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Lag Synchronization of a Class of Time-delayed Chaotic Neural Networks by Impulsive Control

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Abstract

The paper studies the exponential lag synchronization of a class of delayed chaotic neural networks with impulsive effects via the unidirectional linear coupling. Some sufficient conditions are derived by establishing impulsive differential delay inequality and using M-matrix theory. An illustrative example is also provided to show the effectiveness and feasibility of the impulsive control method.

***Keywords**: Impulsive control, lag synchronization, time-delayed, chaotic, neural networks*

1. Introduction

In the past decades, there has been much attention on chaos synchronization due to its potential applications such as secure communications, biological systems, information science, and etc [1-2]. Since Pecora and Carroll originally proposed the drive-response concept for achieving the synchronization of coupled chaotic systems [1], researchers have also proposed a variety of alternative schemes for ensuring the synchronization.

Recently, impulsive control has been widely used to stabilize and synchronize chaotic systems [3-8]. Its necessity and importance lie in that, in some cases, the system cannot be controlled by continuous control. Additionally, impulsive control may give a more efficient method to deal with systems that cannot ensure continuous disturbance. Furthermore, impulsive method can also greatly reduce the control cost [8].

The study of dynamical properties of neural networks appears more and more due to their extensive applications in differential fields such as signal and image processing, combinatorial optimization, pattern recognition and etc [9-12]. In the electronic implementation of a neural network, time delay will occur in the interaction between the neurons inevitably, and will affect the dynamic behavior of the neural network model. In some particular cases, chaotic and hyperchaotic attractors may be generated by the introduction of delays into neural networks [13-15]. Therefore, some time-delayed neural networks could be as a model when we study the chaos synchronization.

For long-distance communication, the transmission time of drive signal is correspondingly long. So the lag synchronization can describe this case more precisely than the complete synchronization. Motivated by the above discussions, the aim of this paper is to study the exponential lag synchronization of chaotic delayed neural networks with impulsive effects. By establishing impulsive differential delay inequality and using M-matrix theory, some criteria for synchronization of impulsive delay neural networks are derived. An illustrative example, along with numerical simulations is also provided to show the effectiveness and feasibility of the developed method.

2. Model Description and Preliminaries

A class of time-delayed chaotic neural networks considered in this letter is described by

$$\dot{x}(t) = -Ax(t) + Bf\left(x(t)\right) + Cf\left(x(t-\tau)\right) + I \, , \qquad (2.1)$$

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Where $x \in R^n$ is the state vector, time-delay $\tau \geq 0$, $A, B, C \in R^{n\times n}$ are constant matrices, $I \in R^n$ represents the external input and the input-output transfer function $f : R^n \to R^n$ is a continuous and nonlinear function.

Remark 2.1. Several chaotic or hyperchaotic neural networks satisfy (2.1). For example, the cellular neural network and the well known Hopfield neural network with or without time delay belong to the class defined by (2.1).

The difference differential Equation (2.1) can be categorized as a kind of functional differential equations, and rewritten as:

$$\dot{x}(t) = -Ax_t(0) + Bf\left(x_t(0)\right) + Cf\left(x_t(-\tau)\right) + I\ , \tag{2.2}$$

Where x_t is a continuous mapping defined on $[-\tau, 0]$ as $x_t(\theta) = x(t+\theta)$, the right-hand side of Equation (2.2) defines a functional mapping $C([-\tau,0], R^n)$ to R^n, where $C([-\tau,0], R^n)$ denotes the set of all continuous mapping from $[-\tau, 0]$ to R^n. The solution space of Equation (2.2) is infinite-dimensional, with initial conditions as any continuous functions defined on the closed interval $[-\tau, 0]$.

We now introduce the impulsive control for chaotic neural networks (2.1) as follows:

$$\begin{cases} \dot{x}(t) = -Ax(t) + Bf\left(x(t)\right) + Cf\left(x(t-\tau)\right) + I, \quad t \neq t_k, \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = -H\,x(t_k), \quad t = t_k, \\ x(t) = \varphi(t), \quad t \in [-\tau, 0], \end{cases} \tag{2.3}$$

Where the discrete set $\{t_k\}$ satisfies $0 \leq t_0 < t_1 < \cdots < t_k < \cdots,\ t_k \to +\infty$ as $k \to +\infty$, $x(t_k^-) = \lim_{t\to t_k - 0} x(t)$, $x(t_k^+) = \lim_{t \to t_k + 0} x(t)$ and $x(t_k) = x(t_k^+)$. H denotes the controller impulsive matrix.

From the unidirectional linear coupling approach, a response system for (2.3) is constructed as follows:

$$\begin{cases} \dot{y}(t) = -Ay(t) + Bf\left(y(t)\right) + Cf\left(y(t-\tau)\right) + I + W\left(x(t-\delta) - y(t)\right), \quad t \neq s_k, t \geq \delta, \\ \Delta y(s_k) = y(s_k^+) - y(s_k^-) = -H\,y(s_k), \quad t = s_k, \\ y(t) = \psi(t), \quad t \in [-\tau+\delta, \delta], \end{cases} \tag{2.4}$$

Where W denotes the controller gain matrix, $s_k = t_k + \delta$ and $\delta < \tau$.

Lag synchronization is characterized by $y(t) = x(t-\delta)$ for some constant $\delta > 0$. Let $e(t) = y(t) - x(t-\delta)$ be the synchronization error. The error system of the impulsive synchronization is given by:

$$\begin{cases} \dot{e}(t) = -(A+W)e(t) + Bg\left(e(t)\right) + Cg\left(e(t-\tau)\right), \quad t \neq s_k, t \geq \delta, \\ \Delta e(s_k) = e(s_k^+) - e(s_k^-) = -H\,e(s_k), \quad t = s_k, \\ e(t) = \phi(t) = \psi(t) - \varphi(t-\delta), \quad t \in [-\tau+\delta, \delta], \end{cases} \tag{2.5}$$

Where $g(e(t)) = f(y(t)) - f(x(t-\delta))$ and $g(e(t-\tau)) = f(y(t-\tau)) - f(x(t-\delta-\tau))$.

Note that the origin is the equilibrium point of system (2.5). If $e(t)$ tends exponentially to origin in evolution, exponential lag synchronization between two systems would be realized. Our aim is to find some criteria on the controller impulsive matrix H and controller gain matrix W such that drive system (2.3) and response system (2.4) are exponentially lag synchronized for any bounded initial condition, $e(¯)=¤(¯), ¯\in[-² +±,±]$ with $\|¤\|=\sup_{¯\in[-²+±,±]}\|¤(¯)\|<\infty$, the notation $\| \ \|$ denotes the Euclidian norm of a vector or a square matrix.

For convenience, we introduce some notations used in this paper.

Usually E denotes an $n\times n$ unit matrix. For $A,B\in R^{m\times n}$ or $A,B\in R^n$, $A\geq B$ means that each pair of corresponding elements of A and B satisfies the inequality "$\geq$". Especially, A is called a nonnegative matrix if $A\geq 0$, and z is called a positive vector if $z>0$.

$PC(I) \square \{\cdot : I\to R^n \mid \cdot(t)=\cdot(t^+)$ for $t\in I$, $\cdot(t^-)$ exists for $t\in(±,+\infty)$, $\cdot(t^-)=\cdot(t)$ for all but points $s_k\in(±,+\infty)\}$, where $I\subset R$ is an interval, $\cdot(t^-)$ and $\cdot(t^+)$ denote the left-hand and right-hand limits of the function $\cdot(t)$, respectively. Especially, let $PC=PC([-² +±,±])$.

For a $m\times n$ matrix A, $|A|$ denotes the absolute value matrix given by $|A|=(|a_{ij}|)_{m\times n}$.

Definition 2.1. A real matrix $D=(d_{ij})_{n\times n}$ is said to be a nonsingular M-matrix if $d_{ij}\leq 0, i,j=1,2,\ldots,n,\quad i\neq j$, and all successive principal minors of D are positive.

Lemma 2.1. Let $D=(d_{ij})_{n\times n}$ with $d_{ij}\leq 0(i\neq j)$, then D is a nonsingular M-matrix if and only if the diagonal elements of D are all positive and there exists a positive vector d such that $Dd>0$ or $D^Td>0$.

For a nonsingular M-matrix D, we denote $\Omega_M(D)\square\{z\in R^n \mid Dz>0, z>0\}$, which is nonempty by the Lemma 1. For a nonnegative matrix $G\in R^{n\times n}$, the spectral radius $°(G)$ is an eigenvalue of G and its eigenspace is denoted by $\Omega_{°}(G)\square\{z\in R^n \mid Gz=°(G)z\}$, which includes all positive eigenvectors of G provided that the nonnegative matrix G has at least one positive eigenvector.

3. Impulsive Lag Synchronization of Chaotic Neural Networks

The following impulsive delay inequality is necessary to develop the main result in this paper.

Theorem 3.1. Let $u(t)\in PC\left([-² +±,\infty)\right)$ satisfy the following impulsive differential delay inequality:

$$\begin{cases} D^+u(t)\leq Pu(t)+Qu(t-²),\ t\neq s_k, t\geq ±, \\ u(s_k)\leq Gu(s_k^-),\ t=s_k, k=1,2,\ldots, \\ u(t)=¤(t),\ -²+±\leq t\leq ±, \end{cases} \tag{3.1}$$

Where $P=(p_{ij})_{n\times n}$ with $p_{ij}\geq 0$ for $i\neq j$, $Q=(q_{ij})_{n\times n}\geq 0, G=(g_{ij})_{n\times n}\geq 0, ¤(t)\in PC$.

We define:

$$\Omega_{DG}=\Omega_M(D)\cap\Omega_{°}(G) \tag{3.2}$$

And assume that:

(H1) $D=-(P+Q)$ is a nonsingular M-matrix.

(H2) Ω_{DG} is nonempty.

Then there exist a positive vector $z=(z_1,z_2,\ldots,z_n)^T\in\Omega_{DG}$ such that:

$$u(t)\leq ¥^{k-1}z\exp(-ª(t-±)),\quad t\in[s_{k-1},s_k),\quad k=1,2,\ldots, \tag{3.3}$$

where $¥\square\max\{1,°(G)\}$ and the positive number $ª$ is determined by the following inequality:

$$[ªE+P+Q\exp(ª²)]z<0. \tag{3.4}$$

Proof. If $G = {\sim} E$, ~ is a constant, then $\Omega_{\circ}(G) = R^n$, so $\Omega_{DG} = \Omega_M(D)$ is nonempty, which shows the assumption (H2) is reasonable. For any $z_1, z_2 \in \Omega_{DG}$, we have $k_1 z_1 + k_2 z_2 \in \Omega_{DG}$ for all $k_1, k_2 > 0$.

Since ¤$(t) \in PC$ is bounded, by the definition of Ω_{DG} and condition (H2), there exists a positive vector $z = (z_1, z_2, \dots, z_n)^T \in \Omega_{DG}$ such that the initial function of (3.1) satisfies.

$$u(t) \le z \exp(-ª(t - ±)), \quad -² + ± \le t \le ±. \tag{3.5}$$

For the vector $z \in \Omega_{DG} \subseteq \Omega_M(D)$, by condition (H1) and the definition of M-matrix, we have $Dz > 0$ or $(P+Q)z < 0$. By using continuity, we know that (3.3) has at least one positive solution ª , i.e.,

$$ª z_i + \Sigma_{j=1}^{n} (p_{ij} + q_{ij} \exp(ª²)) z_j < 0, \quad i = 1, 2, \dots, n. \tag{3.6}$$

Now we shall prove that:

$$u(t) \le z \exp(-ª(t - ±)), \quad t \in [±, s_1). \tag{3.7}$$

To prove (11), we first prove for any given constant $£ > 0$,

$$u_i(t) < (1+£) z_i \exp(-ª(t-±)) \square\ v_i(t), \quad i = 1, 2, \dots, n, \quad t \in [±, s_1) . \tag{3.8}$$

If (3.7) is not true, from the fact that $u(t)$ is continuous in $[±, s_1)$ (since $u(t) \in PC\left([-² + ±, \infty)\right)$), then there must be a $t^* \in [±, s_1)$ and some integer m such that:

$$\begin{aligned} &u_m(t^*) = v_m(t^*), \quad D^+ u_m(t^*) \ge v_m'(t^*), \\ &u_i(t) < v_i(t), \quad -² + ± \le t < t^*, \quad i = 1, 2, \dots, n. \end{aligned} \tag{3.9}$$

By using (3.1), (3.5), (3.7), (3.8) and $p_{ij} \ge 0\, (i \ne j),\ q_{ij} \ge 0$, we obtain:

$$\begin{aligned} D^+ u_m(t^*) &\le \Sigma_{j=1}^{n} \left(p_{mj} u_j(t^*) + q_{mj} u_j(t^* - ²) \right) \\ &\le \Sigma_{j=1}^{n} \left(p_{mj} (1+£) z_j \exp(-ª(t^* - ±)) + q_{mj} (1+£) z_j \exp(-ª(t^* - ² - ±)) \right) \\ &= \Sigma_{j=1}^{n} (p_{mj} + q_{mj} \exp(ª²))(1+£) z_j \exp(-ª(t^* - ±)) \\ &< -ª z_m (1+£) \exp(-ª(t^* - ±)) = v_m'(t^*) \end{aligned} \tag{3.10}$$

Which contradicts the inequality of (3.8), and so (3.7) holds for $t \in [±, s_1)$. Letting $£ \to 0$, then (3.6) holds for $t \in [±, s_1)$. Using (3.1), (3.6), $z \in \Omega_{\circ}(G)$ and the definition of ¥ , we can obtain that:

$$u(s_1) \le G u(s_1^-) \le G z \exp(-ª(s_1 - ±)) = {}^{\circ}(G) z \exp(-ª(s_1 - ±)) \le ¥ z \exp(-ª(s_1 - ±)), \tag{3.11}$$

and so, we have:

$$u(t) \le ¥ z \exp(-ª(t - ±)), \quad t \in [s_1 - ², s_1] . \tag{3.12}$$

By a similar argument of (11), we can use (16) and derive that:

$$u(t) \le ¥ z \exp(-ª(t - ±)), \quad t \in [s_1, s_2) . \tag{3.13}$$

Therefore, by simple induction, we have:

$$u(t) \le ¥^{k-1} z \exp(-ª(t - ±)), \quad t \in [s_{k-1}, s_k), \quad k = 1, 2, \dots . \tag{3.14}$$

The proof is completed.
From Theorem 3.1, we have the following result.

Theorem 3.2. Assume that:
(A1) The neurons activation function f is Lipschitz-continuous, that is, there exists a nonnegative diagonal matrix Γ such that $|f(y)-f(x)|\le \Gamma |y-x|$.
(A2) Let $P=-(A+W)+|B|\Gamma,\ Q=|C|\Gamma,\ G=:|H|;\ D=-(P+Q)$ is a nonsingular M-matrix and Ω_{DG} is nonempty.
(A3) Let $¥ = \max\{1, °(G)\}$, and there exists a constant ¢ such that:

$$k \ln ¥ / (s_k - s_{k-1}) \le ¢ < ª, \quad k = 1, 2, \ldots, \tag{3.15}$$

Where the positive constant ª is determined by the following inequality:

$$[ªE + P + Q\exp(ª²)]z < 0 \text{ for a given } z \in \Omega_{DG}. \tag{3.16}$$

Then the origin of (2.5) is globally exponentially stable, implying that the two systems (2.3) and (2.4) are global impulsive exponential lag synchronization.
Proof. From condition (A2), there exists a positive vector $z=(z_1,z_2,\ldots,z_n)^T \in \Omega_{DG} \subseteq \Omega_M(D)$ such that $Dz>0$, or $(P+Q)z<0$. By using continuity, we know that (3.16) has at least one positive solution ª.

Then calculating the upper right derivative $D^+|e(t)|$ along the solution of (2.5), from the condition (A1) and the definition of P, Q, we have

$$\begin{aligned} D^+|e(t)| &\le -(A+W)|e(t)| + |B[f(y(t)) - f(x(t-±))]| + |C[f(y(t-²)) - f(x(t-²-±))]| \\ &\le -(A+W)|e(t)| + |B|\Gamma|y(t)-x(t-±)| + |C|\Gamma|y(t-²)-x(t-²-±)| \\ &= -(A+W)|e(t)| + |B|\Gamma|e(t)| + |C|\Gamma|e(t-²)| \\ &= P|e(t)| + Q|e(t-²)|, \quad t \ne s_k, \quad t \ge ±, \end{aligned} \tag{3.17}$$

And, $|e(s_k)| \le G|e(s_k^-)|, \quad t = s_k, \quad k = 1, 2, \ldots.$ (3.18)

For the initial condition $¤(t) \in PC$, we can get:

$$|e(t)| \le ¦\,\|¤\| \exp(-ª(t-±)), \quad -² + ± \le t \le ±, \tag{3.19}$$

Where $¦ = z / \min_{1\le i\le n}\{z_i\} \ge (1,1,\ldots,1)^T$. From $z \in \Omega_{DG}$, we have $¦ \in \Omega_{DG}$ and so $¦\,\|¤\| \in \Omega_{DG}$.

All the conditions of Theorem 1 are satisfied by (3.17), (3.18), (3.19), conditions (A2) and (A3). Then we obtain that:

$$|e(t)| \le ¥^{k-1} ¦\,\|¤\| \exp(-ª(t-±)), \quad t \in [s_{k-1}, s_k), \quad k = 1, 2, \ldots. \tag{3.20}$$

By (3.15), we have ¢ < ª and:

$$¥^{k-1} \le \exp\big(¢(s_{k-1} - s_{k-2})\big) \le \exp\big(¢(t-±)\big), \quad t \in [s_{k-1}, s_k), \quad k = 1, 2, \ldots. \tag{3.21}$$

So, from (3.20) and (3.21), we derive that

$$|e(t)| \le ¦\,\|¤\| \exp(-(ª - ¢)(t-±)), \quad t \in [s_{k-1}, s_k), \quad k = 1, 2, \ldots. \tag{3.22}$$

The proof is completed.

4. Numerical Simulations

In this section, we give an example to illustrate the effectiveness of results obtained in the previous sections. Consider a two-dimensional chaotic delayed cellular neural network with impulsive effects.

$$\begin{aligned}
&\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}=-A\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix}+B\begin{pmatrix}f_1(x_1(t))\\ f_2(x_2(t))\end{pmatrix}+C\begin{pmatrix}f_1(x_1(t-1))\\ f_2(x_2(t-1))\end{pmatrix}+I,\ t\neq t_k,\\
&\Delta x(t_k)=x(t_k^+)-x(t_k^-)=-H\,x(t_k),\quad t=t_k,\quad t_k\in\{2,4,6,\ldots\},\\
&x(t)=\P(t),\quad t\in[-1,0],
\end{aligned} \tag{4.1}$$

Where $f_i(x_i)=0.5(|x_i+1|-|x_i-1|),\ i=1,2,\ ^{2}=1.0,\ I=(0,0)^T$, and:

$$A=\begin{pmatrix}1&0\\0&1\end{pmatrix},\ B=\begin{pmatrix}1+®/4&20\\0.1&1+®/4\end{pmatrix},\ C=\begin{pmatrix}-1.3\sqrt{2}®/4&0.1\\0.1&-1.3\sqrt{2}®/4\end{pmatrix}. \tag{4.2}$$

Clearly, the function $f_i(x_i)=0.5(|x_i+1|-|x_i-1|)$ is bounded and satisfies the condition (A1) with diagonal matrix $\Gamma=diag(1,1)$. The controller impulse matrix H is chosen as $H=diag(0.2,0.2)$. The system (4.1) has a chaotic attractor with the initial condition $\P(t)\equiv(0.01,0.1)^T,\ t\in[-1,0]$ (Figure 1).

Viewing (4.1) as drive system, under state coupling, the response system is constructed as follows:

$$\begin{aligned}
&\begin{pmatrix}\dot{y}_1\\ \dot{y}_2\end{pmatrix}=-A\begin{pmatrix}y_1(t)\\ y_2(t)\end{pmatrix}+B\begin{pmatrix}f_1(y_1(t))\\ f_2(y_2(t))\end{pmatrix}+C\begin{pmatrix}f_1(y_1(t-1))\\ f_2(y_2(t-1))\end{pmatrix}+W\begin{pmatrix}x_1(t-\pm)-y_1(t)\\ x_2(t-\pm)-y_2(t)\end{pmatrix},\ t\neq s_k,\ t>\pm,\\
&\Delta y(s_k)=y(s_k^+)-y(s_k^-)=-H\,y(s_k),\quad t=s_k,\quad s_k\in\{2+\pm,4+\pm,6+\pm,\ldots\},\\
&y(t)=¸(t),\quad t\in[-1+\pm,\pm],
\end{aligned} \tag{4.3}$$

Where the initial condition $¸(t)\equiv(-0.01,-0.1)^T,\ t\in[-1+\pm,\pm]$.

Now we take $\pm=0.8$ and the controller gain matrix $W=\begin{pmatrix}5.2&20\\0.1&5.2\end{pmatrix}$. Let $e_i(t)=y_i(t)-x_i(t-0.8)$, then the error system of drive system (4.1) and respond system (4.3) is constructed as follows:

$$\begin{cases}
\dot{e}_1=-(5.2-®/4)e_1(t)-1.3\sqrt{2}®/4e_1(t-1)+0.1e_2(t-1),\\
\dot{e}_2=-(5.2-®/4)e_2(t)+0.1e_1(t-1)-1.3\sqrt{2}®/4e_2(t-1),\ \ t\neq s_k,\ \ t\geq 0.8,\\
\Delta e_1(s_k)=e_1(s_k^+)-e_1(s_k^-)=-0.2e_1(s_k)\\
\Delta e_2(s_k)=e_2(s_k^+)-e_2(s_k^-)=-0.2e_2(s_k),\ \ t=s_k,\ \ s_k\in\{2.8,4.8,6.8,\ldots\},\\
e(t)=¤(t)=¸(t)-\P(t-0.8)\equiv(-0.02,-0.2)^T,\ \ t\in[-0.2,0.8].
\end{cases} \tag{4.4}$$

And we have:

$$\begin{aligned}
&P=\begin{pmatrix}-5.2+®/4&0\\0&-5.2+®/4\end{pmatrix},\qquad Q=\begin{pmatrix}1.3\sqrt{2}®/4&0.1\\0.1&1.3\sqrt{2}®/4\end{pmatrix},\\
&D=\begin{pmatrix}5.2-(1+1.3\sqrt{2})®/4&-0.1\\-0.1&5.2-(1+1.3\sqrt{2})®/4\end{pmatrix},\ G=\begin{pmatrix}0.2&0\\0&0.2\end{pmatrix},
\end{aligned} \tag{4.5}$$

Where $P=-(A+W)+|B|\Gamma,\ Q=|C|\Gamma,\ D=-(P+Q),G=|H|$. One can easily verify that D is a nonsingular M-matrix. By simple calculation, we have,

$$\Omega_M(D)=\left\{(z_1,z_2)^T\,|\,\frac{1}{30}z_2<z_1<30z_2,\ z_1,z_2>0\right\},\ \ \Omega_{\circ}(G)=R^2. \tag{4.6}$$

So $\Omega_M(D)$ and $\Omega_{\circ}(G)$ generate an intersection.

$$\Omega_{DG} = \left\{ (z_1, z_2)^T \mid \frac{1}{30} z_2 < z_1 < 30 z_2,\ z_1, z_2 > 0 \right\}. \quad (4.7)$$

Let $z = (1,2)^T \in \Omega_{DG}$ and ª $= 0.2$, then:

$$[ªE + P + Q\exp(ª²)]z = [0.2E + P + Q\exp(0.2)](1,2)^T = (-2.207, -4.780)^T < 0\ , \quad (4.8)$$

Which implies that the condition (A3) with ¥ $= 1$, ¢ $= 0$ and ª $= 0.2$. Therefore, all the conditions of Theorem 2 are satisfied, we can conclude that systems (4.1) and (4.3) are exponential lag synchronization. The numerical simulations show that lag synchronization could be quickly achieved. The phase plot of response system with the initial condition ¸ $(t) \equiv (-0.01, -0.1)^T$, $t \in [-0.2, 0.8]$ is shown in Figure 2 and the state trajectory of variable x, y is shown in Figure 3. One can find that y retards x by a time period 0.8.

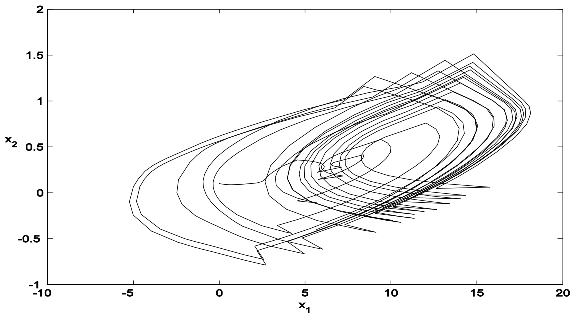

Figure 1. Drive System

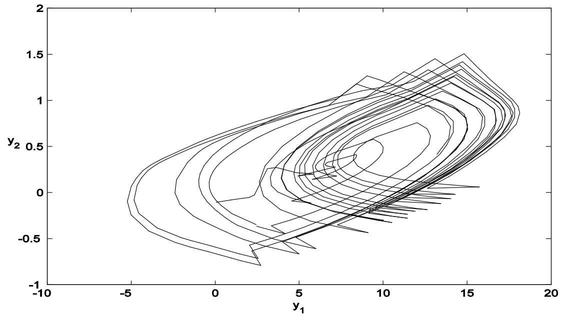

Figure 2. Response System

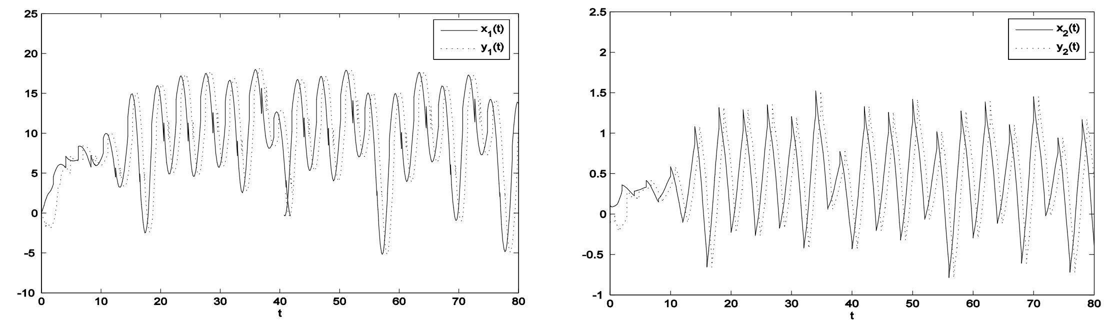

Figure 3. State Trajectory of Variable $x(t)$ and $y(t)$ (± $= 0.8$)

Remark 4.1. It should be noticed that the time delay ² influence the behavior of system. Let ² vary from 0.845 to 1.0, system (4.1) always has a chaotic trajectory.

In the following, we research the relationship between the rate of exponential synchronization ª and time delay parameter ² by numerical calculation, it is shown in Figure 4. And then, we fix ª $= 0.2$, let ² $= 0.85, 0.90, 0.95, 1.0$ respectively, and plot the stable region boundary about parameters w_{11}, w_{12} in controller gain matrix W (Figure 5).

Remark 4.2. Region A_1, A_2 and B represents stable and chaotic region, stable but non-chaotic region, and unstable region respectively. In region A_1 , the rate of exponential synchronization ª will decline with the increasing of time delay ² .

Remark 4.3. For a fixed value of ª , the stable region will enlarge with the decreasing of time delay ² , the range of gain matrix parameters could be broadened by parameter ² accordingly.

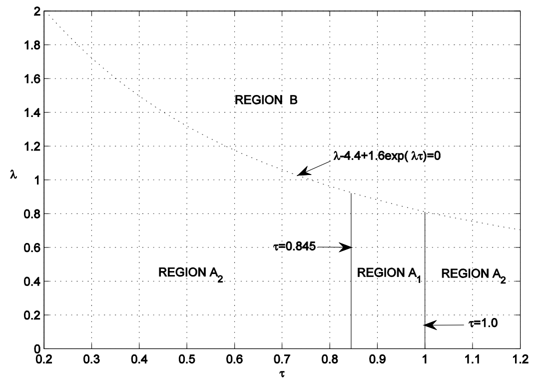

Figure 4. Rate of Exponential Synchronization
a

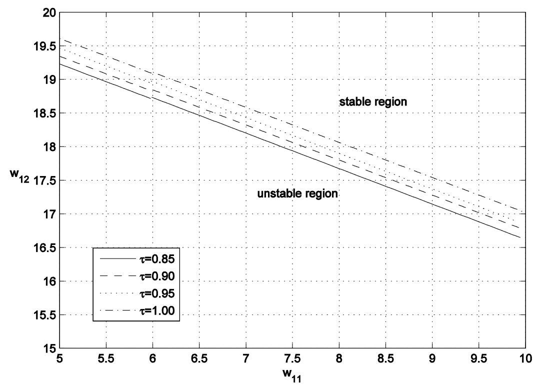

Figure 5. Stable Region Boundary

5. Conclusion

In this paper, the exponential lag synchronization has been investigated via the unidirectional linear coupling approach. Some more comprehensive criteria for the exponential synchronization of a class of chaotic delayed neural networks with impulsive effects are derived. An illustrative example, along with numerical simulations is presented to prove the effectiveness and feasibility of the developed method. The relationships between the rate of exponential synchronization, the range of gain matrix parameters and time delay parameter are shown by numerical calculation.

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