

Entropy-based Evaluation Method of Design Scheme for Helicopter Transmission

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Abstract

To choose the best design scheme of helicopter transmission, and bring out excellent comprehensive performance for helicopter, technical criteria architecture of helicopter transmission is analyzed. Scheme criteria evaluation matrix is standardized and criteria weight vector of every design scheme is optimized. According to the principle of minimum deviation of design scheme evaluation, single scheme evaluation problem is programmed. Uncertainty factors of criteria weight are analyzed, and entropy is introduced to describe these uncertainties. In addition to the maximum entropy theory, an evaluation model of design schemes is formed, which is solved by constructing a Lagrange function and a nonlinear system of equations. The example demonstrates validity of the proposed evaluation method.

Keywords: entropy, design scheme, evaluation method, technical criteria

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1. Introduction

Transmission is one of three key power components of helicopter, which transmits power and speed of the engine to the rotor, tail rotor and attachments in a certain proportion [1]. Transmission property directly influences the helicopter's performance and reliability. According to the requirements from the engine and the helicopter, various transmission schemes can be designed. Scheme evaluation sometimes is described by many technical criteria. How to evaluate synthetically numerous technical criteria and obtain the best transmission design scheme is the critical step to ensure whole excellent performance of helicopter.

There are many common evaluation methods in engineering domain, such as Value Engineering (VE), Fuzzy Evaluation, Gray Relation, Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Entropy [2], and so on. LIN presented an identification method of customer requirement and product design criteria and completed comprehensive evaluation of production design scheme [3]. AYAG introduced fuzzy theory into ANP method to evaluate product design scheme, and the results showed ANP method was more adaptive than AHP method [4]. COVINDALURI put forward a robust design model based on a multi-criteria decision framework [5]. LIN put forward a hybrid model based on advanced DEMATEL-VIKOR algorithm and an evaluation method for product conceptual design scheme [6]. DENG constructed a generic quality evaluation system of conceptual design and used entropy to describe uncertainty factors of quality evaluation criteria weight. Quality evaluation questions were optimized on the basis of maximum entropy principle [7]. LEVNER used the fuzzy Borda method and semantic grades to classify multi-attribute text [8]. KUZGUNKAYA constructed a common structural complexity matrix based on entropy to evaluate manufacturing system scheme [9].

This paper takes technical criteria characters of design scheme for helicopter transmission as groundwork, and introduces entropy to describe uncertainty of criteria weight. The principles of minimum deviation of design scheme evaluation and the maximum entropy theory are applied to the optimization solution of scheme evaluation.

2. Technical Criteria Architecture of Helicopter Transmission

2.1. Analysis of Technical Criteria Architecture of Helicopter Transmission

The technical criteria architecture of helicopter transmission has typical multi-disciplinary coupling characteristics, which has highly systematic and complicated. So it is very difficult to construct the criteria architecture. To build up the technical criteria architecture of helicopter transmission, the principles of science, generalness, conciseness, dynamic and independent must be followed. The purpose of constructing the criteria architecture is applying it to argumentation and scheme evaluation of helicopter transmission. Technical criteria are usually obtained from common statistical information and about research work. Some key criteria which are hard to collect and measure can be instead by quantitative criteria dependent on variables relation.

According to these construction principles for criteria architecture and relations among with technical criteria, the technical criteria architecture of helicopter transmission is formed as Figure 1.

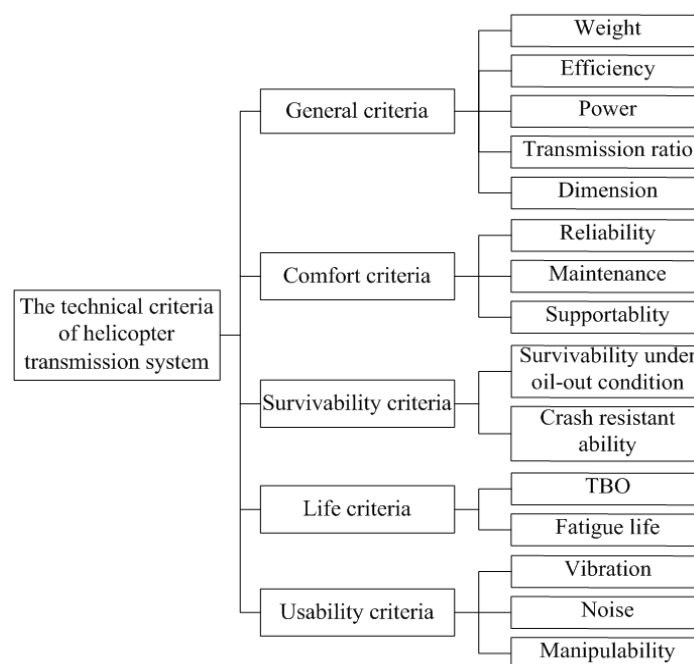


Figure 1. The Technical Criteria Architecture of Helicopter Transmission

2.2. Denotation Definition of Technical Criteria Architecture

We assume that the count of the technical criteria to be evaluated is m , which is expressed as $X = [x_1, x_2, \dots, x_m]$. The count of design schemes to be evaluated is n , which is expressed as $P = [P_1, P_2, \dots, P_n]$. The j number of criterion value corresponding to the scheme is expressed as a_{ij} ($i = 1, 2, \dots, n; j = 1, 2, \dots, m$). So all the technical criteria value of transmission design schemes can be expressed as $A = [a_{ij}]_{n \times m}$, namely:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}$$

We name matrix A as technical criteria evaluation matrix of transmission design schemes collection.

3. Linear Programming for Design Scheme Evaluation

3.1. Standardization of Evaluation Matrix

Meanings and dimensions of every criterion can't be compared with each other because of different criterion properties. To avoid influence to scheme evaluation, criteria value should be transformed, that is transforming different dimensional criterion into non-dimensional standard criterion. Evaluation matrix A will be standardized as $R = [r_{ij}]_{n \times m}$. Because all technical criteria involved this paper are quantitative criteria, so they are just normalized by max and min method. The j number of criteria set of design schemes collection P is $X = [x_1, x_2, \dots, x_n]$. Some technical criteria are better when their values bigger, such as *efficiency*, *durability*, *survivability under oil-out condition*, and so on. The normalization formula for these criteria is expressed as:

$$r_{ij} = \frac{x_i - \min x_i}{\max x_i - \min x_i} \quad (i = 1, 2, \dots, n; j = 1, 2, \dots, m) \quad (1)$$

Other criteria are better when their values smaller, for example *weight*. The normalization formula for these criteria is expressed as:

$$r_{ij} = \frac{\max x_i - x_i}{\max x_i - \min x_i} \quad (i = 1, 2, \dots, n; j = 1, 2, \dots, m) \quad (2)$$

After normalizing treatment, we obtain $0 \leq r_{ij} \leq 1$. So we transform matrix A into standardization matrix R .

3.2. Criteria Weight Vector Optimization of Single Design Scheme

We assume that the weight vector of criteria set is known as $\omega = [\omega_1, \omega_2, \dots, \omega_m]$. The variation scope of criteria weight is $\alpha_j \leq \omega_j \leq \beta_j$ ($\alpha_j \geq 0, \beta_j \geq 0$), and weight coefficient satisfies the condition: $\sum_{j=1}^m \omega_j = 1$.

Comprehensive evaluation value of scheme P_i can be described as the following expressions:

$$v_i = \sum_{j=1}^m \omega_j r_{ij} \quad (i = 1, 2, \dots, n) \quad (3)$$

The essentiality of multi criteria evaluation for design schemes is sorting comprehensive evaluation values of all design schemes. When the value of v_i is bigger, the corresponding design scheme is more excellent. Therefore firstly the criteria weight of every design scheme possessing maximal comprehensive evaluation value should be confirmed, that is to optimize criteria weight vector of every design scheme. For design scheme P_i , we design a single-objective optimization model as:

$$\begin{aligned} & \text{maximize} \quad \sum_{j=1}^m \omega_j r_{ij} \quad (i = 1, 2, \dots, n) \\ & \text{subject to} \quad \alpha_j \leq \omega_j \leq \beta_j; \quad \sum_{j=1}^m \omega_j = 1 \quad (j = 1, 2, \dots, m) \end{aligned} \quad (4)$$

Through solving this optimization model, we can obtain the optimal criteria weight vector of scheme P_i : $\omega^{(i)} = [\omega_1^{(i)}, \omega_2^{(i)}, \dots, \omega_m^{(i)}]$.

3.3. Linear Programming of Design Scheme

The real technical criterion weight of transmission design scheme belongs to a random variable, which can be described as the sum of the average and a random error. The n groups of weights generated from Equation (4) can be regarded as a random sample. The deviation value of comprehensive evaluation of scheme P_i is written as:

$$v_i = \sum_{j=1}^m [(\omega_j - \omega_j^{(i)})r_{ij}]^2 \quad (i = 1, 2, \dots, n) \quad (5)$$

According to the minimum deviation principle, we express a multi-objective linear programming model as:

$$\left. \begin{array}{l} \text{minimize } \{v_i\} \quad (i = 1, 2, \dots, n) \\ \text{subject to } \sum_{j=1}^m \omega_j = 1; \quad \omega_j \geq 0 \quad (j = 1, 2, \dots, m) \end{array} \right\} \quad (6)$$

The multi-objective programming model can be translated into a single-objective programming model as:

$$\left. \begin{array}{l} \text{minimize } \sum_{i=1}^n \sum_{j=1}^m [(\omega_j - \omega_j^{(i)})r_{ij}]^2 \\ \text{subject to } \sum_{j=1}^m \omega_j = 1; \quad \omega_j \geq 0 \quad (j = 1, 2, \dots, m) \end{array} \right\} \quad (7)$$

3.4. Uncertainty of Criteria Weight

Design scheme evaluation weight ω_j is mainly determined by the correlative weightiness between helicopter performance and technical criteria, and the relation among technical criteria.

1) There is existing compact relationship between helicopter performance and technical criteria. Performance requirement of different purpose helicopter will mainly influence weightiness of technical criteria. Helicopter performance requirement reflect probability level of criteria weightiness. Varying helicopter performance leads to weightiness of technical criteria having uncertainty.

2) Complex relationship always exists among technical criteria of transmission. When one technical criterion value changes, it will lead to another technical criterion value change. We name this relationship as criteria relativity. According to influence direction of criteria value with each other, relativity can be classified as positive relativity and negative relativity. Positive relativity denotes same influence direction; however negative relativity denotes inverse influence direction. Criteria relativity is usually not changeless value, which has great uncertainty.

The uncertainty of weightiness of helicopter performance with technical criteria and the uncertainty of criteria relativity induce the uncertainty of criteria weight ω_j .

3.5. Criteria Weight Entropy

Entropy is always used for describing information uncertainty. We introduce entropy to express the uncertainty of criteria weight as:

$$H_\omega = - \sum_{j=1}^m \omega_j \ln \omega_j \quad (8)$$

To eliminate the uncertainty of criteria weight, based on maximum entropy principle, we have:

$$\left. \begin{array}{l} \text{maximize } H_{\omega} = -\sum_{j=1}^m \omega_j \ln \omega_j \\ \text{subject to } \sum_{j=1}^m \omega_j = 1; \quad \omega_j \geq 0 \quad (j = 1, 2, \dots, m) \end{array} \right\} \quad (9)$$

4. Evaluation and Optimization of Design Scheme

To achieve minimum deviation value of comprehensive evaluation and minimum uncertainty of criteria weight at the same time, we thus have an optimization model after combining Equation (7) with Equation (9):

$$\left. \begin{array}{l} \text{minimize } \sum_{i=1}^n \sum_{j=1}^m [(\omega_j - \omega_j^{(i)}) r_{ij}]^2 \\ \text{maximize } H_{\omega} = -\sum_{j=1}^m \omega_j \ln \omega_j \\ \text{subject to } \sum_{j=1}^m \omega_j = 1; \quad \omega_j \geq 0 \quad (j = 1, 2, \dots, m) \end{array} \right\} \quad (10)$$

We translate this linear programming issue into a multi-objective optimization issue, which can be expressed as:

$$\left. \begin{array}{l} \text{minimize } \varepsilon \sum_{i=1}^n \sum_{j=1}^m [(\omega_j - \omega_j^{(i)}) r_{ij}]^2 + (1 - \varepsilon) \sum_{j=1}^m \omega_j \ln \omega_j \\ \text{subject to } \sum_{j=1}^m \omega_j = 1; \quad \omega_j \geq 0 \quad (j = 1, 2, \dots, m) \end{array} \right\} \quad (11)$$

Where ε is a balance coefficient, whose value is depended on particular condition. To solve this optimization model, we construct a Lagrange function as:

$$L(\omega_j, k) = \varepsilon \sum_{i=1}^n \sum_{j=1}^m [(\omega_j - \omega_j^{(i)}) r_{ij}]^2 + (1 - \varepsilon) \sum_{j=1}^m \omega_j \ln \omega_j - k \left(\sum_{j=1}^m \omega_j - 1 \right) \quad (12)$$

According to the necessary condition that Equation (12) has extremums, we can derive:

$$\frac{\partial L}{\partial \omega_j} = 2\varepsilon \sum_{i=1}^n (\omega_j - \omega_j^{(i)}) r_{ij}^2 + (1 - \varepsilon) (\ln \omega_j + 1) - k = 0 \quad (13)$$

$$\frac{\partial L}{\partial k} = 1 - \sum_{j=1}^m \omega_j = 0 \quad (14)$$

For simplifying equation expression, we assume:

$$s_j = \sum_{i=1}^n r_{ij}^2, \quad t_j = \sum_{i=1}^n \omega_j^{(i)} r_{ij}^2 \quad (15)$$

Combining Equation (13) with Equation (14) we can derive a nonlinear system of equations as:

$$\begin{cases} 2\varepsilon_j \omega_j + (1 - \varepsilon) \ln \omega_j - k + 1 - 2\varepsilon t_j - \varepsilon = 0 \\ \sum_{j=1}^m \omega_j = 1 \end{cases} \quad (j = 1, 2, \dots, m) \quad (16)$$

The count of the equations or the variables in the system of equations is $m+1$. All the variables are $\omega_j (j=1, 2, \dots, m)$ and k . The equations set can be solved through numerical solving methods, such as Newton iteration method. We then can derive the optimal criteria weight of the design schemes. Put the optimal criteria weight into Equation (3), we can obtain every comprehensive evaluation value of design schemes. After sorting all evaluation values, the design scheme corresponding to the maximal evaluation value is the optimal one [10-11].

5. Example Research

We take same type helicopter transmission as example. In this paper five types technical criteria are used to evaluated, which are *weight*, *efficiency*, *MTBF*, *MTBR* and *survivability under oil-out condition*. There are four design schemes, whose technical criterion values are shown in Table 1.

Table 1. Technical Criterion Values of Helicopter Transmission Design Schemes

Design scheme	Weight (kg)	Efficiency	MTBF (hour)	MTBR (hour)	Survivability under oil-out condition (min)
P_1	575	0.988	4500	2000	35
P_2	530	0.966	4320	1880	29
P_3	620	0.985	4550	1990	34
P_4	590	0.979	4350	1980	32

Criteria values of efficiency, *MTBF*, *MTBR* and *survivability under oil-out condition* are handled by Equation (1), however criterion value of *weight* is handled by Equation (2). Matrix A composed of data in Table 1 thus transmits standardization matrix R as:

$$R = \begin{vmatrix} 0.5 & 1 & 0.783 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.864 & 1 & 0.917 & 0.833 \\ 0.333 & 0.591 & 0.13 & 0.833 & 0.5 \end{vmatrix}$$

Assuming the bounds of every criteria weight are: $0.1 \leq \omega_1 \leq 0.5$, $0.1 \leq \omega_2 \leq 0.35$, $0.1 \leq \omega_3 \leq 0.3$, $0.1 \leq \omega_4 \leq 0.25$, $0.1 \leq \omega_5 \leq 0.2$.

From Equation (4) we can derive the optimal criteria weight vector of every design scheme as:

$$\begin{aligned} \omega^{(1)} &= [\omega_1^{(1)}, \omega_2^{(1)}, \omega_3^{(1)}, \omega_4^{(1)}, \omega_5^{(1)}] = (0.1, 0.35, 0.1, 0.25, 0.2), \\ \omega^{(2)} &= [\omega_1^{(2)}, \omega_2^{(2)}, \omega_3^{(2)}, \omega_4^{(2)}, \omega_5^{(2)}] = (0.5, 0.2, 0.1, 0.1, 0.1), \\ \omega^{(3)} &= [\omega_1^{(3)}, \omega_2^{(3)}, \omega_3^{(3)}, \omega_4^{(3)}, \omega_5^{(3)}] = (0.1, 0.25, 0.3, 0.25, 0.1), \\ \omega^{(4)} &= [\omega_1^{(4)}, \omega_2^{(4)}, \omega_3^{(4)}, \omega_4^{(4)}, \omega_5^{(4)}] = (0.1, 0.35, 0.1, 0.25, 0.2). \end{aligned}$$

From Equation (15) we can obtain:

$$\begin{aligned} s_1 &= 1.361, \quad s_2 = 2.095, \quad s_3 = 1.630, \quad s_4 = 2.535, \quad s_5 = 1.944, \\ t_1 &= 0.535, \quad t_2 = 0.659, \quad t_3 = 0.378, \quad t_4 = 0.633, \quad t_5 = 0.319. \end{aligned}$$

We assume $\varepsilon = 0.5$ and put these results into Equation (16). Using solving program of nonlinear system of equations developed under Matlab software environment, we can derive the optimal criteria weight vector $\omega = [0.228, 0.22, 0.186, 0.2, 0.166]$. From Equation (3) comprehensive evaluation values of design schemes can be obtained as $\nu = [0.846, 0.228, 0.697, 0.48]$. We can sort the four design schemes according to these data as $P_1 > P_3 > P_4 > P_2$. Finally we get the result that scheme 1 is the optimal design scheme.

6. Conclusion

This paper presented the technical criteria architecture of helicopter transmission. The criteria evaluation matrix was standardized by max and min method. Criteria weight vector of every design scheme was programmed. We put forward describing the uncertainty of criteria weight with entropy. Taking minimum deviation value of comprehensive evaluation and maximum uncertainty of criteria weight as objective, we constructed the optimization model of design scheme evaluation. Adopting Lagrange function we solved the optimization model, and then obtained the optimal criteria weight vector. Finally we computed every design scheme comprehensive evaluation value, and chose the best design scheme.

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References

- [1] Andrew TB. Drive System Design Methodology for a Single Main Rotor Heavy Lift Helicopter. *Georgia Institute of Technology*. 2005.
- [2] Shannon CE. A Mathematical Theory of Communication. *Bell System Technical Journal*. 1984; 27: 379-423.
- [3] Lin MC, Wang CC, Chen MS, et al. Using AHP and TOPSIS Approaches in Customer-driven Product Design Process. *Computers in Industry*. 2008; 59(1): 17-31.
- [4] Ayag Z, Ozdemir RG. A Hybrid Approach to Concept Selection through Fuzzy Analytic Network Process. *Computers & Industrial Engineering*. 2009; 56(1):368-379.
- [5] Covindaluri SM, Cho BR. Robust Design Modeling with Correlated Quality Characteristics Using a Multicriteria Decision Framework. *International Journal of Advanced Manufacturing Technology*. 2007; 32(5): 423-433.
- [6] Lin XH, Feng YX, Tan JR, et al. Product Concept Evaluation Based on Hybrid Model of Advanced DEMATEL-VIKOR Algorithm. *Computer Integrated Manufacturing Systems*. 2011; 17(12): 2552-2561.
- [7] Deng J, Yu ZH, Wu ZT. Quality Evaluation of Quality Design Based on Entropy. *Journal of Zhejiang University*. 2009; 43(8): 1480-1484.
- [8] Levner E, Alcaide D, Sicilia J. *Multi-attribute Text Classification Using the Fuzzy Borda Method and Semantic Grades*. Proceedings of the 7th International workshop on Fuzzy Logic and Applications. Berlin, Germany: Springer. 2007; 422-429.
- [9] Kuzgunkaya O, Eimaraghy HA. Assessing the Structural Complexity of Manufacturing Systems Configurations. *International Journal of Flexible Manufacturing Systems*. 2006; 18(2): 147-171.
- [10] Wu Fengping, Pan Wenwen. Research on Investment Risk Evaluation of Water Conservancy Project based on Support Vector Machine. *IJACT: International Journal of Advancements in Computing Technology*. 2013; 5(2): 801-808.
- [11] Jingwen Tian, Meijuan Gao, Chuichao Kong. Sludge Compost Quality Evaluation Based on Improved Genetic Algorithm and Radial Basis Function Neural Network. *JCIT: Journal of Convergence Information Technology*. 2013; 8(1): 802-810.