

---

## Using Variable Perturbation Method to Study the Stability of Torsional Self-Excited Vibration in Main Drive System of Rolling Mill

Fenglan Wang  
Shenyang University  
e-mail: flwang1965@sina.com

### Abstract

*The self-excited vibration of blooming mill is a kind of torsional vibration, which occurs only when the rolling machine slips under appropriate conditions. Once in place, this may cause the larger peak pressure of each component in the main drive system; reduce the service life of the components, cause components' damage directly. A large number of studies have shown that, at present there are not effective methods for diagnosis, monitoring of slipping and take proper measures in time to stop slipping so as to avoid the occurrence of malignant accidents. In this paper, author set out from another angle, take the main drive system of rolling mill as an example, to study stability of self-excited vibration. The conditions of the stable vibration are gained. By combining with the actual working conditions of blooming mill, author has put forward some effective measures to meet the stable conditions in order to make the blooming mill work in the stable state. Practice research has proved that the effective measures can contribute to reduce structural damage directly caused by the torsional vibration when "slipping" phenomenon occurred and to extend the service life of the components.*

**Keywords:** *blooming mill, main drive system, self-excited vibration, stability, variable perturbation method, torsional vibration*

**Copyright © 2013 Universitas Ahmad Dahlan. All rights reserved.**

### 1. Introduction

The self-excited vibration of blooming mill usually occurs when it subjects to impact load during the biting steel or in the rolling process. This may cause the larger peak pressure of each component in the main drive system; reduce the service life of the components, so as to cause structural damage directly. The research about the malignant accidents in some domestic rolling mills in recent years has showed that the process of the accident is often accompanied by slipping phenomenon. The analysis data about a malignant accident of a 1150 rolling mill in Baotou Steel Corp in the last century 80's proved this result [1-4]. About this kind of accident, the domestic and foreign scholars thought to be caused by the torsional vibration of main drive system. The torsional vibration of blooming mill can be divided into two categories in summary; one category is transient forced torsional vibration as a result of impact excitation torque, which is often the case. Another kind is the self-excited vibration; this is "torsional vibration" under appropriate conditions when the slip occurred ". Therefore, it must immediately be stopped in case "slipping" phenomenon occurred, so as not to cause large economic loss. In recent years, many scholars made much thorough research about the mechanism of self-excited vibration of blooming mill, and put forward many methods of calculating the torque. They also put forward some methods of reducing torsional vibration level and the idea of monitoring and anglicizing the torque amplification factor (TAF). People intended use these methods to diagnosis the self-excited vibration of blooming mill and stop skid phenomenon in order to avoid accidents [4-6]. However, some literatures showed that, the damage of the components is due to frequent occurrence of torsional vibration [7-8]. So, one or several calculation or measurement of TAF value can not realize the diagnosis and monitoring of "skid" phenomenon. At present, the study on the stability -one of characteristics in torsional self-excitation vibration is very lack. In this paper, author has taken main drive system of a blooming mill for example and studied the stability of its vibration theoretically, and obtained the stable conditions of stable vibration. By combining with the actual working conditions of blooming mill, author has put forward some

effective measures to meet the stable conditions in order to make the blooming mill work in the stable state. Practice research has proved that the effective measures can contribute to reduce structural damage directly caused by the torsional vibration when “slipping” phenomenon occurred and to extend the service life of the components.

## 2. The System is Simplified and Motion Equation Established

Main drive system of a blooming mill mainly consists of motor rotor, universal connection shaft, roller and other components. Assuming the moment of inertia of the motor, coupling and the roll are  $J_1$ ,  $J_2$  and  $J_3$ , the stiffness coefficient between the motor and the coupling shaft is  $K_1$ , the stiffness coefficient between the coupling to the roll is  $K_2$ , then the system can be simplified to the following mechanical models [4]:

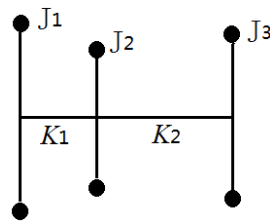


Figure1. Mechanical Model of Main Transmission System in a Rolling Mill

When the slip phenomenon occurred, the external torques applied to the main drive system includes motor driving torque  $M_1$ , and the friction torque  $M_3$  between the steel in the rolls. Then differential equation of the system can be written as:

$$[J]\{\varphi''\} + [C]\{\varphi'\} + [K]\{\varphi\} = \{M\}$$

In which,

$$[J] = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$$

$$\{M\} = \begin{Bmatrix} M_1 \\ 0 \\ M_3 \end{Bmatrix} \quad \{\varphi\} = \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{Bmatrix}$$

$$[K] = \begin{bmatrix} K_1 & -K_2 & 0 \\ -K_2 & K_1 + K_2 & -K_2 \\ 0 & -K_2 & K_2 \end{bmatrix}$$

Damping matrix can be approximately assumed:

$$[C] = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$$

For such a multiple degrees of freedom vibration equation, its natural frequency and vibration model of matrix can be obtained by Matrix Iteration Method (the results omitted). Thus through the coordinate transformation, the original equation becomes the differential equation of principal coordinate form. Ignoring high order differential and secondary factors, the following differential equations can be obtained [4]:

$$\begin{cases} \ddot{\theta}_{p_1} = a_0 - a_1 \dot{\theta}_{p_2} - a_2 \theta_{p_3} \\ \ddot{\theta}_{p_2} + \omega_2^2 \theta_{p_2} = b_0 - b_1 \dot{\theta}_{p_2} - b_2 \dot{\theta}_{p_3} \\ \ddot{\theta}_{p_3} + \omega_3^2 \theta_{p_3} = c_0 - c_1 \dot{\theta}_{p_2} - c_2 \dot{\theta}_{p_3} \end{cases} \quad (1)$$

Among them, every coefficient a, b, c (i=0, 1, 2) in the right-hand side have relation with the characteristic parameters of conversion mode in the system [3].

Because the moment of inertia of motor rotor  $J_1$  is much larger than the moment of inertia of the connecting shaft and the roll  $J_2, J_3$ , and motor drive torque M1 is constant, so we suppose motor rotor according to the approximate constant speed  $\dot{\phi}_1$ , that is  $\dot{\phi}_1 = \omega_0 = 0$ ; In addition, the rotor of the motor, connecting shaft and roll are mounted on the same shaft, so under normal circumstances three rotate at the same speed. When the M3 changes, the angular velocity  $\dot{\phi}_2, \dot{\phi}_3$  of  $J_2, J_3$  makes small changes with the angular speed  $\omega_0$ . Therefore, angular velocity change of  $J_2, J_3$  can be neglected in consideration of the first approximation of the angular velocity  $\dot{\theta}_{p_1}$ , that is, we regard  $\dot{\phi}_1 = \dot{\phi}_2 = \dot{\phi}_3 = \omega_0$ , then  $\dot{\theta}_{p_1} = \omega_0$  can be obtained, so that each coefficient becomes known parameters in right-hand side of (1).

Actually, (1) becomes:

$$\begin{cases} \ddot{\theta}_{p_2} + \omega_2^2 \theta_{p_2} = b_0 - b_1 \dot{\theta}_{p_2} - b_2 \dot{\theta}_{p_3} \\ \ddot{\theta}_{p_3} + \omega_3^2 \theta_{p_3} = c_0 - c_1 \dot{\theta}_{p_2} - c_2 \dot{\theta}_{p_3} \end{cases} \quad (2)$$

### 3. Using Variable Perturbation Method to Study the Stability of the Vibration Equations (2)

The equations (2) can be written as:

$$\begin{cases} \frac{d\theta_{p_2}}{dt} = \beta \\ \frac{d\beta}{dt} + \omega_2^2 \theta_{p_2} = b_0 - b_1 \beta - b_2 \gamma \\ \frac{d\theta_{p_3}}{dt} = \gamma \\ \frac{d\gamma}{dt} + \omega_3^2 \theta_{p_3} = c_0 - c_1 \beta - c_2 \gamma \end{cases} \quad (3)$$

Assume variables perturbation solution of (4):

$$\begin{cases} \theta_{p_2} = A_1(t) \cos \omega_2 t + B_1(t) \sin \omega_2 t + \sum_{q=1}^{\infty} \varepsilon^q \theta_{p_2}^{(q)} \\ \beta = \omega_2 [-A_1(t) \sin \omega_2 t + B_1(t) \cos \omega_2 t] + \sum_{q=1}^{\infty} \varepsilon^q \frac{d\theta_{p_2}^{(q)}}{dt} \\ \theta_{p_3} = A_2(t) \cos \omega_3 t + B_2(t) \sin \omega_3 t + \sum_{q=1}^{\infty} \varepsilon^q \theta_{p_3}^{(q)} \\ \gamma = \omega_3 [-A_2(t) \sin \omega_3 t + B_2(t) \cos \omega_3 t] + \sum_{q=1}^{\infty} \varepsilon^q \frac{d\theta_{p_3}^{(q)}}{dt} \end{cases} \quad (4)$$

The first two on the right-hand side are the solutions of variable section, the last one is part of the solution of perturbation. Take (4) into (3), and retain a first-order approximation until a power of  $\varepsilon$ , thus obtained:

$$\left\{ \begin{array}{l}
 \frac{dA_1}{dt} \cos \omega_2 t + \frac{dB_1}{dt} \sin \omega_2 t = 0 \\
 -\omega_2 \frac{dA_1}{dt} \sin \omega_2 t + \omega_2 \frac{dB_1}{dt} \cos \omega_2 t + \varepsilon \left( \frac{d^2 \theta_{p_2}^{(1)}}{dt^2} + \omega_2^2 \theta_{p_2}^{(1)} \right) \\
 = b_0 - b_1 \left( -\omega_1 A_2 \sin \omega_1 t + \omega_2 B_1 \cos \omega_1 t + \varepsilon \frac{d\theta_{p_2}^{(1)}}{dt} \right) \\
 - b_2 \left( -\omega_3 A_2 \sin \omega_3 t + B_2 \omega_3 \cos \omega_3 t + \varepsilon \frac{d\theta_{p_3}^{(1)}}{dt} \right) \\
 \frac{dA_2}{dt} \cos \omega_3 t + \frac{dB_2}{dt} \sin \omega_3 t = 0 \\
 -\omega_3 \frac{dA_2}{dt} \sin \omega_3 t + \omega_3 \frac{dB_2}{dt} \cos \omega_3 t + \varepsilon \left( \frac{d^2 \theta_{p_3}^{(1)}}{dt^2} + \omega_3^2 \theta_{p_3}^{(1)} \right) \\
 = c_0 - c_1 \left( -\omega_2 A_1 \sin \omega_2 t + \omega_2 B_1 \cos \omega_2 t + \varepsilon \frac{d\theta_{p_2}^{(1)}}{dt} \right) \\
 - c_2 \left( -\omega_3 A_2 \sin \omega_3 t + B_2 \omega_3 \cos \omega_3 t + \varepsilon \frac{d\theta_{p_3}^{(1)}}{dt} \right)
 \end{array} \right. \quad (5)$$

As following, the vibration stability discussed according to the near resonance ( $\omega_3 = \omega_2 + \varepsilon \sigma$ ) and internal resonance ( $\omega_3 = 3\omega_2 + \varepsilon \sigma$ ).

### 3.1. The Near Resonance ( $\omega_3 = \omega_2 + \varepsilon \sigma$ )

Assuming  $\omega_3 = \omega_2 + \varepsilon \sigma$ , the equations of the variable parts become into:

$$\left\{ \begin{array}{l}
 \frac{dA_1}{dt} \cos \omega_2 t + \frac{dB_1}{dt} \sin \omega_2 t = 0 \\
 -\frac{dA_1}{dt} \sin \omega_2 t + \frac{dB_1}{dt} \cos \omega_2 t = \frac{b_0}{\omega_2} + (b_1 + \varepsilon b_1)(A_1 \sin \omega_2 t - B_1 \cos \omega_2 t) \\
 + \frac{(b_2 + \varepsilon b_2)\omega_3}{\omega_2} (A_2 \sin \omega_3 t - B_2 \cos \omega_3 t) \\
 \frac{dA_2}{dt} \cos \omega_3 t + \frac{dB_2}{dt} \sin \omega_3 t = 0 \\
 -\frac{dA_2}{dt} \sin \omega_3 t + \frac{dB_2}{dt} \cos \omega_3 t = \frac{c_0}{\omega_3} + \frac{(c_1 + \varepsilon c_1)\omega_2}{\omega_3} (A_1 \sin \omega_2 t - B_1 \cos \omega_2 t) \\
 + (c_2 + \varepsilon c_2)(A_2 \sin \omega_3 t - B_2 \cos \omega_3 t)
 \end{array} \right. \quad (6)$$

Solving the above four equations and taking the average value in the period T by Kryloff-Bogolinboff-Vander Pol method ( $\omega_2 t, \omega_3 t$  in the period  $2\pi$ ), we can obtain:

$$\left\{ \begin{array}{l}
 \frac{dA_1}{dt} = -\frac{b_1}{2}(1 + \varepsilon)A_1 - \frac{b_2}{2}(1 + \varepsilon)(A_2 \cos \varepsilon \sigma t + B_2 \sin \varepsilon \sigma t) \frac{\omega_3}{\omega_2} \\
 \frac{dB_1}{dt} = -\frac{b_1}{2}(1 + \varepsilon)B_1 + \frac{b_2}{2}(1 + \varepsilon)(A_2 \sin \varepsilon \sigma t - B_2 \cos \varepsilon \sigma t) \frac{\omega_3}{\omega_2} \\
 \frac{dA_2}{dt} = -\frac{c_1}{2}(1 + \varepsilon)(A_1 \cos \varepsilon \sigma t - B_1 \sin \varepsilon \sigma t) \frac{\omega_2}{\omega_3} - \frac{c_2}{2}(1 + \varepsilon)A_2 \\
 \frac{dB_2}{dt} = -\frac{c_1}{2}(1 + \varepsilon)(A_1 \sin \varepsilon \sigma t + B_1 \cos \varepsilon \sigma t) \frac{\omega_2}{\omega_3} - \frac{c_2}{2}(1 + \varepsilon)B_2
 \end{array} \right. \quad (7)$$

To solve the above four equations, assume  $x_1 = A_1 + B_1 i, y_1 = A_2 + B_2 i, x_2 = A_1 - B_1 i, y_2 = A_2 - B_2 i$ . And further assuming:

$$\begin{aligned} x_1 &= x_{10} e^{pt-0.5\varepsilon\sigma t} & x_2 &= x_{20} e^{qt+0.5\varepsilon\sigma t} \\ y_1 &= y_{10} e^{pt+0.5\varepsilon\sigma t} & y_2 &= y_{20} e^{qt-0.5\varepsilon\sigma t} \end{aligned} \quad (8)$$

Among,  $x_{10}, y_{10}, x_{20}, y_{20}$  are constants.

$$\text{Then: } \begin{cases} x_{10}(p - \frac{1}{2}\varepsilon\sigma) = \frac{b_1}{2}(1+\varepsilon)x_{10} - \frac{b_2}{2}(1+\varepsilon)\frac{\omega_3}{\omega_2}y_{10} \\ y_{10}(p + \frac{1}{2}\varepsilon\sigma) = -\frac{c_2}{2}(1+\varepsilon)y_{10} - \frac{c_1}{2}(1+\varepsilon)\frac{\omega_2}{\omega_3}x_{10} \\ x_{20}(q + \frac{1}{2}\varepsilon\sigma) = \frac{b_1}{2}(1+\varepsilon)x_{20} - \frac{b_2}{2}(1+\varepsilon)\frac{\omega_3}{\omega_2}y_{20} \\ y_{20}(q - \frac{1}{2}\varepsilon\sigma) = -\frac{c_2}{2}(1+\varepsilon)y_{20} - \frac{c_1}{2}(1+\varepsilon)\frac{\omega_2}{\omega_3}x_{20} \end{cases} \quad (9)$$

By Solving the above equation group and referring  $x_1, y_1, x_2, y_2$  assumed, the condition of the system stable vibration is:

$$\frac{b_1 c_2}{c_1 b_2 - b_1 c_2} < 0$$

### 3.2. Internal Resonance ( $\omega_3=3\omega_2+\varepsilon\sigma$ )

Similarly we can obtain:

$$\begin{cases} A_1 = A_{01} \exp[-\frac{b_1}{2}(1+\varepsilon)t] \\ B_1 = B_{02} \exp[-\frac{b_1}{2}(1+\varepsilon)t] \\ A_2 = A_{02} \exp[-\frac{c_2}{2}(1+\varepsilon)t] \\ B_2 = B_{02} \exp[-\frac{c_2}{2}(1+\varepsilon)t] \end{cases}$$

Among  $A_{01}, B_{01}, A_{02}, B_{02}$  are constants decided by initial conditions. Obviously, because  $\varepsilon$  is a small parameter, if  $b_1$  and  $c_2$  are positive, then  $A_1, B_1, A_2, B_2$  did not increase with time, and gradually decay. Take the above four into the second and the fourth of (IV), we can know that the self-excited vibration is attenuated and stable. So, we can determine the vibration stability in the neighborhood of  $\omega_3=3\omega_2$ .

## 4. Conclusion

In order to eliminate slippage or weaken self-excited vibration, or let the self-excited vibration time short and stable, according to the previous stability conditions derived, we find out:

- (1) Make the work piece head a cone shape or arc shape to reduce the motor torque. We can know according to the stability conditions, if  $b_1$  is increased, increasing rate of the right side in equation is greater than left. Thus the self-excited vibration stability conditions are more easily satisfied.
- (2) Increase the coefficient of friction between the steel and the roller as far as possible, such as cleaning up the oxidation iron sheet, which is, making the steel surface without loosening layer. Thus  $b_1, c_1, b_0$  and  $c_0$  will enlarge. Thus the self-excited vibration stability conditions are more likely satisfied.
- (3) Other methods, such as reducing the rolling speed, reducing the pressure volume can also provide favorable conditions for weakening the self-excited vibration.

To thoroughly solve the slipping problem, the study about the mechanism of self-excited vibration needs strengthening. The method of diagnosis and monitoring for slip should be found fundamentally. This remains to be studied.

### References

- [1] Chen Yushu, Ding Qian. Some Problems in the Study of Nonlinear Dynamics of Large Scale Rotor System. *ICVE'98*. 1998: 36-39.
- [2] He Chengbing, Gu Yujiong. An Increment Transfer Matrix Method for the Coupled Bending and Torsional Vibrations of Turbo-Generator Shafts with Rub-Impact. *AISS: Advances in Information Sciences and Service Sciences*. 2012; 4(18):189-196.
- [3] Lin He, Kang Zuli. The torsional vibration response caused by slip in blooming mill rolling. *Journal of Beijing University of Science and Technology*. 1991; 13(1): 31-36
- [4] Wang tieguang, Guo wanzhen. The torsion self-excited vibration analysis of main drive system in 1100 rolling mill. *Journal of Northeastern University*. 1989.
- [5] Xu Yang, Chaonan Tong. Nonlinear Friction Compensation Control of Cold Rolling Main Drive System with Parameters Uncertainty. *JCIT: Journal of Convergence Information Technology*. 2012; 7(20): 17-24.
- [6] Yu heji, Kouhui, Yuan peixin. Condition monitoring and fault diagnosis of rolling machine. China: Metallurgical Industry Press. 1993.
- [7] Chen Changming, Chen Yingying, Li Zhen, Lian Xigang. Blooming mill torsional vibration of main drive system. *Metal Journal*. 1985; 21(5): 261-271.
- [8] Xiang Xu, Ruiping Zhou, Mengsheng Wang. Research on the Marine Shafting Torsional Vibration with Gears Transmission. *IJEI*. 2012; 3(1): 99-111.