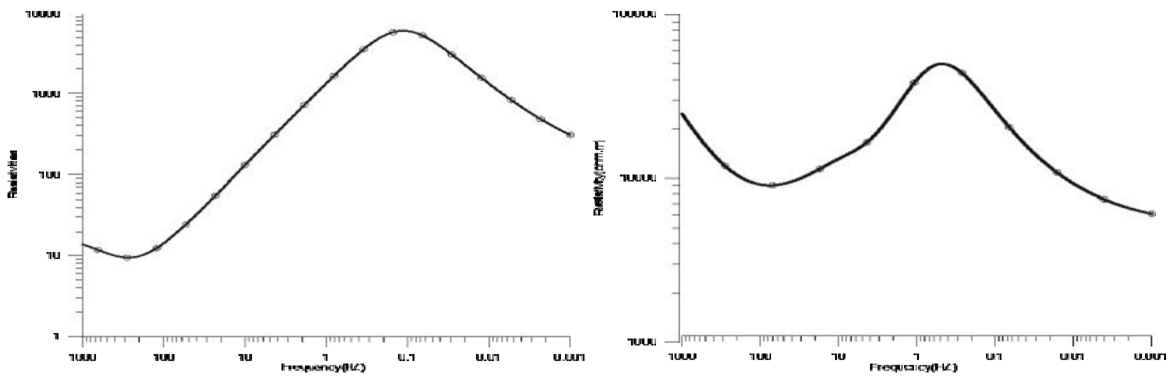

Figure 4. Forward Apparent Resistivity Curve of K Type

H $°_1 > °_2 < °_3$ element nodes of the first layer may define as $° = [10000, 20]\Omega \cdot m$, then the second $° = [20, 5000]\Omega \cdot m$,and the last is $° = 5000\Omega \cdot m$, We also can set $h = [0,100]m$, $h = [100,10000]m$, and the results can be shown in Figure 3.

3.2. Five Layer Model

From $N = 2^{n-1}$ we know there are 15 results if $n = 5$. And here we discuss one of the situations: $°_1 = [°_{top}, °_{below}] = [10,5]$, $°_2 = [°_{top}, °_{below}] = [10,100]$ $°_3 = [°_{top}, °_{below}] = [500,10]$ $°_4 = [°_{top}, °_{below}] = [20,1000]$, $°_5 = [°_{top}, °_{below}] = [1000,1000]$,
$[h_1, h_2, h_3, h_4, h_5] = [1000, 2000, 7000, 90000, +\infty]$

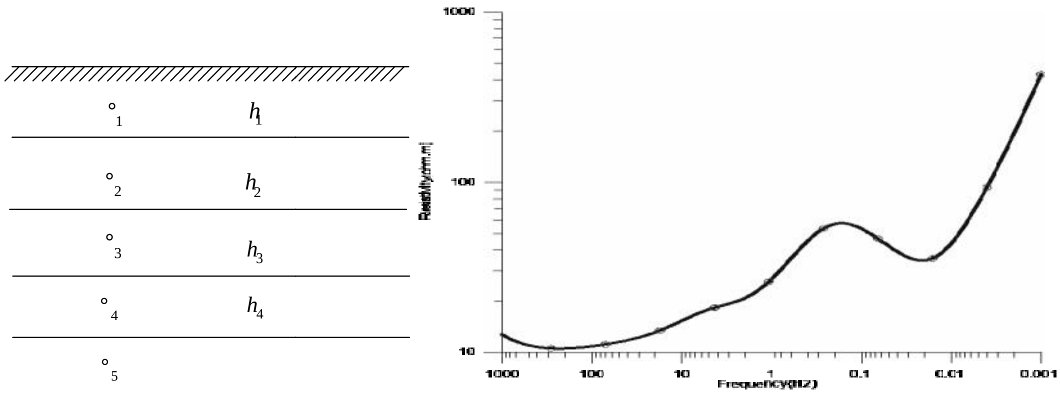

Figure 5. Five Layer Model and Forward Apparent Resistivity Curve Detection Result

3.3. 2-D Model

As shown in right of Figure 6, in the surrounding rock formation of $100\Omega\cdot m$ which contains anomaly body with $10\Omega\cdot m$, the result of the apparent resistivity pseudo section shown in Figure 6. The top of the depth of the designed model is 600m and the abnormal morphology is rectangular second degree with a width of 300m and length of 400m.Results can be concluded according to the model in the right of the figure6: low resistivity anomaly can be well reflected in the apparent resistivity section diagram in TE model.

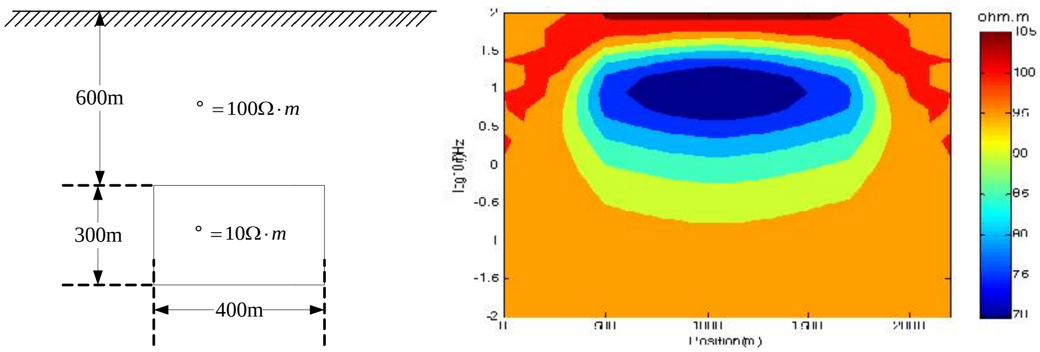

Figure 6. Two-dimensional Resistivity Model and Numerical Simulation Detection Results

4. Conclusion

Based upon the previous work, this article uses variation method to deduce a dimensional continuum of corresponding functional problems, making use of Finite Element Method, to get the stiffness matrix by linear interpolation, then uses MATLAB to establish the corresponding procedure, and has carried on the simulation to typical models. The result has confirmed the procedure accurate.The main conclusions are as follows:

Frist, Founded on the results of numerical simulation in the lower frequency and higher frequency of the numerical solution is relatively large in error, the reason may be concluded on the mesh or algorithm design that is not perfect;

Second, although numerical simulation of continuous medium has spent more computing time, but the calculation accuracy improved to value the better results.

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