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A note on complex fuzzy subfield

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ABSTRACT

In this paper, we introduce idea of complex fuzzy subfield and discuss its various algebraic aspects. We prove that every complex fuzzy subfield generate two fuzzy fields and shows that intersection of two complex fuzzy subfields is also complex fuzzy subfields. We also present the concept of level subsets of complex fuzzy subfield and shows that level subset of complex fuzzy subfield form subfield. Furthermore, we extend this idea to define the notion of the direct product of two complex fuzzy subfields and also investigate the homomorphic image and inverse image of complex fuzzy subfield.

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1. INTRODUCTION

A field is an algebraic structure which play a significant role in number theory, algebra, and many other areas of mathematics. Fields serve as development notions in various mathematical domains. The fuzzy set theory is founded on the doctrine of concerning relative graded membership basing human mechanism of cognition as well as perception.

Zahed [1] published his maiden well acknowledged research paper about fuzzy sets in 1965. Many mathematician have applied various hybrid models of fuzzy sets and intuitionistic fuzzy sets to several algebraic structure such as non-associative ring [2, 3], and time series [4, 5]. Malik and Mordeson [6] studied fuzzy subfield and its basic properties. Moredson [7, 8] studied fuzzy field extensions and also described a link between fuzzy set and finite fields. The development about fuzzy algebraic structure may be viewed in [9-11].

Ramot et al [12] started the conception of complex fuzzy sets in 2002. The enlargement of fuzzy sets to complex fuzzy sets is comparable to the extension of real numbers to complex numbers. This idea becomes more effective for researcher and quite different from fuzzy complex numbers innovated by Buckley [13]. In 2009, the various operations of complex fuzzy sets were founded by Zhang et al [14]. Sharma [15] described α-fuzzy subgroups and discussed their several algebraic properties in 2012 Thirunavukarasu et al [16] illustrated a possible application including complex fuzzy representation of solar activity, forecasting problems, time series, signal processing application and compare the two national economies by using the concept of complex fuzzy relation. Trevijano et al [17] portrayed the annihilator of group by using the conception of fuzzy sets. Anandh and Giri [18] examined the notion of intuitionistic fuzzy subfield with respect to (T, S) norm in 2017. Makaba and Murali [19] discussed fuzzy subgroups of finite groups. Al-Tahan, Davvaz [20] introduced the concept of complex H_v subgroups and discussed various characterization of these groups. Rasuli [21] discussed Q-fuzzy subring with respect to t-norm in 2018. The Q-fuzzy subgroup in algebra was discussed in [22]. More development about fuzzy subgroup may be viewed in [23, 24]. Shafei et al [25] studied the fuzzy logic control systems for demand management in airports and energy efficiency by using 3D simulator.

We will extended the discussion of complex fuzzy sets to developing a new concept of complex fuzzy subfield by adding a second dimension in membership function of fuzzy set. Many field theory problems can be handled by complex fuzzy set. This theory will be useful for mathematician in future research work.

This paper is organized as: Section 2 contains the introductory definition of fuzzy subfield and related result which play a key role for our further discussion. In Section 3 we define π-complex fuzzy subfields, complex fuzzy subfields and also prove that level subset of complex fuzzy subfield is subfield of field F, and vice versa. In Section 4, we prove that the product of two complex fuzzy subfields is complex fuzzy subfield and develop some results of the product of two complex fuzzy subfields. The image and inverse image of complex fuzzy subfield under field homomorphism is described in Section 5.

2. PRELIMINARIES

We recall first the elementary notion of fuzzy sets and fuzzy subfield which play a key role for our further analysis.

Definition (2.1) [1]: A fuzzy set δ of a nonempty set P is a mapping

$$\delta : P \to [0, 1].$$

Definition (2.2) [6]: A fuzzy subset $\aleph$ of a field $(F, +, \cdot)$ is called a fuzzy subfield if

1. $\aleph(m - n) \geq \min\{\aleph(m), \aleph(n)\}$,
2. $\aleph(mn) \geq \min\{\aleph(m), \aleph(n)\}$
3. $\aleph(m^{-1}) \geq \aleph(m)$, for all $m, n \in F$

Definition (2.3)[12]: A complex fuzzy set A of universe of discourse P is identify with the membership function $\theta_A(m) = \eta_A(m)e^{i\varphi_A(m)}$ and is defined as

$$\theta_A : P \to \{x \in C : |x| \leq 1\}$$

This membership function receive all membership value from the unit circle of complex plane, where $i = \sqrt{-1}$, both $\eta_A(m)$ and $\varphi_A(m)$ are real valued such that $\eta_A(m) \in [0,1]$ and $\varphi_A(m) \in [0,2\pi]$. Throughout this paper we take membership function of complex fuzzy sets A and B such as $\theta_A(m) = \eta_A(m)e^{i\varphi_A(m)}$ and $\theta_B(m) = \eta_B(m)e^{i\varphi_B(m)}$, respectively.

Definition (2.4) [20] Let $A = \{(m, A(m)) : m \in P\}$ be a fuzzy subset. Then the set

$A_\pi = \{(m, A_\pi(m)) : A_\pi(m) = 2\pi A(m), m \in P\}$ is called a π-fuzzy subset.

Definition (2.5)[20]: Let A and be B CFS of P. Then

1. A complex fuzzy set A is homogeneous complex fuzzy set, if for all $m, n \in P$, we have

$\eta_A(m) \leq \eta_A(n)$ if and only if $\varphi_A(m) \leq \varphi_A(n)$

2. A complex fuzzy set A is homogeneous complex fuzzy set with B, if for all $m, n \in P$, we have $\eta_A(m) \leq \eta_B(m)$ if and only if $\varphi_A(m) \leq \varphi_B(m)$. In this paper we take complex fuzzy set as homogeneous complex fuzzy set.

Definition (2.6) [14] Let A be complex fuzzy sets of set P, for all $m \in P$. The complement of complex fuzzy set A is specified by a function

$$\theta_{A^c}(m) = \eta_{A^c}(m)e^{i\varphi_{A^c}(m)} = \{1 - \eta_A(m)\}e^{i\{2\pi - \varphi_A(m)\}}$$

Definition 2.7 [14] Let A and B two complex fuzzy sets of universe of discourse P. The Cartesian product of complex fuzzy sets A and B is defined by a function

$$\theta_{A\times B}(m,n) = \eta_{A\times B}(x,y)e^{i\varphi_{A\times B}(m,n)} = \min\{\eta_A(m), \eta_B(n)\}e^{i\min\{\varphi_A(m), \varphi_B(n)\}}$$

Theorem 2.8: Intersection of two fuzzy subfield of field F is fuzzy subfield of F.

Definition 2.9: Let $g: P_1 \to P_2$ be a homomorphism. Let A and B be two complex fuzzy set of P_1 and P_2. The image $g(A)$ and pre-image $g^{-1}(B)$ of A and B respectively and defined as

1. $g(\theta_A)(n) = \begin{cases} \sup\{\theta_A(m), \text{if } g(m) = n, g^{-1}(n) \neq \emptyset \\ 0, \text{otherwise} \end{cases}$ for all $n \in P_2$
2. $g^{-1}(\theta_A)(m) = \theta_A(g(m))$, for all $m \in P_1$

3. PROPERTIES OF COMPLEX FUZZY SUBFIELD

This section devoted the study of π-fuzzy subfield and complex fuzzy subfields. We also found that a complex fuzzy subfield generates two fuzzy subfields. We also show that level subset of complex fuzzy subfield form subfield of field. We define the concept of direct product of complex fuzzy subfield and prove that direct product of two complex fuzzy subfields is complex fuzzy subfield and investigate some fundamental properties of these fields.

Definition 3.1: Let $A_\pi = \{(m, \varphi_A(m)) : m \in F\}$ be a π-fuzzy set of field $(F, +, \cdot)$ is called π-fuzzy subfield of F, for all $m, n \in F$, if

1. $\varphi_{A_\pi}(m - n) \geq \min\{\varphi_{A_\pi}(m), \varphi_{A_\pi}(n)\}$
2. $\varphi_{A_\pi}(mn) \geq \min\{\varphi_{A_\pi}(m), \varphi_{A_\pi}(n)\}$
3. $\varphi_{A_\pi}(m^{-1}) \geq \varphi_{A_\pi}(m)$.

Theorem 3.2: Let $A = \{(m, \theta_A(m)) : m \in F\}$ be π-fuzzy set of $(F, +, \cdot)$. Then A is π-fuzzy subfield if and only if A is fuzzy subfield.

Definition 3.3: A complex fuzzy set A of field $(F, +, \cdot)$ is called complex fuzzy subfield of F if

1. $\theta_A(m - n) \geq \min\{\theta_A(m), \theta_A(n), \}$,
2. $\theta_A(mn) \geq \min\{\theta_A(m), \theta_A(n), \}$,
3. $\theta_A(m^{-1}) \geq \theta_A(m)$, for all $m, n \in F$.

Theorem 3.4: Let $A = \{(m, \theta_A(m)) : m \in F\}$ be complex fuzzy set of field F. Then A is a complex fuzzy subfield of F iff:

1. The fuzzy set $B = \{(m, \eta_A(m)) : m \in F, \eta_A(m) \in [0,1]\}$ is a fuzzy subfield.
2. The π-fuzzy set $C = \{(m, \varphi_A(m)) : m \in F, \varphi_A(m) \in [0,2\pi]\}$ is a π-fuzzy subfield.

Proof: Suppose that A be complex fuzzy subfield, for all $m, n \in F$, then we have

$$\begin{aligned} \eta_A(m-n)e^{i\varphi_A(m-n)} &= \theta_A(m-n) \\ &\geq \min\{\theta_A(m), \theta_A(n)\} \\ &= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(m)e^{i\varphi_A}(n)\} \\ &= \min\{\eta_A(m), \eta_A(n)\}e^{i\min\{\varphi_A(m), \varphi_A(n)\}} \end{aligned}$$

As A is homogeneous,

$\eta_A(m - n) \geq \min\{\eta_A(m), \eta_A(n)\}$ and $\varphi_A(m - n) \geq \min\{\varphi_A(m), \varphi_A(n)\}$

Moreover,

$$\begin{aligned} \eta_A(mn)e^{i\varphi_A(mn)} &= \theta_A(mn) \\ &\geq \min\{\theta_A(m), \theta_A(n)\} \\ &= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\} \\ &= \min\{\eta_A(m), \eta_A(n)\}\, e^{i\min\{\varphi_A(m), \varphi_A(n)\}} \end{aligned}$$

Implies that $\eta_A(mn) \geq \min\{\eta_A(m), \eta_A(n)\}$ and $\varphi_A(mn) \geq \min\{\varphi_A(m), \varphi_A(n)\}$

Moreover,

$$\eta_A(m^{-1})e^{i\varphi_A(m^{-1})} = \theta_A(m^{-1}) \geq \theta_A(m) = \eta_A(m)e^{i\varphi_A(m)}$$

Implies that $\eta_A(m^{-1}) \geq \eta_A(m)$ and $\varphi_A(m^{-1}) \geq \varphi_A(m)$.

Hence B and C are fuzzy subfield and π-fuzzy subfield,respectively.

Conversely, suppose that B and C is fuzzy subfield and π-fuzzy subfield. Then we have

$\eta_A(m - n) \geq \min\{\eta_A(m), \eta_A(n)\}$ and $\varphi_A(m - n) \geq \min\{\varphi_A(m), \varphi_A(n)\}$,

$\eta_A(mn) \geq \min\{\eta_A(m), \varphi_A(n)\}$ and $\varphi_A(mn) \geq \min\{\varphi_A(m), \varphi_A(n)\}$,

$\eta_A(m^{-1}) \geq \eta_A(m)$ and $\varphi_A(m^{-1}) \geq \varphi_A(m)$.

As A is homogeneous,

$$\begin{aligned} \text{So, } \theta_A(m-n) &= \eta_A(m-n)e^{i\varphi_A(m-n)} \\ &\geq \min\{\eta_A(m), \eta_A(n)\}e^{i\min\varphi_A(m), \varphi_A(n)}\} \\ &= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\} \end{aligned}$$

$$\theta_A(m-n) \geq \min\{\theta_A(m), \theta_A(n)\}$$

Moreover, $\theta_A(mn) = \eta_A(mn)e^{i\varphi_A(mn)}$
$\geq \min\{\eta_A(m), \eta_A(n)\}\, e^{imin\{\varphi_A(m),\varphi_A(n)\}}$
$= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\}$
$= \min\{\theta_A(m), \theta_A(n)\}$
Also, $\theta_A(m^{-1}) = \eta_A(m^{-1})e^{i\varphi_A(m^{-1})} \geq \eta_A(m)e^{i\varphi_A(m)} = \theta_A(m)$.
Hence, A is complex fuzzy subfield.

Theorem 3.5: Intersection of two complex fuzzy subfield of field $(F, +,\cdot)$ is also complex fuzzy subfield of $(F, +, .)$.
Proof: Let $A = \{(m, \theta_A(m)): m \in F\}$ and $B = \{(m, \theta_B(m)): m \in F\}$ be two complex fuzzy subfield of F. Note that, $\eta_A(m)$ and $\varphi_A(m)$ are fuzzy subfield and π-fuzzy subfield. From theorem (3.2) and (2.9), we have $\eta_{A\cap B}(m)$ and $\varphi_{A\cap B}(m)$ are fuzzy subfield and π-fuzzy subfield.
Consider, $\theta_{A\cap B}(m-n) = \eta_{A\cap B}(m-n)e^{i\varphi_{A\cap B}(m-n)}$
$\geq \min\{\eta_{A\cap B}(m), \eta_{A\cap B}(n)\}\, e^{\mathrm{imin}\{\varphi_{A\cap B}(m),\varphi_{A\cap B}(n)\}}$
$= \min\{\eta_{A\cap B}(m)e^{i\varphi_{A\cap B}(m)}, \eta_{A\cap B}(n)e^{i\varphi_{A\cap B}(n)}\}$
$\theta_{A\cap B}(m-n) \geq \min\{\theta_{A\cap B}(m), \theta_{A\cap B}(n)\}$.
Moreover, $\theta_{A\cap B}(mn) = \eta_{A\cap B}(mn)e^{i\varphi_{A\cap B}(mn)}$
$\geq \min\{\eta_{A\cap B}(m), \eta_{A\cap B}(n)\}\, e^{\mathrm{imin}\{\varphi_{A\cap B}(m),\varphi_{A\cap B}(n)\}}$
$= \min\{\eta_{A\cap B}(m)e^{i\varphi_{A\cap B}(m)}, \eta_{A\cap B}(n)e^{i\varphi_{A\cap B}(n)}\}$
Therefore, $\theta_{A\cap B}(mn) \geq \min\{\theta_{A\cap B}(m), \theta_{A\cap B}(n)\}$.
Thus, $\theta_{A\cap B}(m^{-1}) = \eta_{A\cap B}(m^{-1})e^{i\varphi_{A\cap B}(m^{-1})} \geq \eta_{A\cap B}(m)e^{i\varphi_{A\cap B}(m)} = \theta_{A\cap B}(m)$.
Hence, proved our claim.

Theorem 3.6: If $A = \{(m, \theta_A(m)): m \in F\}$ is a complex fuzzy subfield of field F, for all $m \in F$. Then
1. $\eta_A(0) \geq \eta_A(m)$, $\varphi_A(0) \geq \varphi_A(m)$,
2. $\eta_A(1) \geq \eta_A(m)$, $\varphi_A(1) \geq \varphi_A(m)$,

where 0 and 1 is identity element of F.
Proof: (1). Consider, $\eta_A(0)e^{i\varphi_A(0)} = \eta_A(mm^{-1})e^{i\varphi_A(mm^{-1})}$
$= \theta_A(mm^{-1})$
$\geq \min\{\theta_A(m), \theta_A(m)\}$
$= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(m)e^{i\varphi_A(m)}\}$
$\geq \min\{\eta_A(m), \eta_A(m)\}\, e^{\min\{\varphi_A(m),\varphi_A(m)\}}$
$\eta_A(0)e^{i\varphi_A(0)} \geq \eta_A(m)e^{i\varphi_A(m)}$
As A is homogeneous
Thus, $\eta_A(0) \geq \eta_A(m)$, and $\varphi_A(0) \geq \varphi_A(m)$.
Similarly, we can prove that the second part of this theorem.

Theorem 3.7: Let $A = \{(m, \theta_A(m)): m \in F\}$ be complex fuzzy set of field F. Then the following are equivalent:
1. A is complex fuzzy subfield of F.
2. A^c is complex fuzzy subfield of F.

Proof: $(1) \Rightarrow (2)$
Suppose A is complex fuzzy subfield of F. Then we have
$\theta_A(m-n) \geq \min\{\theta_A(m), \theta_A(n)\}$
$= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\}$
$= \min\{\eta_A(m), \eta_A(n)\}e^{\mathrm{imin}\{\varphi_A(m),\varphi_A(n)\}}$
$\theta_{A^c}(m-n) \leq (1 - \min\{\eta_A(m), \eta_A(n)\})e^{\mathrm{i}(2\pi - \min\{\varphi_A(m),\varphi_A(n)\})}$
$= \max\{1-\eta_A(m), 1-\eta_A(n)\}\, e^{imin\{2\pi-\varphi_A(m),2\pi-\varphi_A(n)\}}$
$= \max\{\eta_{A^c}(m), \eta_{A^c}(n)\}\, e^{imin\{\varphi_{A^c}(m),\varphi_{A^c}(n)}$
$= \max\{\eta_{A^c}(m)e^{i\varphi_{A^c}(m)}, \eta_{A^c}(n)e^{i\varphi_{A^c}(n)}\}$.
Moreover,
$\theta_A(mn) \geq \min\{\theta_A(m), \theta_A(n)\}$
$= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\}$
$= \min\{\eta_A(m), \eta_A(n)\}e^{\mathrm{imin}\{\varphi_A(m),\varphi_A(n)\}}$

$$\theta_{A^c}(mn) \leq (1 - \min\{\eta_A(m), \eta_A(n)\})e^{\mathrm{i}(2\pi - \min\{\varphi_A(m), \varphi_A(n)\})}$$
$$= \max\{1 - \eta_A(m), 1 - \eta_A(n)\}\, e^{imin\{2\pi - \varphi_A(m), 2\pi - \varphi_A(n)\}}$$
$$= \max\{\eta_{A^c}(m), \eta_{A^c}(n)\}\, e^{imin\{\varphi_{A^c}(m), \varphi_{A^c}(n)}$$
$$= \max\{\eta_{A^c}(m)e^{i\varphi_{A^c}(m)}, \eta_{A^c}(n)e^{i\varphi_{A^c}(n)}\}$$

As a result, $\theta_{A^c}(mn) \leq \max\{\theta_{A^c}(m), \theta_{A^c}(n)\}$.
Further, $\theta_A(m^{-1}) \geq \theta_A(m) = \eta_A(m)e^{i\varphi_A(m)}$
Implies that, $\theta_{A^c}(m^{-1}) \leq (1 - \eta_A(m))e^{i(2\pi - \varphi_A(m))} = \eta_{A^c}(m)e^{i\varphi_{A^c}(m)} = \theta_{A^c}(m)$
Conversely, assume that A^c is a complex fuzzy subfield of F. Then we have

$$\theta_{A^c}(m - n) \leq \max\{\theta_{A^c}(m), \theta_{A^c}(n)\}$$
$$= \max\{\eta_{A^c}(m)e^{i\varphi_{A^c}(m)}, \eta_{A^c}(n)e^{i\varphi_{A^c}(n)}\}$$
$$= \max\{\eta_{A^c}(m), \eta_{A^c}(n)\}\, e^{imin\{\varphi_{A^c}(m), \varphi_{A^c}(n)}$$
$$= \max\{1 - \eta_A(m), 1 - \eta_A(n)\}\, e^{imin\{2\pi - \varphi_A(m), 2\pi - \varphi_A(n)\}}$$
$$\theta_{A^c}(m - n) \geq (1 - min\{\eta_A(m), \eta_A(n)\})e^{\mathrm{i}(2\pi - \min\{\varphi_A(m), \varphi_A(n)\})}$$
$$\theta_A(m - n) \geq \min\{\eta_A(m), \eta_A(n)\}e^{\mathrm{imin}\{\varphi_A(m), \varphi_A(n)\}}$$
$$= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\}$$
$$\theta_A(m - n) \geq \min\{\theta_A(m), \theta_A(n)\}$$

Moreover,$\theta_{A^c}(mn) \leq \max\{\theta_{A^c}(m), \theta_{A^c}(n)\}$

$$= \max\{\eta_{A^c}(m)e^{i\varphi_{A^c}(m)}, \eta_{A^c}(n)e^{i\varphi_{A^c}(n)}\}$$
$$= \max\{\eta_{A^c}(m), \eta_{A^c}(n)\}\, e^{imin\{\varphi_{A^c}(m), \varphi_{A^c}(n)}$$
$$= \max\{1 - \eta_A(m), 1 - \eta_A(n)\}\, e^{imin\{2\pi - \varphi_A(m), 2\pi - \varphi_A(n)\}}$$
$$\theta_{A^c}(mn) \geq (1 - min\{\eta_A(m), \eta_A(n)\})e^{\mathrm{i}(2\pi - \min\{\varphi_A(m), \varphi_A(n)\})}$$
$$\theta_A(mn) \geq \min\{\eta_A(m), \eta_A(n)\}e^{\mathrm{imin}\{\varphi_A(m), \varphi_A(n)\}}$$
$$= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\}$$
$$\theta_A(mn) \geq \min\{\theta_A(m), \theta_A(n)\}$$

Further, $\theta_{A^c}(m^{-1}) \leq \theta_{A^c}(m) = \eta_{A^c}(m)e^{i\varphi_{A^c}(m)} = (1 - \eta_A(m))e^{i(2\pi - \varphi_A(m))}$
Implies that $\theta_A(m^{-1}) \geq \eta_A(m)e^{i\varphi_A(m)} = \theta_A(m)$
Thus conclude the proof.

Definition 3.8: Let $A = \{(m, \theta_A(m)): m \in P, \theta_A = \eta_A(m)e^{i\varphi_A(m)}\}$ be a complex fuzzy set of universe of discourse P. For $r \in [0,1]$, and $t \in [0,2\pi]$ the level subset of complex fuzzy set A is defined by

$$A_{(r,t)} = \{x \in P: \eta_A(m) \geq r, \varphi_A(n) \geq t\}$$

For $t = 0$, we obtain the lower level subset $A_r = \{m \in P: \eta_A(m) \geq r\}$ and for $t = 0$, then we obtain the lower level subset $A_t = \{m \in P: \varphi_A(m) \geq t\}$.

Theorem 3.9: Let $A = \{m, \theta_A(m): m \in F\}$ be complex fuzzy set of field F. Then A is a complex fuzzy subfield of F if and only $A_{(r,t)}$ is a subfield of field F, for all $r \in [0,1]$ and $t \in [0,2\pi]$. Where $\eta_A(0) \geq r \leq \eta_A(1), \varphi_A(0) \geq t \leq \varphi_A(1)$.
Proof: Obviously $A_{(r,t)}$ is nonempty, as $0,1 \in A_{(r,t)}$
Let $m, n \in A_{(r,t)}$ be any two elements. Then

$$\eta_A(m) \geq t, \varphi_A(m) \geq t \text{ and } \eta_A(n) \geq r, \varphi_A(m) \geq t$$

Consider, $\eta_A(m - n)e^{i\varphi_A(m-n)} = \theta_A(m - n)$

$$\geq \min\{\theta_A(m), \theta_A(n)\}$$
$$= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\}$$
$$= \min\{\eta_A(m), \eta_A(n)\}e^{\mathrm{imin}\{\varphi_A(m), \varphi_A(n)\}}$$

(As A is homogeneous)

$$\eta_A(m - n) \geq \min\{\eta_A(m), \eta_A(n)\} = \min\{r, r\} = r$$
$$\varphi_A(m - n) \geq \min\{\varphi_A(m), \varphi_A(n)\} = \min\{t, t\} = t$$
$$\Rightarrow m - n \in A_{(r,t)}$$

Further, $\eta_A(mn)e^{i\varphi_A(mn)} = \theta_A(mn) \geq \min\{\theta_A(m), \theta_A(n)\}$

$$= \min\{\eta_A(m)e^{i\varphi_A(m)}, \eta_A(n)e^{i\varphi_A(n)}\}$$
$$= \min\{\eta_A(m), \eta_A(n)\}e^{\mathrm{imin}\{\varphi_A(m), \varphi_A(n)\}}$$

(As A is homogeneous)

$$\eta_A(mn) \geq \min\{\eta_A(m), \eta_A(n)\} = \min\{r, r\} = r$$
$$\varphi_A(mn) \geq \min\{\varphi_A(m), \varphi_A(n)\} = \min\{t, t\} = t$$

$$\Rightarrow mn \in A_{(r,t)}$$

Moreover,

$$\eta_A(m^{-1})e^{i\varphi_A(m^{-1})} = \theta_A(m^{-1}) \geq \theta_A(m) = \eta_A(m)e^{i\varphi_A(m)}$$

(by homogeneity)

$\eta_A(m^{-1}) \geq \eta_A(m) \geq r$, and $\varphi_A(m^{-1}) \geq \varphi_A(m) \geq t$

$$\Rightarrow m^{-1} \in A_{(r,t)}$$

Hence $A_{(r,t)}$ is subfield.

Conversely, Let $A_{(r,t)}$ is a subfield of F and let $\min\{\eta_A(m), \eta_A(n)\} = r$ and $\min\{\varphi_A(m), \varphi_A(n)\} = t$. Then we have

$$\eta_A(m) \geq r , \eta_A(n) \geq r, \text{and } \varphi_A(m) \geq t, \varphi_A(n) \geq t$$

$$\eta_A(m) \geq r, \varphi_A(m) \geq t \text{ and } \eta_A(n) \geq r, \varphi_A(n) \geq t$$

This implies that $m \in A_{(r,t)}$ $n \in A_{(r,t)}$

As $A_{(r,t)}$ is subfield. So $m - n, mn \in A_{(r,t)} \Rightarrow \eta_A(m-n) \geq r, \varphi_A(m-n) \geq t$

Implies that, $\eta_A(m-n) \geq \min\{\eta_A(m), \eta_A(n)\}$ and $\varphi_A(m-n) \geq \min\{\varphi_A(m), \varphi_A(n)\}$

A is homogeneous, then we have,

$$\theta_A(m-n) \geq \min\{\theta_A(m), \theta_A(n)\}$$

Also, we have $\eta_A(mn) \geq r, \varphi_A(mn) \geq t$

Implies that, $\eta_A(mn) \geq \min\{\eta_A(m), \eta_A(n)\}$ and $\varphi_A(mn) \geq \min\{\varphi_A(m), \varphi_A(n)\}$

A is homogeneous, then we have,

$$\theta_A(mn) \geq \min\{\theta_A(m), \theta_A(n)\}$$

Further, let $m \in F$ be any element. Let $\eta_A(m) = r$, and $\varphi_A(m) = t$.

Then, $\eta_A(m) \geq r$, and $\varphi_A(m) \geq t$ is true. Implies that $m \in A_{(r,t)}$

As $A_{(r,t)}$ is subfield. So, $m^{-1} \in (A \times B)_{((r,t))}$

$\Rightarrow \eta_A(m^{-1}) \geq r$, and $\varphi_A(m^{-1}) \geq t$

$\Rightarrow \eta_A(m^{-1}) \geq \eta_A(m)$ and $\varphi_A(m^{-1}) \geq \varphi_A(m)$

Consequently,

$$\theta_A(m^{-1}) \geq \theta_A(m).$$

Hence proved the theorem.

Definition 3.10: Let $A = \{(m, \theta_A(m)) : m \in F\}$ and $B = \{(m, \theta_{\boldsymbol{B}}(m)) : m \in F\}$ be any two π-fuzzy sets of sets F_1 and F_2 respectively. The Cartesian product of π- fuzzy sets A and B is defined as

$$A_\pi \times B_\pi(m, n) = \min\{A_\pi(m), B_\pi(n)\}, \text{for all } m \in F_1 and\ n \in F_2$$

Remark 3.11: Let $A = \{(m, \theta_A(m)) : m \in F\}$ and $B = \{(m, \theta_B(m)) : m \in F\}$ be two π- fuzzy subfields of F_1 and F_2, respectively. Then $A \times B$ is π- fuzzy subfield of $F_1 \times F_2$

Definition 3.12: Let $A = \{(m, \theta_A(m)) : m \in F\}$ and $B = \{(m, \theta_B(m)) : m \in F\}$ two complex fuzzy sets of fields F_1 and F_2. The Cartesian product of complex fuzzy sets A and B is defined by a function

$$\theta_{A\times B}(m, n) = \eta_{A\times B}(m, n)e^{i\varphi_{A\times B}(m,n)} = \min\{\eta_A(m), \eta_B(n)\}e^{i\min\{\varphi_A(m), \varphi_B(n)\}}$$

Theorem 3.13: Let $A = \{(m, \theta_A(m)) : m \in F\}$ and $B = \{(m, \theta_B(m)) : m \in F\}$ be two complex fuzzy subfield of F_1 and F_2 respectively. Then $A \times B$ is complex fuzzy subfield of $F_1 \times F_2$.

Proof: Let $m, x \in F_1$ and $n, y \in F_2$ be an elements. Then $(m, n), (x, y) \in F_1 \times F_2$.

Consider

$$\begin{aligned}
\theta_{A\times B}((m, n) - (x, y)) &= \theta_{A\times B}(m - x, n - y) \\
&= \eta_{A\times B}(m - x, n - y)e^{i\varphi_{A\times B}(m-x,n-y)} \\
&= \min\{\eta_A(m - x), \eta_B(n - y)\}\, e^{i\min\{\varphi_A(m-x),\varphi_B(n-y)\}} \\
&= \min\{\eta_A(m - x)e^{i\varphi_A(m-x)}, \eta_B(n - y)e^{i\varphi_A(n-y)}\} \\
&= \min\{\theta_A(m - x), \theta_B(n - y)\} \\
&\geq \min\{\min\{\theta_A(m), \theta_A(x)\}, \min\{\theta_B(n), \theta_B(y)\}\} \\
&= \min\{\min\{\theta_A(m), \theta_B(n)\}, \min\{\theta_A(x), \theta_B(y)\}\}
\end{aligned}$$

$$\theta_{A\times B}((m, n) - (x, y)) \geq \min\{\theta_{A\times B}(m, n), \theta_{A\times B}(x, y)\}$$

Also, $\theta_{A\times B}((m, n)(x, y)) = \theta_{A\times B}(mx, ny)$

$$\begin{aligned}
&= \eta_{A\times B}(mx, ny)e^{i\varphi_{A\times B}(mx,ny)} \\
&= \min\{\eta_A(mx), \eta_B(ny)\}\, e^{i\min\{\varphi_A(mx),\varphi_B(ny)\}}
\end{aligned}$$

$$= \min\{\eta_A(mx)e^{i\varphi_A(mx)}, \eta_B(ny)e^{i\varphi_A(ny)}\}$$
$$= \min\{\theta_A(mx), \theta_B(ny)\}$$
$$\geq \min\{\min\{\theta_A(m), \theta_A(x)\}, \min\{\theta_B(n), \theta_B(y)\}\}$$
$$= \min\{\min\{\theta_A(m), \theta_B(n)\}, \min\{\theta_A(x), \theta_B(y)\}\}$$
$$\theta_{A\times B}((m,n)(x,y)) \geq \min\{\theta_{A\times B}(m,n), \theta_{A\times B}(x,y)\}$$

Further, $\theta_{A\times B}(m^{-1},n^{-1}) = \eta_{A\times B}(m^{-1},n^{-1})e^{i\varphi_{A\times B}(m^{-1},n^{-1})}$
$$= \min\{\eta_A(m^{-1}), \eta_B(n^{-1})\}\, e^{i\min\{\varphi_A(m^{-1}),\varphi_B(n^{-1})\}}$$
$$= \min\{\eta_A(m^{-1})e^{i\varphi_A(m^{-1})}, \eta_B(n^{-1})e^{i\varphi_A(n^{-1})}\}$$
$$= \min\{\theta_A(m^{-1}), \theta_B(n^{-1})\}$$
$$\geq \min\{\theta_A(m), \theta_B(n)\}$$
Consequently,
$$\theta_{A\times B}(m^{-1},n^{-1}) \geq \theta_{A\times B}(m,n)$$
Thus conclude the proof.

Corollary 3.14: Let $A_1, A_2 \dots A_n$ be complex fuzzy subfields of $F_1, F_2, \dots F_n$ respectively. Then $A_1 \times A_2 \times \dots \times A_n$ is complex fuzzy subfields of $F_1 \times F_2 \times \dots \times F_n$.

4. HOMOMORPHISM OF COMPLEX FUZZY SUBFIELD

In this section, we define the homomorphic image and pre image of complex fuzzy subfield. We prove some results of complex fuzzy subfield under field homomorphism.

Definition 4.1: Let $g: F_1 \to F_2$ be a homomorphism from field F_1 to field F_2. Let $A = \{(m, \theta_A(m)): m \in F\}$ and $B = \{(m, \theta_B(m)): m \in F\}$ be two fuzzy sets of fields F_1 and F_2 respectively, for all $m \in F_1$ and for all $n \in F_2$ The image $g(A)$ and pre-image $g(B)$ of A and B respectively and defined as
$$g(\theta_A)(n) = \begin{cases} \min\{\theta_A(m), \text{if } g(m) = n, m \in g^{-1}(n) \neq \emptyset \\ 0, \text{otherwise} \end{cases}, n \in F_2,$$
$$g^{-1}(\theta_B)(m) = \theta_B(g(m)), m \in F_1.$$

Theorem 4.2: Let $g: F_1 \to F_2$ be homomorphism from field F_1 to field F_2. Let A be fuzzy subfield of F_1 and B be fuzzy subfield of F_2. Then $g(A)$ is fuzzy subfield of F_2 and $g^{-1}(B)$ is fuzzy subfield of F_1.

Lemma 4.3: Let $g: F_1 \to F_2$ be a homomorphism from field F_1 to field F_2. Let $A = \{(m, \theta_A(m)): m \in F_1\}$ and $B = \{(m, \theta_B(m)): m \in F_2\}$ be two complex fuzzy subfields. Then
1. $g(\theta_A)(n) = g(\eta_A)(n)e^{ig(\varphi_A)(n)}$, for all $n \in F_2$
2. $g^{-1}(\theta_B)(m) = g^{-1}(\eta_B)(m)e^{ig^{-1}(\varphi_B)(m)}$, for all $m \in F_1$

Proof: Consider
$$g(\theta_A)(n) = \min\{(\theta_A)(m), \text{if } g(m) = n\}$$
$$= \min\{(\eta_A)(m)e^{i(\varphi_A)(m)}, \text{if } g(m) = (n)\}$$
$$= \min\{(\eta_A)(m), \text{if } g(m) = n\}\, e^{i\min\{(\varphi_A)(m), \text{if } g(m)=n\}}$$
$$= g(\eta_A)(n)e^{ig(\varphi_A)(n)}$$
Hence, $g(\theta_A)(n) = g(\eta_A)(n)e^{ig(\varphi_A)(n)}$

Consider, $g^{-1}(\theta_B)(m) = \theta_B(g(m)) = \eta_B(g(m))e^{i\varphi_B(g(m))} = g^{-1}(\eta_B)(m)e^{ig^{-1}(\varphi_B)(m)}$
$$g^{-1}(\theta_B)(m) = g^{-1}(\eta_B)(m)e^{ig^{-1}(\varphi_B)(m)}.$$

Theorem 4.4: Let $g: F_1 \to F_2$ be a field homomorphism from F_1 to F_2. Let $A = \{(m, \theta_A(m)): m \in F\}$ be complex fuzzy subfield of F_1. Then $g(A)$ is complex fuzzy subfield of F_2.

Proof: Obviously, $\eta_A(m)$ and $\varphi_A(m)$ are fuzzy subfield and π-fuzzy subfield respectively. From Theorem 3.2 and Theorem 4.2 the homomorphic image of $\eta_A(m)$ and $\varphi_A(m)$ are fuzzy subfield and π-fuzzy subfield, respectively, for all $m, n \in F_2$. Then we have
$$g(\eta_B)(m-n) \geq \min\{g(\eta_B)(m), g(\eta_B)(n)\}$$
$$g(\eta_B)(mn) \geq \min\{g(\eta_B)(m), g(\eta_B)(n)\}$$
$$g(\eta_B)(m^{-1}) \geq g(\eta_B)(m)$$
$$g(\varphi_B)(m-n) \geq \min\{g(\varphi_B)(m), g(\varphi_B)(n)\}$$
$$g(\varphi_B)(mn) \geq \min\{g(\varphi_B)(m), g(\varphi_B)(n)\}$$
$$g(\varphi_B)(m^{-1}) \geq g(\varphi_B)(m)$$

Consider

$$\begin{aligned} g(\theta_A)(m-n) &= g(\eta_A)(m-n)e^{ig(\varphi_A)(m-n)}, \forall\, m,n \in F_2 \\ &\geq \min\{g(\eta_A)(m), g(\eta_A)(n)\}\, e^{i\{g(\varphi_A)(m), g(\varphi_A)(n)\}} \\ &= \min\{g(\eta_A)(m)e^{ig(\varphi_A)(m)}, g(\eta_A)(n)e^{ig(\varphi_A)(n)}\} \\ &= \min\{g(\theta_A)(m), g(\theta_A)(n)\} \end{aligned}$$

Moreover,

$$\begin{aligned} g(\theta_A)(mn) &= g(\eta_A)(mn)e^{ig(\varphi_A)(mn)}, \forall\, m,n \in F_2 \\ &\geq \min\{g(\eta_A)(m), g(\eta_A)(n)\}\, e^{i\{g(\varphi_A)(m), g(\varphi_A)(n)\}} \\ &= \min\{g(\eta_A)(m)e^{ig(\varphi_A)(m)}, g(\eta_A)(n)e^{ig(\varphi_A)(n)}\} \\ &= \min\{g(\theta_A)(m), g(\theta_A)(n)\} \end{aligned}$$

Consequently, $g(\theta_A)(mn) \geq \min\{g(\theta_A)(m), g(\theta_A)(n)\}$

Further,

$$\begin{aligned} g(\theta_A)(m^{-1}) &= g(\eta_A)(m^{-1})e^{ig(\varphi_A)(m^{-1})}, \text{for all } m \in F_2 \\ &\geq g(\eta_A)(m)e^{ig(\varphi_A)(m)} \\ &= g(\theta_A)(m) \end{aligned}$$

Thus, $g(\theta_A)(m^{-1}) \geq g(\theta_A)(m)$

This establishes the proof.

Theorem 4.5: Let $g: F_1 \to F_2$ be a homomorphism from field F_1 to field F_2. Let $B = \{(m, \theta_B(m)): m \in F\}$ be two complex fuzzy subfields of F_2. Then $g^{-1}(B)$ is complex fuzzy subfield of F_1.

Proof: Obviously, $\eta_B(m)$ and $\varphi_B(m)$ are fuzzy subfield and π-fuzzy subfield respectively. Then From Theorem 3.2 and Theorem 4.2 the inverse image of $\eta_B(m)$ and $\varphi_B(m)$ are fuzzy subfield and π-fuzzy subfield, respectively, for all $m, n \in F_1$. Then we have

$g^{-1}(\eta_B)(m-n) \geq \min\{g^{-1}(\eta_B)(m), g^{-1}(\eta_B)(n)\}$
$g^{-1}(\eta_B)(mn) \geq \min\{g^{-1}(\eta_B)(m), g^{-1}(\eta_B)(n)\}$
$g^{-1}(\eta_B)(m^{-1}) \geq g^{-1}(\eta_B)(m)$
$g^{-1}(\varphi_B)(m-n) \geq \min\{g^{-1}(\varphi_B)(m), g^{-1}(\varphi_B)(n)\}$
$g^{-1}(\varphi_B)(mn) \geq \min\{g^{-1}(\varphi_B)(m), g^{-1}(\varphi_B)(n)\}$
$g^{-1}(\varphi_B)(m^{-1}) \geq g^{-1}(\varphi_B)(m)$

Consider

$$\begin{aligned} g^{-1}(\theta_B)(m-n) &= g^{-1}(\eta_B)(m-n)e^{ig^{-1}(\varphi_B)(m-n)}, \text{for all } m,n \in F_1 \\ &\geq \min\{g^{-1}(\eta_B)(m), g^{-1}(\eta_B)(n)\}\, e^{i\{g^{-1}(\varphi_B)(m), g^{-1}(\varphi_B)(n)\}} \\ &= \min\{g^{-1}(\eta_B)(m)e^{ig^{-1}(\varphi_B)(m)}, g^{-1}(\eta_B)(n)e^{ig^{-1}(\varphi_B)(n)}\} \\ &= \min\{g^{-1}(\theta_B)(m), g^{-1}(\theta_B)(n)\} \end{aligned}$$

$$\begin{aligned} g^{-1}(\theta_B)(mn) &= g^{-1}(\eta_B)(mn)e^{ig^{-1}(\varphi_B)(mn)}, \text{for all } m,n \in F_1 \\ &\geq \min\{g^{-1}(\eta_B)(m), g^{-1}(\eta_B)(n)\}\, e^{i\{g^{-1}(\varphi_B)(m), g^{-1}(\varphi_B)(n)\}} \\ &= \min\{g^{-1}(\eta_B)(m)e^{ig^{-1}(\varphi_B)(m)}, g^{-1}(\eta_B)(n)e^{ig^{-1}(\varphi_B)(n)}\} \\ &= \min\{g^{-1}(\theta_B)(m), g^{-1}(\theta_B)(n)\} \end{aligned}$$

Therefore,

$$g^{-1}(\theta_B)(mn) \geq \min\{g^{-1}(\theta_B)(m), g^{-1}(\theta_B)(n)\}$$

Further,

$$\begin{aligned} g^{-1}(\theta_B)(m^{-1}) &= g^{-1}(\eta_B)(m^{-1})e^{ig^{-1}(\varphi_B)(m^{-1})}, \text{for all } m \in F_1 \\ &\geq g^{-1}(\eta_B)(m)\, e^{ig^{-1}(\varphi_B)(m)} \\ &= g^{-1}(\theta_B)(m) \end{aligned}$$

Consequently, $g^{-1}(\theta_B)(m^{-1}) = g^{-1}(\theta_B)(m)$

This conclude the proof.

5. CONCLUSION

Up to this point we have introduced the π-fuzzy subfield, complex fuzzy subfield and lower level subset of complex fuzzy subfield and have proved that level subset of complex fuzzy subfield is subfield of field F. We have also defined product of two complex fuzzy subfields and have proved that the product two complex fuzzy subfields is also complex fuzzy subfield and discussed various algebraic properties. Further, we have studied the behavior of homomorphic image and inverse image of these complex fuzzy subfields.

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REFERENCES

[1] L.A, Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338-353, 1965.

[2] N. Kausar, B. U. Islam, M. Y. Javid, S. A. Ahmed, and U. Ijaz, "Characterization of non-associative rings by the properties of their fuzzy ideals," *Journal of Taibah University for Science*, vol. 13, no. 1, pp. 820-833, 2019.

[3] N. Kausar, "Direct product of finite intuitionistic fuzzy normal subrings over non-associative rings," *European Journal of Pure and Applied Mathematics*, vol. 12, no. 2, pp. 622-648, 2019.

[4] M. N. Alemu, "Fuzzy model for chaotic time series prediction," *International Journal of Innovative Computing, Information and Control*, vol. 14, no. 5, pp. 1767-1786, 2018.

[5] D. Shan, W. Lu and J. Yang, "The data-driven fuzzy cognitive map model and its application to prediction of time series," *International Journal of Innovative Computing, Information and Control*, vol. 14, no. 5, pp.1583-1602, 2018.

[6] D. S. Malik and J. N. Mordeson, "Fuzzy subfields," *Fuzzy Sets and Systems*, vol. 37, no. 3, pp. 383-388, 1990.

[7] J. N. Mordeson, "Fuzzy algebraic fields extensions," *Fuzzy Sets and Systems*, vol. 45, no. 3, pp. 359-365, 1992.

[8] J. N. Mordeson, "Fuzzy subfields of finite fields," *Fuzzy sets and Systems*, vol. 52, no. 1, pp. 93-96, 1992.

[9] A. Rosenfeld, "Fuzzy groups," *Journal of Mathematical Analysis and Application*, vol. 35, no. 3 pp. 512−517, 1971.

[10] W. J. Liu, "Fuzzy invariant subgroups and fuzzy ideals," *Fuzzy Sets And Systems*, vol. 8, no. 2, pp. 133−139, 1982.

[11] V. N. Dixit, R. Kumar, N. Ajmal, "Level subgroups and union of fuzzy subgroup," *Fuzzy Sets and Systems*, vol. 37, no. 3, pp. 359-371, 1990.

[12] D. Ramot, R. Milo, M. Friedman, A. Kandel, "Complex fuzzy sets", *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 2, pp.450-461, 2002.

[13] J. J. Buckley, "Fuzzy complex numbers," *Fuzzy Sets and Systems*, vol. 33, no. 3, pp. 333-345, 1989.

[14] G. Q. Zhang, T. S. Dillon, K. Y. Cai, J. Ma, J. Lu, "Operation properties and δ-equalities of complex fuzzy sets," *International Journal of Approximate Reasoning*, vol. 50, no. 8, pp. 1227-1249, 2009.

[15] P. K. Sharma," α-Fuzzy subgroups", *International Journal of Fuzzy Mathematics and Systems*, vol. 3, no. 1, pp. 47-59, 2013.

[16] P. Thirunavukarasu, R. Suresh, P. Thamilmani, "Application of complex fuzzy sets," *JP Journal of Applied Mathematics*, vol. 6, no. 1, pp. 5-22, 2013.

[17] S. A. Trevijano, M. J. Chasco and J. Elorza, "The annihilator of fuzzy subgroups," *Fuzzy Sets Syst.*, vol. 369, pp. 122-131, Aug. 2019.

[18] B. Anandh, and R. Giri, "Intuitionistic fuzzy subfields of a field with respect to (T,S)-norm," *Advanced in Fuzzy Mathematics*, vol. 12, no. 4, pp. 1007-1015, 2017.

[19] B. B. Makamba, V. Murali, "A class of fuzzy subgroups of finite reflection groups," *Journal of Intelligent and Fuzzy Systems*, vol. 33, no. 2, pp.979-983,2017.

[20] Al-Tahan, B. Davvaz, "Generalized complex fuzzy and anti-fuzzy H_v-subgroups of a H_v-subgroup," *International Journal of Analysis and Applications*, vol. 16, no. 5, pp.628-642, 2018.

[21] R. Rasuli, "Characterization of Q-fuzzy subrings (Anti Q-fuzzy subrings) with respect to a T-norm (T-conorm)," *Journal of Information and Optimization Sciences*, vol. 39, no. 4, pp. 827-837,2018.

[22] Dr. R. Jahir Hussain, "A Review On Q-fuzzy subgroup in Algebra," *International Journal of Applied Engineering Research*, vol. 14, no. 3, pp. 60-63,2019.

[23] U. Shuaib, M. Shaheryar and W. Asghar, "On some characterization of o-fuzzy subgroups," *Internationa journal of Mathematics and Computer Science*, vol. 13, no. 2, pp. 119-131,2018.

[24] M. Gulzar, G. Abbas, F. Dilawar, "Algebraic properties of ω-Q-fuzzy subgroup," *International journal of Mathematics and Computer Science, vol*. 15, no. 1 pp. 265-274,2020.

[25] M. A. R. Shafei, M. A. Tawfik and D. K. Ibrahim, "Fuzzy control scheme for energy efficiency and demand management in airports using 3D simulator," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 20, no. 2, pp. 583-592, 2020.