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A new 2-D multi-stable chaotic attractor and its MultiSim electronic circuit design

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ABSTRACT

A new multi-stable system with a double-scroll chaotic attractor is developed in this paper. Signal plots are simulated using MATLAB and multi-stability is established by showing two different coexisting double-scroll chaotic attractors for different states and same set of parameters. Using integral sliding control, synchronized chaotic attractors are achieved between drive-response chaotic attractors. A MultiSim circuit is designed for the new chaotic attractor, which is useful for practical engineering realizations.

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1. INTRODUCTION

Chaotic dynamical models with double-scroll attractors have been analyzed in the science literature [1], [2]. These attractors resemble like butterfly wings due to their double-scroll shape. Especially, the dynamical plants exhibiting multi-stability and cohappening chaotic attractors have been studied [3], [4]. Engineering fields have many utilizations of chaotic attractors [2]. Some common utilizations are enlisted such as oscillations [5], [6], vibrations [7], neuron models [8], [9], control and memristor models [10]-[12], mechanical attractors [13], [14].

In the control literature, there are many control techniques available for the control and synchronization of chaotic systems [2]. Bahoo and Poria [13] used active control method for food chain model. Mustafa *et al.* [14] used chaos-enhanced cuckoo search for economic dispatch with valve point effects. Vaidyanathan and Rasappan [15] used active control for the hybrid synchronization of hyperchaotic Qi and Lü systems. Vaidyanathan [16] used active control for stabilizing the state trajectories of a new hyperchaotic system with three quadratic nonlinearities. Medhaffar *et al.* [17] investigated the stabilization of unstable periodic orbits of continuous time chaotic systems using adaptive fuzzy controllers. Boubellouta and

Boulkroune [18] investigated the problem of chaos synchronization based on fractional-order intelligent sliding-mode control approach for a class of fractional-order chaotic optical systems with unknown dynamics and disturbances. Vaidyanathan [19] studied the global chaos synchronization of Tokamak chaotic systems with symmetric and magnetically confined plasma. Khan and Kumar [20] studied the T–S fuzzy observed based design and synchronization of chaotic and hyper-chaotic dynamical systems.

The novelty of this work is the modelling a new double-scroll chaotic attractor with interesting dynamic properties. The signal plots, dynamical properties and multi-stability with cohappening chaotic attractors are reported for the new chaotic attractor. For practical realizations, an electronic circuit is immensely useful after the modelling of a new chaotic attractor [21]-[26]. A MultiSim electronic circuit model of the new chaotic attractor is carried out and a good match between the MultiSim circuit outputs and the MATLAB signal plots has been found.

2. A NEW DOUBLE-SCROLL MULTI-STABLE CHAOTIC ATTRACTOR

We first give the dynamics of a new system described as follows:

$$\begin{cases} \dot{p}_1 = \alpha(p_2 - p_1) + p_2 p_3 \\ \dot{p}_2 = \beta p_2 - p_1 p_3 \\ \dot{p}_3 = p_1 p_2 - \gamma p_3 + \delta | p_2 | \end{cases} \tag{1}$$

We note that $\Lambda = (\alpha, \beta, \gamma, \delta)$ is the parameter and $P = (p_1, p_2, p_3)$ is the phase vector.
Using Wolf's approach [27], we will show that the model (1) will exhibit a chaotic attractor for

$$\Lambda = (\alpha, \beta, \gamma, \delta) = (40, 26, 5, 0.2). \tag{2}$$

For MATLAB simulations, the initial phase vector is chosen as $P(0) = (0.1, 0.3, 0.2)$.
Then the Lyapunov indices of (1) are estimated using Wolf's approach [27] as follows:

$$LE_1 = 3.9125,\ LE_2 = 0,\ LE_3 = -22.9125 \tag{3}$$

Using (3), it is concluded that the model (1) has chaoticity and dissipativity.
The double-scroll attractor of the model (1) is simulated in various planes in Figure 1.

The balance points of the new double-scroll attractor (1) for $(\alpha, \beta, \gamma, \delta) = (40, 26, 5, 0.2)$ are calculated as below:

$$P_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ P_1 = \begin{bmatrix} 11.3022 \\ 7.8017 \\ 17.9473 \end{bmatrix},\ P_2 = \begin{bmatrix} -11.3022 \\ -7.8017 \\ 17.9473 \end{bmatrix} \tag{4}$$

By finding spectral values of the linearization matrices of the double-scroll system (1), it can be ascertained that the balance point P_0 is a saddle point, and P_1, P_2 are saddle-foci.

We next demonstrate that the new double-scroll system (1) has cohappening chaotic attractors.

When selecting $(\alpha, \beta, \gamma, \delta) = (40, 26, 5, 0.2)$, and the initial phase vectors $P_0 = (0.1, 0.3, 0.2)$ (blue) and $Q_0 = (-0.5, -0.3, -0.5)$ (red), the new double-scroll chaotic attractor (1) depicts cohappening chaotic attractor (blue) and chaotic attractor (red) as plotted in Figure 2.

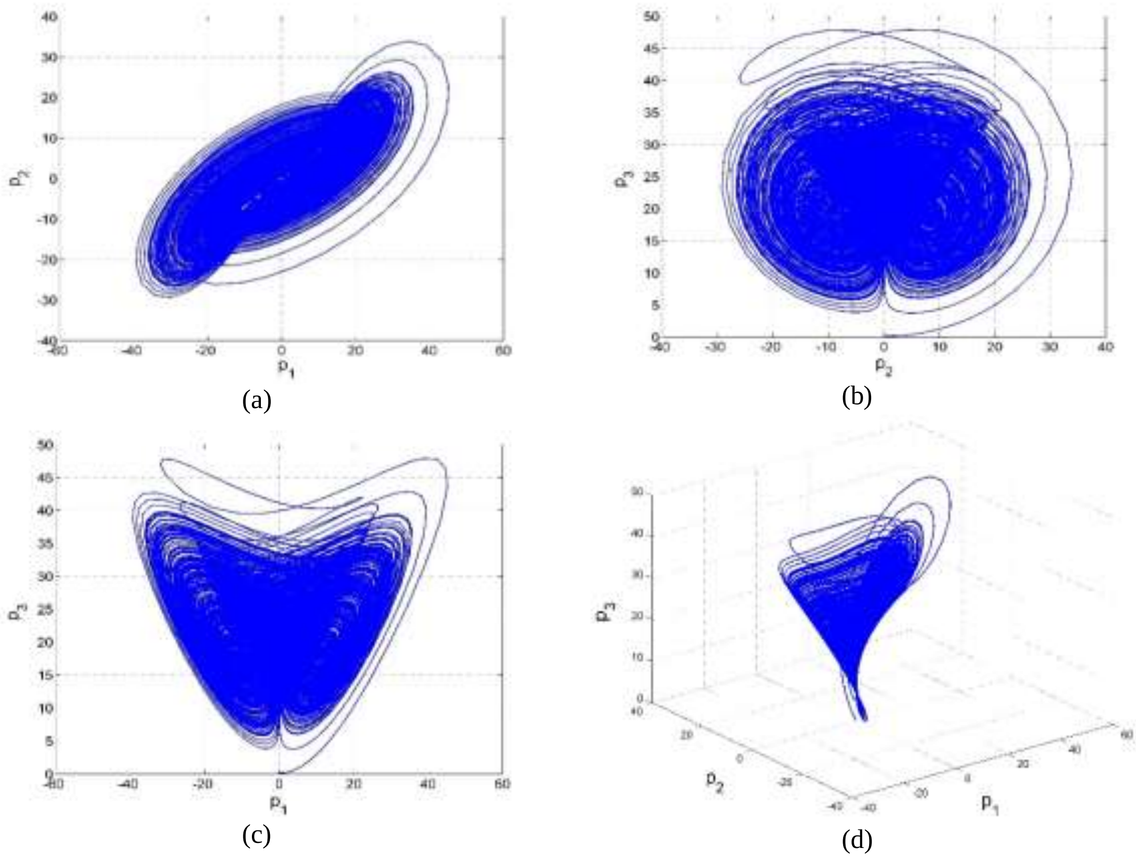

Figure 1. MATLAB phase plots showing double-scroll chaotic attractor of the model (1)

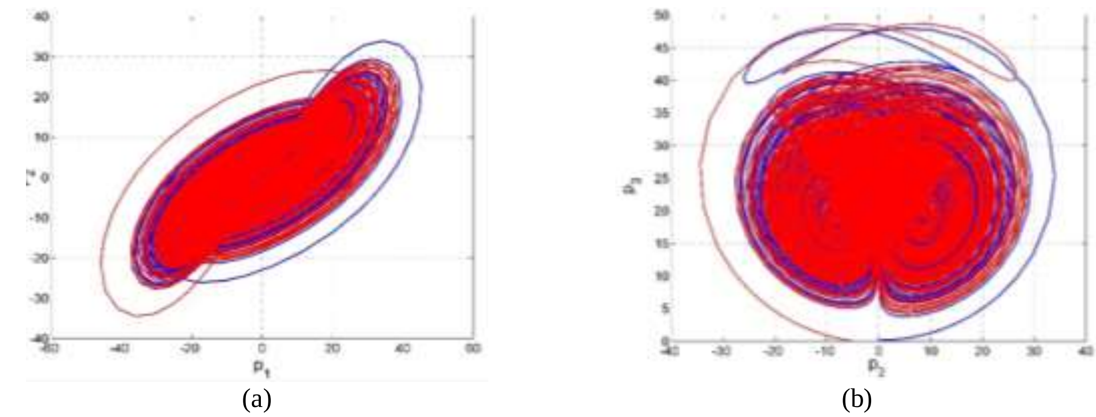

Figure 2. Multi-stability of the new double-scroll attractor (1): Cohappening chaotic attractors

3. INTEGRAL SLIDING CONTROL DESIGN FOR THE PHASE SYNCHRONIZATION OF DOUBLE-SCROLL CHAOTIC ATTRACTORS

For the phase synchronization of double-scroll chaotic attractor, we consider a pair of drive-response chaotic attractors listed as follows.

$$\begin{cases} \dot{p}_1 = \alpha(p_2 - p_1) + p_2 p_3 \\ \dot{p}_2 = \beta p_2 - p_1 p_3 \\ \dot{p}_3 = p_1 p_2 - \gamma p_3 + \delta \, | \, p_2 \, | \end{cases} \tag{5}$$

$$\begin{cases} \dot{q}_1 = \alpha(q_2 - q_1) + q_2 q_3 + v_1 \\ \dot{q}_2 = \beta q_2 - q_1 q_3 + v_2 \\ \dot{q}_3 = q_1 q_2 - \gamma q_3 + \delta |q_2| + v_3 \end{cases} \tag{6}$$

The phase synchronization error between (5) and (6) can be defined as below:

$$\begin{cases} \varepsilon_1 = q_1 - p_1 \\ \varepsilon_2 = q_2 - p_2 \\ \varepsilon_3 = q_3 - p_3 \end{cases} \tag{7}$$

A simple calculation pinpoints the dynamics of the phase synchronization error as below:

$$\begin{cases} \dot{\varepsilon}_1 = \alpha(\varepsilon_2 - \varepsilon_1) + q_2 q_3 - p_2 p_3 + v_1 \\ \dot{\varepsilon}_2 = \beta \varepsilon_2 - q_1 q_3 + p_1 p_3 + v_2 \\ \dot{\varepsilon}_3 = -\gamma \varepsilon_3 + q_1 q_2 - p_1 p_2 + \delta\left(|q_2| - |p_2|\right) + v_3 \end{cases} \tag{8}$$

We define the integral sliding surface associated with each error variable as below:

$$\begin{cases} z_1 = \left[\dfrac{d}{dt} + \psi_1\right]\left[\int_0^t \varepsilon_1(\tau) d\tau\right] = \varepsilon_1 + \psi_1 \int_0^t z_1(\tau) d\tau \\ z_2 = \left[\dfrac{d}{dt} + \psi_2\right]\left[\int_0^t \varepsilon_2(\tau) d\tau\right] = \varepsilon_2 + \psi_2 \int_0^t z_2(\tau) d\tau \\ z_3 = \left[\dfrac{d}{dt} + \psi_3\right]\left[\int_0^t \varepsilon_3(\tau) d\tau\right] = \varepsilon_3 + \psi_3 \int_0^t z_3(\tau) d\tau \end{cases} \tag{9}$$

Taking time-derivative of all the of (9), we obtain as below:

$$\begin{cases} \dot{z}_1 = \dot{\varepsilon}_1 + \psi_1 \varepsilon_1 \\ \dot{z}_2 = \dot{\varepsilon}_2 + \psi_2 \varepsilon_2 \\ \dot{z}_3 = \dot{\varepsilon}_3 + \psi_3 \varepsilon_3 \end{cases} \tag{10}$$

We take ψ_1, ψ_2, ψ_3 as positive constants.
Next, we set the dynamics of the sliding variables as follows:

$$\begin{cases} \dot{z}_1 = -\theta_1 \operatorname{sgn}(z_1) - \mu_1 z_1 \\ \dot{z}_2 = -\theta_2 \operatorname{sgn}(z_2) - \mu_2 z_2 \\ \dot{z}_3 = -\theta_3 \operatorname{sgn}(z_3) - \mu_3 z_3 \end{cases} \tag{11}$$

From (10) and (11), we deduce the following:

$$\begin{cases} \dot{e}_1 + \lambda_1 e_1 = -\varphi_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ \dot{e}_2 + \lambda_2 e_2 = -\varphi_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ \dot{e}_3 + \lambda_3 e_3 = -\varphi_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{12}$$

Combining (8) and (12), we obtain the following:

$$\begin{cases} \alpha(\varepsilon_2 - \varepsilon_1) + q_2 q_3 - p_2 p_3 + v_1 + \psi_1 \varepsilon_1 = -\theta_1 \operatorname{sgn}(z_1) - \mu_1 z_1 \\ \beta \varepsilon_2 - q_1 q_3 + p_1 p_3 + v_2 + \psi_2 \varepsilon_2 = -\theta_2 \operatorname{sgn}(z_2) - \mu_2 z_2 \\ -\gamma \varepsilon_3 + q_1 q_2 - p_1 p_2 + \delta\left(|q_2| - |p_2|\right) + v_3 + \psi_3 e_3 = -\theta_3 \operatorname{sgn}(z_3) - \mu_3 z_3 \end{cases} \tag{13}$$

The integral sliding controls are deduced from (13) as below:

$$\begin{cases} v_1 = -\alpha(\varepsilon_2 - \varepsilon_1) - q_2 q_3 + p_2 p_3 - \psi_1 \varepsilon_1 - \theta_1 \operatorname{sgn}(z_1) - \mu_1 z_1 \\ v_2 = -\beta e_2 + q_1 q_3 - p_1 p_3 - \psi_2 \varepsilon_2 - \theta_2 \operatorname{sgn}(z_2) - \mu_2 z_2 \\ v_3 = \gamma e_3 - q_1 q_2 + p_1 p_2 - \delta\left(|q_2| - |p_2|\right) - \psi_3 \varepsilon_3 - \theta_3 \operatorname{sgn}(z_3) - \mu_3 z_3 \end{cases} \tag{14}$$

Theorem 1. The integral sliding control law defined by (14) achieves the global phase chaos synchronization between the new double-scroll attractors (5) and (6), where the constants ψ_i, θ_i, μ_i, $(i = 1,2,3)$ are all positive.

Proof. First, as a positive definite Liapunov function candidate, we choose the function

$$W(z_1, z_2, z_3) = 0.5\left(z_1^2 + z_2^2 + z_3^2\right) \tag{15}$$

We calculate the time-derivative of W as below:

$$\dot{W} = z_1 \dot{z}_1 + z_2 \dot{z}_2 + z_3 \dot{z}_3 \tag{16}$$

Combining (11) and (16), we get

$$\dot{W} = z_1[-\theta_1 \operatorname{sgn}(z_1) - \mu_1 z_1] + z_2[-\theta_2 \operatorname{sgn}(z_2) - \mu_2 z_2] + z_3[-\theta_3 \operatorname{sgn}(z_3) - \mu_3 z_3] \tag{17}$$

Simplifying (17), we obtain

$$\dot{W} = -\theta_1 |z_1| - \mu_1 z_1^2 - \theta_2 |z_2| - \mu_2 z_2^2 - \theta_3 |z_3| - \mu_3 z_3^2 \tag{18}$$

Since $\theta_1, \theta_2, \theta_3 > 0$ and $\mu_1, \mu_2, \mu_3 > 0$, we see that $\dot{W}$ is a negative definite function.

From Liapunov stability theory [28], we find $(z_1(t), z_2(t), z_3(t)) \to (0,0,0)$ as $t \to \infty$.

Hence, we observe that $(\varepsilon_1(t), \varepsilon_2(t), \varepsilon_3(t)) \to (0,0,0)$ as $t \to \infty$. ■

For MATLAB simulations, we assume the parameter vector as in the chaotic case, viz. $(\alpha, \beta, \gamma, \delta) = (40, 26, 5, 0.2)$. We also assume the gains as $\psi_i = 0.1$, $\theta_i = 0.1$, and $\mu_i = 20$ for $i = 1,2,3$.

The initial state of the drive system (5) is picked as $(p_1(0), p_2(0), p_3(0)) = (3, -0.5, 2)$ and the initial state of the response system (6) is picked as $(q_1(0), q_2(0), q_3(0)) = (1.5, 0.9, 4.2)$.

Figure 3 shows the phase synchronization error between systems (5) and (6).

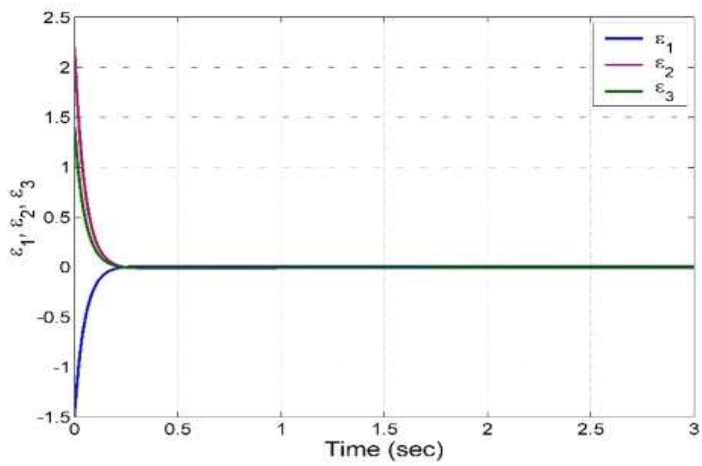

Figure 3. The phase synchronization error between the systems (5) and (6)

4. MULTISIM CIRCUIT DESIGN OF THE NEW DOUBLE-SCROLL ATTRACTOR

The MultiSim electronic circuit of the new double-scroll attractor (1) is realized by using off-the-shelf components such as resistors, capacitors, operational amplifiers and analog multipliers. The phases p_1, p_2, p_3 of the double-scroll attractor (1) are the voltages across the capacitors C_1, C_2 and C_3, respectively. The electronic circuit of the new double-scroll attractor is realized in MultiSim by 19 resistors, 8 operational amplifiers (TL082CD), 3 multipliers (AD633JN), 2 diodes (1N4148) and 3 capacitors. By the use of Kirchhoff's circuit laws into the circuit in Figure 4, its circuital equations are obtained as (19):

$$\begin{cases} \dot{p}_1 = \dfrac{1}{C_1R_1}p_2 - \dfrac{1}{C_1R_2}p_1 + \dfrac{1}{10C_1R_3}p_2p_3 \\ \dot{p}_2 = \dfrac{1}{C_2R_4}p_2 - \dfrac{1}{10C_2R_5}p_1p_3 \\ \dot{p}_3 = \dfrac{1}{10C_3R_6}p_1p_2 - \dfrac{1}{C_3R_7}p_3 + \dfrac{1}{C_3R_8}|\,p_2\,| \end{cases} \tag{19}$$

We selected $R_1 = R_2 = 10$ kΩ, $R_3 = R_5 = R_6 = 40$ kΩ, $R_4 = 15.384$ kΩ, $R_7 = 80$ kΩ, $R_8 = 2$ MΩ, $R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = 100$ kΩ, $C_1 = C_2 = C_3 = 3.2$ nF.

Figures 5-7 with MultiSim outputs of the double-scroll chaotic attractor (19) exhibit a good match with the MATLAB outputs of the double-scroll chaotic attractor (1) shown in Figure 1.

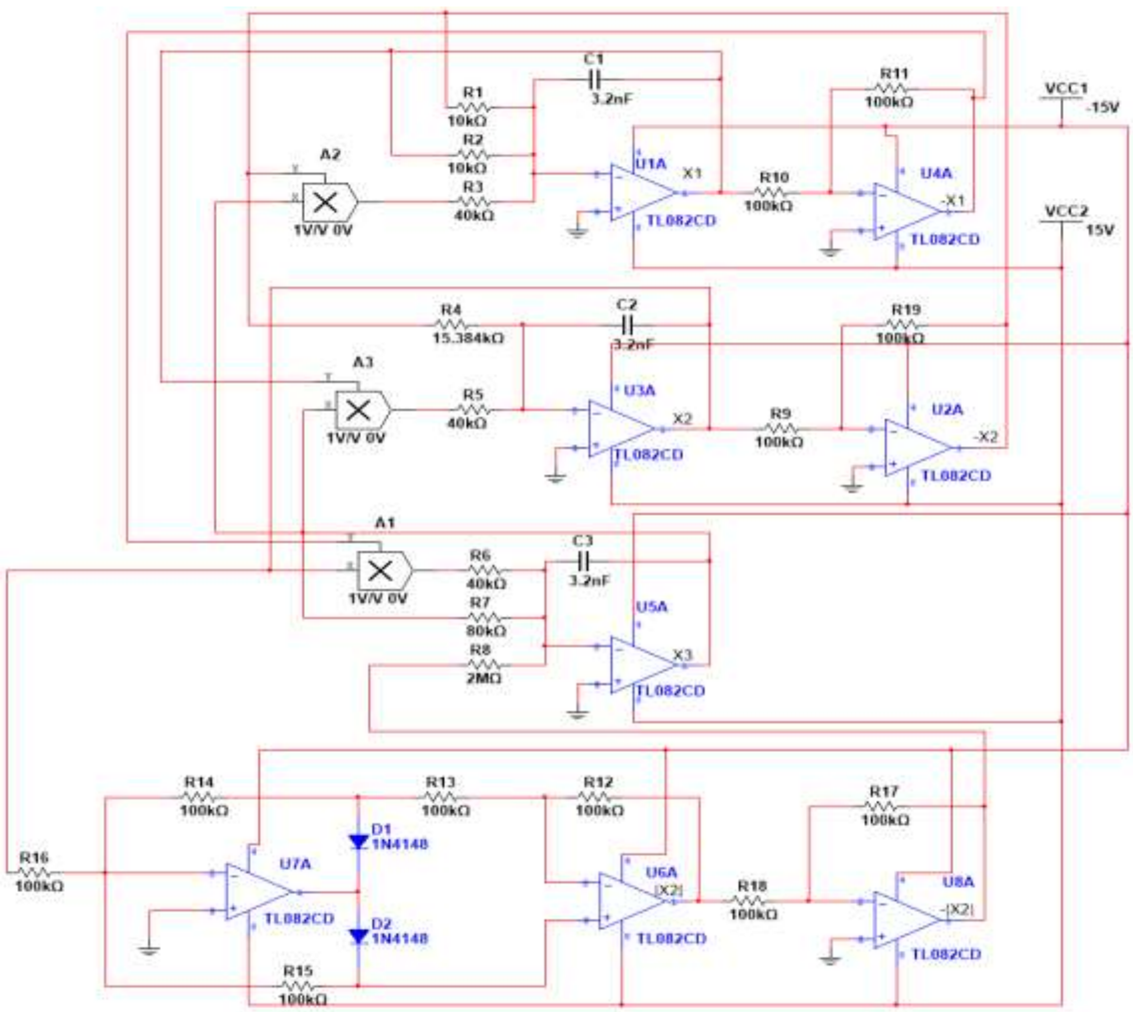

Figure 4. The circuit schematic of the double-scroll chaotic attractor (19) (Note: p_1, p_2, $p_3 = x_1$, x_2, x_3)

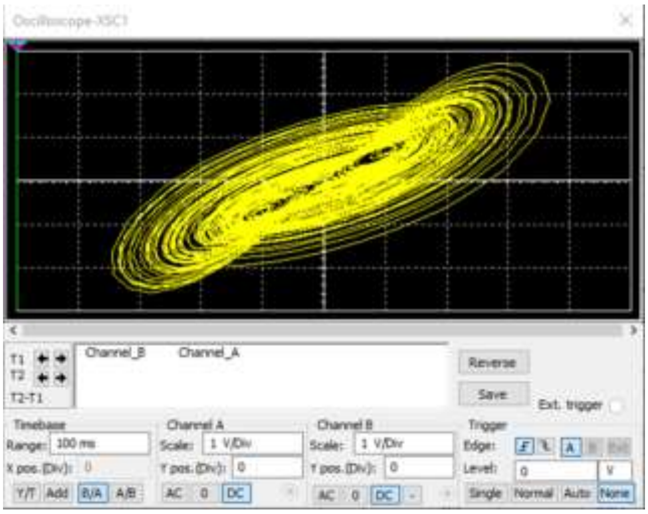

Figure 5. MultiSim output of the double-scroll circuit (19) in (p_1, p_2) – plane

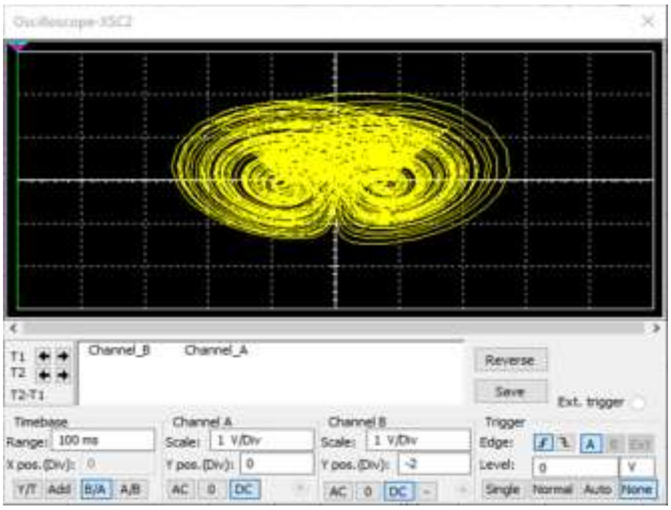

Figure 6. MultiSim output of the double-scroll circuit (19) in (p_2, p_3) – plane

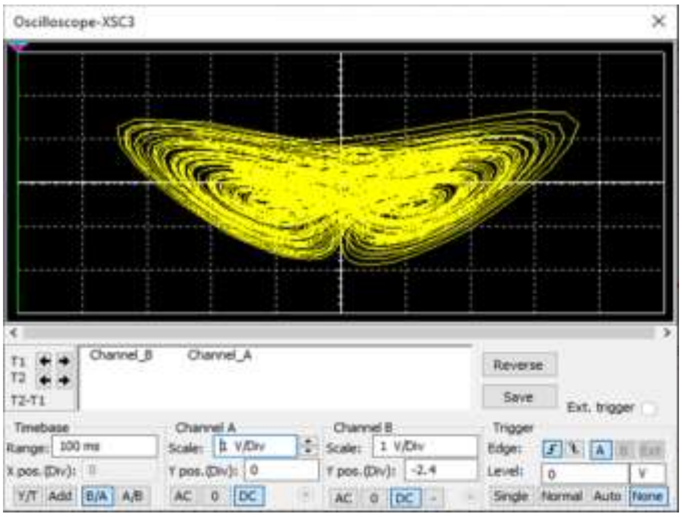

Figure 7. MultiSim output of the chaotic circuit (19) in (p_1, p_3) – plane

5. CONCLUSION

In this paper, a new multi-stable system with a double-scroll chaotic attractor is developed and detailed. Signal plots were simulated using MATLAB and multi-stability was established by showing two different coexisting double-scroll chaotic attractors for different states and same set of parameters. Using integral sliding control, synchronized chaotic attractors are achieved between drive-response chaotic attractors. A MultiSim electronic circuit was designed for the new double-scroll attractor, which is useful for practical engineering realizations.

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