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Pythagorean Fuzzy N-Soft Groups

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ABSTRACT

We elaborate in this paper a new structure Pythagorean fuzzy N-soft groups which is the generalization of intuitionistic fuzzy soft group initiated by Karaaslan in 2013. In Pythagorean fuzzy N-soft sets concepts of fuzzy sets, soft sets, N-soft sets, fuzzy soft sets, intuitionistic fuzzy sets, intuitionistic fuzzy soft sets, Pythagorean fuzzy sets, Pythagorean fuzzy soft sets are generalized. We also talk about some elementary basic concepts and operations on Pythagorean fuzzy N-soft sets with the assistance of illusions. We additionally define three different sorts of complements for Pythagorean fuzzy N-soft sets and examined a few outcomes not hold in Pythagorean fuzzy N-soft sets complements as they hold in crisp set hypothesis with the assistance of counter examples. We further talked about (α, β, γ)-cut of Pythagorean fuzzy N-soft set and their properties. We likewise talk about some essential properties of Pythagorean fuzzy N-soft groups like groupoid, normal group, left and right cosets, (α, β, γ)-cut subgroups and some fundamental outcomes identified with these terms. Pythagorean fuzzy N-soft sets is increasingly efficient and adaptable model to manage uncertainties. The proposed models of Pythagorean fuzzy N-soft groups can defeat a few disadvantages of the existing statures.

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1. INTRODUCTION

To handel complex genuine issues the techniques generally utilized in traditional mathematics are not constantly beneficial because of a number of sorts of vagueness and uncertainties involve in such issues. There are various procedures, similar to the interval mathematics, hypothesis of probability, rough set hypothesis, and fuzzy set hypothesis which we can be consider of as numerical models for adapting to vulnerabilities. Unfortunately, every one of these structures have their own down sides and troubles. For instance, in the genuine the words lovely, best, renowned are ambiguous. The criteria for wonderful, best, renowned fluctuates from individual to individual. To handel such type of ambiguous and uncertain information initiated the the possibility of fuzzy set augmentation of the customary new sort with the assistance of participation function which assign an enrollment grade in the unit shut interval $[0, 1]$ to every alternative in the universe [1]. The possibility of intuitionistic fuzzy sets as an generalization of fuzzy set by introducing the idea of enrollment and non-

participation grades [2, 3]. The idea of a latest sort or new sort of set also called soft set, as a numerical model to manage uncertainties. The possibility of soft sets have opened new entryways for the scientists because of their applications in different genuine issues. Fuzzy sets and soft sets are independent generalizations of crisp sets [4]. Pythagorean fuzzy set introduced in [5]. In [6–8], certain outcomes for Pythagorean fuzzy sets and some Pythagorean fuzzy information measures and their implementations investigated.

Soft sets give binary object evaluation and other numerics models like fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets partner esteems in the interval $[0, 1]$. These models fail to handle the situation while modelling partner non-binary decisions on true issues. In addition , non-binary assessments in the rating or ranking positions are expected. The ranking can be communicated in multinary values in the type of number of stars, spots, grades or any generalized notation. Propelled by these concerns, in [9], initiated the possibility of N-soft set as an extended model of soft set, in request to depict the importance of evaluations, in actuality. The concept of fuzzy sets to algebra and developed a group of fuzzy subgroup in [10]. In [11], introduced the concept of level subsets, which [12] used earlier. To show that its level subgroups can define almost all of the global notions of the fuzzy subgroup. The normal fuzzy soft group concept, also described a normal fuzzy soft subgroup's level subsets, and discussed some of its properties [13]. The normal fuzzy soft group definition and described a normal fuzzy soft subgroup's level subsets [14]. In [15], define the intuitionist fuzzy group. In [16], attempts to research some algebraic existence of intuitionistic fuzzy subgroups and their properties with the assistance of their (α, β)-cut sets. Soft group and their basic properties in [17]. Abelian soft group, soft group coset, normal soft group,soft group product, cyclic soft group. After that, soft factor group, normal maximum soft group, simple soft group are defined and some important results are proved [18]. In [19], extended the intuitive Fuzzy Soft Set to semigroup. Some remarkable work on the extension of fuzzy sets and fuzzy algebraic structure have been investigated [20] - [28].

The intention of paper is to inaugurate Pythagorean fuzzy N-soft groups as the generalization of intuitionistic fuzzy group and soft group. So as to take care of this present reality issues that intuitionistic fuzzy soft group unable to manage the circumstance that the total of enrollment degree and non-participation level of the parameter is bigger than 1. It makes the restricted , and affects the ideal choice. Pythagorean fuzzy N-soft set gives an enormous number of uses to the for such genuine world issues.

The paper is sorted out as: Section 2 introduces our newly model of Pythagorean fuzzy N-soft sets to deal with the situations in which we need to give Pythagorean fuzzy characterization and non-binary evaluations of objects of universe with respect to the parameters. Likewise established basic tasks and crucial properties of of Pythagorean fuzzy N-soft sets. Section 3 gives three different kinds of complements of Pythagorean fuzzy N-soft sets and examined a few outcomes not hold in Pythagorean fuzzy N-soft sets complements as they hold in crisp set hypothesis with the assistance of counter examples. Section 4 we further talked about (α, β, γ)-cut of Pythagorean fuzzy N-soft set and their properties. Section 5 we introduced the notions of Pythagorean fuzzy N-soft groups. We likewise talk about some essential properties of Pythagorean fuzzy N-soft groups like groupoid, normal group, left and right cosets, (α, β, γ)-cut subgroups and some fundamental outcomes identified with these terms. Finally, At last, a solid end is summed up in section 6.

2. PYTHAGOREAN FUZZY N-SOFT SET

In this portion, we concentrate some essential ideas, principal properties and arithmetic operations on *Pythagorean fuzzy N-soft set* (PFNS set).

Definition 2..1 *Let X be a universal set and E be the collection of parameters. Let $A \subseteq E$ be non-empty and PF^X denote the collection of all Pythagorean fuzzy subsets over X. Let $\mathfrak{L} = \{0, 1, 2, \cdots, N-1\}$ be the set of grading values. A (PFNS set) on X is expressed as (Ψ, A, N) or Ψ_A^N, where $\Psi : A \to \mathbf{P}(PF^X \times \mathfrak{L})$ is a mapping i.e.*

$$(\Psi, A, N) \quad = \quad \left\{ \left(e, \left\{ \langle \rho, \mathfrak{l}_e(\rho), (\mu_e(\rho), \nu_e(\rho)) \rangle \right\} \right) : e \in A, \rho \in X, \mathfrak{l}_e(\rho) \in \mathfrak{L} \right\}$$

where $\mu_e : X \to [0, 1]$ and $\nu_e : X \to [0, 1]$ are mappings as well as the property

$$0 \leq \mu_e^2(\rho) + \nu_e^2(\rho) \leq 1$$

In particular, $\mu_e(\rho)$ represents the degree of membership, $\nu_e(\rho)$ denotes degree of non-membership and $\mathfrak{l}_e(\rho)$ denotes the grading value of the element $\rho \in X$ corresponding to the attribute $e \in A$ to the set (Ψ, A, N).

The set of all PFNSSs over X and with set of parameters from E is called PFNS class and is designated as $PFNS(X, E, N)$.

If we write $a_{ij} = \mu_{e_j}(\rho_i)$, $b_{ij} = \nu_{e_j}(\rho_i)$ and $c_{ij} = \mathfrak{l}_{e_j}(\rho_i)$ where $i = i, 2..., m$ and $i = i, 2..., m$, then the PFNS set Ψ_A^N may be expressed in tabular form as

Ψ_A^N	e_1	e_2	$\cdots$	e_n
ρ_1	$\langle c_{11}, (a_{11}, b_{11}) \rangle$	$\langle c_{12}, (a_{12}, b_{12}) \rangle$	$\cdots$	$\langle c_{1n}, (a_{1n}, b_{1n}) \rangle$
ρ_2	$\langle c_{21}, (a_{21}, b_{21}) \rangle$	$\langle c_{22}, (a_{22}, b_{22}) \rangle$	$\cdots$	$\langle c_{2n}, (a_{2n}, b_{2n}) \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
ρ_m	$\langle c_{m1}, (a_{m1}, b_{m1}) \rangle$	$\langle c_{m2}, (a_{m2}, b_{m2}) \rangle$	$\cdots$	$\langle c_{mn}, (a_{mn}, b_{mn}) \rangle$

and in matrix form as

$$
\begin{aligned}
(\Psi, A, N) &= \left[\langle c_{ij}, (a_{ij}, b_{ij}) \rangle \right]_{m \times n} \\
&= \begin{pmatrix}
\langle c_{11}, (a_{11}, b_{11}) \rangle & \langle c_{12}, (a_{12}, b_{12}) \rangle & \cdots & \langle c_{1n}, (a_{1n}, b_{1n}) \rangle \\
\langle c_{21}, (a_{21}, b_{21}) \rangle & \langle c_{22}, (a_{22}, b_{22}) \rangle & \cdots & \langle c_{2n}, (a_{2n}, b_{2n}) \rangle \\
\vdots & \vdots & \ddots & \vdots \\
\langle c_{m1}, (a_{m1}, b_{m1}) \rangle & \langle c_{m2}, (a_{m2}, b_{m2}) \rangle & \cdots & \langle c_{mn}, (a_{mn}, b_{mn}) \rangle
\end{pmatrix}
\end{aligned}
$$

This matrix is referred to as the Pythagorean fuzzy N-soft(PFNS) matrix.

Definition 2..2 *A PFNS set Ψ_A^N over X is known as null PFNS set, symbolized as Ψ_ϕ^0 and defined as*

$$
\Psi_\phi^0 = \left\{ \left(e, \{ \langle \rho, \mathfrak{l}_\phi(\rho), (\mu_\phi(\rho), \nu_\phi(\rho)) \rangle \} \right) : e \in A, \rho \in X, \mathfrak{l}_\phi(\rho) \in \mathfrak{L} \right\}
$$

where $\mu_\phi(\rho) = 0$, $\nu_\phi(\rho) = 1$ and $\mathfrak{l}_\phi(\rho) = 0$.

Definition 2..3 *A PFNS set Ψ_A^N over X is said to be absolute PFNS set, symbolized as Ψ_E^{N-1} and defined as*

$$
\Psi_E^{N-1} = \left\{ \left(e, \{ \langle \rho, \mathfrak{l}_E(\rho), (\mu_E(\rho), \nu_E(\rho)) \rangle \} \right) : e \in E, \rho \in X, \mathfrak{l}_E(\rho) \in \mathfrak{L} \right\}
$$

where $\mu_E(\rho) = 1$, $\nu_E(\rho) = 0$ and $\mathfrak{l}_E(\rho) = N - 1$.

Example 2..4 *Let $X = \{\rho_i : i = 1, 2, \cdots, 7\}$, $E = \{e_i : i = 1, 2, \cdots, 5\}$ and $\mathfrak{L} = \{0, 1, 2, \cdots, 8\}$. Assume that $A = \{e_1, e_3\}$. Then,*

$$
\begin{aligned}
\Psi_A^9 = \Big\{ &\left(e_1, \{ \langle \rho_1, 4, (0.3, 0.9) \rangle, \langle \rho_4, 6, (0.5, 0.2) \rangle, \langle \rho_7, 8, (0.7, 0.4) \rangle \} \right), \\
&\left(e_3, \{ \langle \rho_2, 3, (0.2, 0.8) \rangle, \langle \rho_3, 2, (0.1, 0.6) \rangle, \langle \rho_5, 2, (0.3, 0.7) \rangle \} \right) \Big\}
\end{aligned}
$$

is a PF9S set over X. The representation in tabular form of Ψ_A^9 is

Ψ_A^9	e_1	e_2	e_3	e_4	e_5
ρ_1	$\langle 4,(0.3,\ 0.9) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$
ρ_2	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 3,(0.2,\ 0.8) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$
ρ_3	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 2,(\ 0.1,\ 0.6) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$
ρ_4	$\langle 6,(0.5,\ 0.2) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$
ρ_5	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 2,\ (0.3,\ 0.7) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$
ρ_6	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$
ρ_7	$\langle 8,(\ 0.7,\ 0.4) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$	$\langle 0,(0,\ 1) \rangle$

The corresponding PF9S matrix is

$$
\begin{aligned}
(\Psi, A, 9) &= [a_{ij}, b_{ij}]_{7\times 5} \\
&= \begin{pmatrix}
\langle 4, (0.3, 0.9)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle \\
\langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 3, (0.2, 0.8)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle \\
\langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 2, (0.1, 0.6)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle \\
\langle 6, (0.5, 0.2)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle \\
\langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 2, (0.3, 0.7)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle \\
\langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle \\
\langle 8, (0.7, 0.4)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle & \langle 0, (0,1)\rangle
\end{pmatrix}
\end{aligned}
$$

Definition 2..5 *Let $\Psi_A^{N_1}$ and $\Psi_B^{N_2}$ be PFNS set over X. Then their extended union is defined as* $\Psi_C^{N^*} = \Psi_A^{N_1} \widetilde{\cup}_E \Psi_B^{N_2}$,

$$
\Psi_C^{N^*} = (\Psi, C, N^*) = \left\{ \left(e, \{ \langle \rho, \mathfrak{l}_C(\rho), (\mu_C(\rho), \nu_C(\rho)) \rangle \} \right) : e \in C, \rho \in X, \mathfrak{l}_C(\rho) \in \mathfrak{L} \right\}
$$

where, $C = A \cup B$, $N^* = \max\{N_1, N_2\}$, $\mu_C(\rho) = \max\{\mu_A(\rho), \mu_B(\rho)\}$, $\nu_C(\rho) = \min\{\nu_A(\rho), \nu_B(\rho)\}$ and$\mathfrak{l}_C(\rho) = \max\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\} \ \forall \ e \in C$.

Definition 2..6 *Let $\Psi_A^{N_1}$ and $\Psi_B^{N_2}$ be PFNS set over X. Then their restricted union is defined as* $\Psi_C^{N^*} = \Psi_A^{N_1} \widetilde{\cup}_R \Psi_B^{N_2}$,

$$
\Psi_D^{N^*} = (\Psi, D, N^*) = \left\{ \left(e, \{ \langle \rho, \mathfrak{l}_D(\rho), (\mu_D(\rho), \nu_D(\rho)) \rangle \} \right) : e \in D, \rho \in X, \mathfrak{l}_D(\rho) \in \mathfrak{L} \right\}
$$

where, $D = A \cap B$, $N^* = \max\{N_1, N_2\}$, $\mu_D(\rho) = \max\{\mu_A(\rho), \mu_B(\rho)\}$, $\nu_D(\rho) = \min\{\nu_A(\rho), \nu_B(\rho)\}$ and $\mathfrak{l}_D(\rho) = \max\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\} \ \forall \ e \in D$.

Definition 2..7 *Let $\Psi_A^{N_1}$ and $\Psi_B^{N_2}$ be PFNS set over X. Then their extended intersection is defined as* $\Psi_C^{N^*} = \Psi_A^{N_1} \widetilde{\cap}_E \Psi_B^{N_2}$,

$$
\Psi_C^{N_*} = (\Psi, C, N_*) = \left\{ \left(e, \{ \langle \rho, \mathfrak{l}_C(\rho), (\mu_C(\rho), \nu_C(\rho)) \rangle \} \right) : e \in C, \rho \in X, \mathfrak{l}_C(\rho) \in \mathfrak{L} \right\}
$$

where, $C = A \cup B$, $N_* = \min\{N_1, N_2\}$, $\mu_C(\rho) = \min\{\mu_A(\rho), \mu_B(\rho)\}$, $\nu_C(\rho) = \max\{\nu_A(\rho), \nu_B(\rho)\}$ and $\mathfrak{l}_C(\rho) = \min\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\} \ \forall \ e \in C$.

Definition 2..8 *Let $\Psi_A^{N_1}$ and $\Psi_B^{N_2}$ be PFNS set over X. Then their restricted intersection is defined as* $\Psi_C^{N^*} = \Psi_A^{N_1} \widetilde{\cap}_R \Psi_B^{N_2}$,

$$
\Psi_D^{N_*} = (\Psi, D, N_*) = \left\{ \left(e, \{ \langle \rho, \mathfrak{l}_D(\rho), (\mu_D(\rho), \nu_D(\rho)) \rangle \} \right) : e \in D, \rho \in X, \mathfrak{l}_D(\rho) \in \mathfrak{L} \right\}
$$

where, $D = A \cap B$, $N_* = \min\{N_1, N_2\}$, $\mu_D(\rho) = \min\{\mu_A(\rho), \mu_B(\rho)\}$, $\nu_D(\rho) = \max\{\nu_A(\rho), \nu_B(\rho)\}$ and $\mathfrak{l}_D(\rho) = \min\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\} \ \forall \ e \in D$.

Proposition 2..9 *If $\Psi_A^{N_1}$ and $\Psi_B^{N_2}$ are PFNS sets over X, then*

(i) $\left(\Psi_A^{N_1} \widetilde{\cap}_E \Psi_B^{N_2} \right) \widetilde{\subseteq} \Psi_A^{N_1} \widetilde{\subseteq} \left(\Psi_A^{N_1} \widetilde{\cup}_E \Psi_B^{N_2} \right)$

(ii) $\left(\Psi_A^{N_1} \widetilde{\cap}_E \Psi_B^{N_2} \right) \widetilde{\subseteq} \Psi_B^{N_2} \widetilde{\subseteq} \left(\Psi_A^{N_1} \widetilde{\cup}_E \Psi_B^{N_2} \right)$.

(iii) $\left(\Psi_A^{N_1} \widetilde{\cap}_R \Psi_B^{N_2} \right) \widetilde{\subseteq} \Psi_A^{N_1} \widetilde{\subseteq} \left(\Psi_A^{N_1} \widetilde{\cup}_E \Psi_B^{N_2} \right)$

(iv) $\left(\Psi_A^{N_1} \widetilde{\cap}_R \Psi_B^{N_2} \right) \widetilde{\subseteq} \Psi_B^{N_2} \widetilde{\subseteq} \left(\Psi_A^{N_1} \widetilde{\cup}_E \Psi_B^{N_2} \right)$.

3. COMPLEMENTS OF PYTHAGOREAN N-SOFT SET

Definition 3..1 *Let Ψ_A^N be PFNS set over X. Then their weak complement is represented as $\left(\Psi_A^N\right)^{wc}$ and defined by*

$$\left(\Psi_A^N\right)^{wc} = \left\{ \left(e, \left\{\langle \rho, \mathfrak{r}_A^{wc}(\rho), (\mu_A^{wc}(\rho), \nu_A^{wc}(\rho))\rangle\right\}\right) : e \in A, \rho \in X, \mathfrak{r}_A^{wc}(\rho) \in \mathfrak{L}\right\}$$

where, $\mu_A^{wc}(\rho) = \nu_A(\rho), \nu_A^{wc}(\rho) = \mu_A(\rho)$ and $\mathfrak{r}_A^{wc}(\rho) \cap \mathfrak{l}_A(\rho) = \phi \ \forall\, e \in A$.

Definition 3..2 *Let Ψ_A^N be PFNS set over X. Then their top weak complement is represented as $\left(\Psi_A^N\right)^{twc}$ and defined by*

$$\left(\Psi_A^N\right)^{twc} = \left\{ \left(e, \left\{\langle \rho, \mathfrak{r}_A^{twc}(\rho), (\mu_A^{twc}(\rho), \nu_A^{twc}(\rho))\rangle\right\}\right) : e \in A, \rho \in X, \mathfrak{r}_A^{twc}(\rho) \in \mathfrak{L}\right\}$$

where, $\mu_A^{twc}(\rho) = \nu_A(\rho), \nu_A^{twc}(\rho) = \mu_A(\rho)$ and

$$\mathfrak{r}_A^{twc}(\rho) = \begin{cases} N-1, & if\, \mathfrak{l}_A(\rho) < N-1 \\ 0, & if\, \mathfrak{l}_A(\rho) = N-1 \ \forall\, e \in A. \end{cases}$$

Definition 3..3 *Let Ψ_A^N be PFNS set over X. Then their bottom weak complement is represented as $\left(\Psi_A^N\right)^{bwc}$ and defined by*

$$\left(\Psi_A^N\right)^{bwc} = \left\{ \left(e, \left\{\langle \rho, \mathfrak{r}_A^{bwc}(\rho), (\mu_A^{bwc}(\rho), \nu_A^{bwc}(\rho))\rangle\right\}\right) : e \in A, \rho \in X, \mathfrak{r}_A^{bwc}(\rho) \in \mathfrak{L}\right\}$$

where, $\mu_A^{bwc}(\rho) = \nu_A(\rho), \nu_A^{bwc}(\rho) = \mu_A(\rho)$ and

$$\mathfrak{r}_A^{bwc}(\rho) = \begin{cases} N-1, & if\, \mathfrak{l}_A(\rho) = 0 \\ 0, & if\, \mathfrak{l}_A(\rho) > 0 \ \forall\, e \in A. \end{cases}$$

Theorem 3..4 Let Ψ_A^N be PFNS set, Ψ_ϕ^0 be null PFNS and Ψ_E^{N-1} absolute PFNS over X, then following results that hold in crisp set theory but not hold in PFNS set theory

   (i) $\left(\Psi_\phi^0\right)^{wc} \neq \Psi_E^{N-1}$.

   (ii) $\left(\Psi_E^{N-1}\right)^{wc} \neq \Psi_\phi^0$.

   (iii) $\left(\left(\Psi_A^N\right)^{wc}\right)^{wc} \neq \Psi_A^N$

Proposition 3..5 Let Ψ_A^N be PFNS set, Ψ_ϕ^0 be null PFNS and Ψ_E^{N-1} absolute PFNS over X, then

   (i) $\left(\Psi_\phi^0\right)^{twc} = \Psi_E^{N-1}$.

   (ii) $\left(\Psi_E^{N-1}\right)^{twc} = \Psi_\phi^0$.

Theorem 3..6 Let Ψ_A^N be PFNS set, over X, then

   (i) $\left(\left(\Psi_A^N\right)^{bwc}\right)^{bwc} \neq \Psi_A^N$

4. (α, β, γ)-CUT OF PYTHAGOREAN FUZZY N-SOFT SET AND THEIR PROPERTIES

Definition 4..1 *Let Ψ_A^N be a PFNS set on X. Then (α, β, γ)-cut of Ψ_A^N is a crisp subset $\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$ of PFNS Ψ_A^N is given by*

$$\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N) = \left\{ \rho_i : \rho_i \in X \text{ such that } \mu_A(\rho_i) \geq \alpha, \nu_A(\rho_i) \leq \beta, \mathfrak{l}_A(\rho_i) \geq \gamma \right\}$$

where $\alpha, \beta \in [0, 1]$ with $0 \leq \alpha^2 + \beta^2 \leq 1$ and $\gamma \in \{0, 1, 2, \cdots N - 1\}$.

Proposition 4..2 *If Ψ_A^N and Ψ_B^N be two PFNS set's on X, then following holds.*

1. $\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N) \subseteq \mathfrak{C}_{\sigma,\theta,\phi}(\Psi_A^N)$ *if* $\alpha \geq \sigma, \beta \leq \theta$ *and* $\gamma \geq \phi$

2. $\mathfrak{C}_{1-\beta,\beta,\gamma}(\Psi_A^N) \subseteq \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N) \subseteq \mathfrak{C}_{\alpha,1-\alpha,\gamma}(\Psi_A^N)$

3. $\Psi_A^N \subseteq \Psi_B^N \Rightarrow \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N) \subseteq \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_B^N)$

4. $\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N \widetilde{\cup_E} \Psi_B^N) \supseteq \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N) \widetilde{\cup_E} \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_B^N)$

5. $\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N \widetilde{\cap_R} \Psi_B^N) = \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N) \widetilde{\cap_R} \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_B^N)$ *equality hold if* $\alpha^2 + \beta^2 = 1$

6. $\mathfrak{C}_{\alpha,\beta,\gamma}(\widetilde{\cap_R} \Psi_{A_i}^N) = \widetilde{\cap_R} \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_{A_i}^N)$

7. $\mathfrak{C}_{0,1,0}(\Psi_A^N) = X$

5.   PYTHAGOREAN FUZZY N-SOFT GROUPS

In this section, we introduce notion of pythagorean fuzzy N-soft group (PFNS-group) by inspiring from the Intuitionistic fuzzy soft groups of Karaaslan *et.al* [23]. In this section $\mathcal{G}$ denotes an arbitrary group with identity element $\mathfrak{e}$.

Definition 5..1 *Let $\mathcal{G}$ be a group and Ψ_A^N be a PFNS set on X. $(\Psi_A^N)_\mathcal{G}$ is called PFNS-groupoid if $(\Psi_A^N)_\mathcal{G}(\rho_i.\rho_j)$ $\widetilde{\supseteq}(\Psi_A^N)_\mathcal{G}(\rho_i) \sqcap (\Psi_A^N)_\mathcal{G}(\rho_j) \; \forall \rho_i, \rho_j \in \mathcal{G}$. i.e.*
$\mu_A(\rho_i.\rho_j) \leq \mu_A(\rho_i) \wedge \mu_A(\rho_j), \; \nu_A(\rho_i.\rho_j) \geq \nu_A(\rho_i) \vee \nu_A(\rho_j)$ and $\mathfrak{l}_A(\rho_i.\rho_j) \leq \mathfrak{l}_A(\rho_i) \wedge \mathfrak{l}_A(\rho_j) \; \forall \rho_i, \rho_j \in \mathcal{G}$.

Definition 5..2 *Let $\mathcal{G}$ be a group and Ψ_A^N be a PFNS set on X. $(\Psi_A^N)_\mathcal{G}$ is called PFNS-group if $(\Psi_A^N)_\mathcal{G}(\rho_i.\rho_j)$ $\widetilde{\supseteq}(\Psi_A^N)_\mathcal{G}(\rho_i) \sqcap (\Psi_A^N)_\mathcal{G}(\rho_j)$ and $(\Psi_A^N)_\mathcal{G}(\rho_i^{-1}) \widetilde{\subseteq} (\Psi_A^N)_\mathcal{G}(\rho_i) \; \forall \rho_i, \rho_j \in \mathcal{G}$. i.e.*
$\mu_A(\rho_i.\rho_j) \leq \mu_A(\rho_i) \wedge \mu_A(\rho_j), \; \mu_A(\rho_i^{-1}) \leq \mu_A(\rho_i), \; \nu_A(\rho_i.\rho_j) \geq \nu_A(\rho_i) \vee \nu_A(\rho_j), \; \nu_A(\rho_i^{-1}) \geq \nu_A(\rho_i)$ and $\mathfrak{l}_A(\rho_i.\rho_j) \leq \mathfrak{l}_A(\rho_i) \wedge \mathfrak{l}_A(\rho_j) \; \forall \rho_i, \rho_j \in \mathcal{G}. \; \mathfrak{l}_A(\rho_i^{-1}) \leq \mathfrak{l}_A(\rho_i)$.

Example 5..3 *Let $S_3 = <\alpha, \beta| \, \alpha^3 = \beta^2 = (\alpha\beta)^2 = e >$ be a symmetric group w.r.t multiplication, $X = A = \{x_1, \; x_2, \; x_3, x_4, x_5, x_6\}$ and Ψ_A^7 be a PF7S set on X given below.*

$(\Psi_A^7)_{S_3}$	e	α	α^2	β	$\alpha\beta$	$\alpha^2\beta$
x_1	$\langle 3, (0.2, 0.2) \rangle$	$\langle 3, (0.2, 0.4) \rangle$	$\langle 3, (0.2, 0.4) \rangle$	$\langle 3, (0.2, 0.3) \rangle$	$\langle 3, (0.2, 0.4) \rangle$	$\langle 3, (0.2, 0.4) \rangle$
x_2	$\langle 5, (0.6, 0.3) \rangle$	$\langle 5, (0.6, 0.5) \rangle$	$\langle 5, (0.6, 0.5) \rangle$	$\langle 5, (0.6, 0.4) \rangle$	$\langle 5, (0.6, 0.5) \rangle$	$\langle 5, (0.6, 0.5) \rangle$
x_3	$\langle 6, (0.9, 0.1) \rangle$	$\langle 6, (0.9, 0.3) \rangle$	$\langle 6, (0.9, 0.3) \rangle$	$\langle 6, (0.9, 0.2) \rangle$	$\langle 6, (0.9, 0.3) \rangle$	$\langle 6, (0.9, 0.3) \rangle$
x_4	$\langle 4, (0.5, 0.4) \rangle$	$\langle 4, (0.5, 0.6) \rangle$	$\langle 4, (0.5, 0.6) \rangle$	$\langle 4, (0.5, 0.5) \rangle$	$\langle 4, (0.5, 0.6) \rangle$	$\langle 4, (0.5, 0.6) \rangle$
x_5	$\langle 2, (0.2, 0.5) \rangle$	$\langle 2, (0.2, 0.5) \rangle$	$\langle 2, (0.2, 0.5) \rangle$	$\langle 2, (0.2, 0.6) \rangle$	$\langle 2, (0.2, 0.5) \rangle$	$\langle 2, (0.2, 0.5) \rangle$
x_6	$\langle 1, (0.1, 0.7) \rangle$	$\langle 1, (0.1, 0.9) \rangle$	$\langle 1, (0.1, 0.9) \rangle$	$\langle 1, (0.1, 0.8) \rangle$	$\langle 1, (0.1, 0.9) \rangle$	$\langle 1, (0.1, 0.9) \rangle$

which satisfied all the condition of $(\Psi_A^7)_{S_3}$

Definition 5..4 *Let $\mathcal{G}$ be a group and Ψ_A^N be a PFNS set on X. $(\Psi_A^N)_\mathcal{G}$ is called PFNS normal group if $\mu_A(\rho_i.\rho_j) = \mu_A(\rho_j.\rho_i), \; \nu_A(\rho_i.\rho_j) = \nu_A(\rho_j.\rho_i), \; \mathfrak{l}_A(\rho_i.\rho_j) = \mathfrak{l}_A(\rho_j.\rho_i) \; \; \forall \rho_i, \rho_j \in \mathcal{G}$.*

$(\Psi_A^7)_{S_3}$ is not PF7S normal group because $\nu_{x_1}(\alpha.\alpha^2\beta) = 0.4, \; \nu_{x_1}(\alpha^2\beta.\alpha) = 0.3$. This implies $\nu_{x_1}(\alpha.\alpha^2\beta) \neq \nu_{x_1}(\alpha^2\beta.\alpha)$.

Proposition 5..5 *A necessary and sufficient condition for PFNS set Ψ_A^N of a group $\mathcal{G}$ to be a PFNSG of $\mathcal{G}$ is that $\forall \ \rho_i, \rho_j \in \mathcal{G}, \mu_A(\rho_i.\rho_j^{-1}) \leq \mu_A(\rho_i) \wedge \mu_A(\rho_j), \nu_A(\rho_i.\rho_j^{-1}) \geq \nu_A(\rho_i) \vee \nu_A(\rho_j)$ and$\mathfrak{l}_A(\rho_i.\rho_j^{-1}) \leq \mathfrak{l}_A(\rho_i) \wedge \mathfrak{l}_A(\rho_j).$*

Proof: Proof of this preposition is obvious from Definition 5..2.

Example 5..6 *Let $S_3 = <\alpha, \beta| \alpha^3 = \beta^2 = (\alpha\beta)^2 = e >$ be a symmetric group w.r.t multiplication, $X = A = \{x_1, \ x_2, \ x_3, x_4, x_5, x_6\}$ and Ψ_A^7 be a PF7S subset H on X given below.*

$(\Psi_A^7)_H$	e	α	α^2
x_1	$\langle 3, (0.2, 0.2)\rangle$	$\langle 3, (0.2, 0.4)\rangle$	$\langle 3, (0.2, 0.4)\rangle$
x_2	$\langle 5, (0.6, 0.3)\rangle$	$\langle 5, (0.6, 0.5)\rangle$	$\langle 5, (0.6, 0.5)\rangle$
x_3	$\langle 6, (0.9, 0.1)\rangle$	$\langle 6, (0.9, 0.3)\rangle$	$\langle 6, (0.9, 0.3)\rangle$
x_4	$\langle 4, (0.5, 0.4)\rangle$	$\langle 4, (0.5, 0.6)\rangle$	$\langle 4, (0.5, 0.6)\rangle$
x_5	$\langle 2, (0.2, 0.5)\rangle$	$\langle 2, (0.2, 0.5)\rangle$	$\langle 2, (0.2, 0.5)\rangle$
x_6	$\langle 1, (0.1, 0.7)\rangle$	$\langle 1, (0.1, 0.9)\rangle$	$\langle 1, (0.1, 0.9)$

which satisfied all the condition of PFNS normal subgroup $(\Psi_A^7)_H$.

Definition 5..7 *Let $(\Psi_A^N)_\mathcal{G}$ be PFNSG of a group $\mathcal{G}$. For any $\rho_j \in \mathcal{G}$, the PFNS set $\rho_j.(\Psi_A^N)_\mathcal{G}$ is called left coset of $(\Psi_A^N)_\mathcal{G}$ if $\left(\rho_j.(\Psi_A^N)_\mathcal{G}\right)(\rho_i) = (\Psi_A^N)_\mathcal{G}(\rho_j^{-1}.\rho_i) \ \forall \ \rho_i, \rho_j \in \mathcal{G}$, i.e*
$\rho_j.\mu_A(\rho_i) = \mu_A(\rho_j^{-1}.\rho_i), \ \rho_j.\nu_A(\rho_i) = \nu_A(\rho_j^{-1}.\rho_i)$ and $\rho_j.\mathfrak{l}_A(\rho_i) = \mathfrak{l}_A(\rho_j^{-1}.\rho_i).$

Definition 5..8 *Let $(\Psi_A^N)_\mathcal{G}$ be PFNSG of a group $\mathcal{G}$. For any $\rho_j \in \mathcal{G}$, the PFNS set $(\Psi_A^N)_\mathcal{G}.\rho_j$ is called right coset of $(\Psi_A^N)_\mathcal{G}$ if $\left((\Psi_A^N)_\mathcal{G}.\rho_j\right)(\rho_i) = (\Psi_A^N)_\mathcal{G}(\rho_i.\rho_j^{-1}) \ \forall \ \rho_i, \rho_j \in \mathcal{G}$, i.e*
$\mu_A(\rho_i).\rho_j = \mu_A(\rho_i.\rho_j^{-1}), \ \nu_A(\rho_i).\rho_j = \nu_A(\rho_i.\rho_j^{-1})$ and $\mathfrak{l}_A(\rho_i).\rho_j = \mathfrak{l}_A(\rho_i.\rho_j^{-1}).$

Proposition 5..9 *A necessary and sufficient condition for PFNSG $(\Psi_A^N)_\mathcal{G}$ of a group $\mathcal{G}$ to be a PFNS normal group of $\mathcal{G}$ is that $\forall \ \rho_i, \rho_j \in \mathcal{G}, \ \mu_A(\rho_j^{-1}.\rho_i.\rho_j) = \mu_A(\rho_i), \ \nu_A(\rho_j^{-1}.\rho_i.\rho_j) = \nu_A(\rho_i)$ and $\mathfrak{l}_A(\rho_j^{-1}.\rho_i.\rho_j) = \mathfrak{l}_A(\rho_i).$*

Proof: Let $\rho_j \in \mathcal{G}$ be any element, then

$$\mu_A(\rho_j^{-1}.\rho_i.\rho_j) = \mu_A(\rho_j^{-1}.(\rho_i.\rho_j)) = \mu_A(\rho_i.(\rho_j^{-1}.\rho_j)) = \mu_A(\rho_i.\mathfrak{e}) = \mu_A(\rho_i),$$

$$\nu_A(\rho_j^{-1}.\rho_i.\rho_j) = \nu_A(\rho_j^{-1}.(\rho_i.\rho_j)) = \nu_A(\rho_i.(\rho_j^{-1}.\rho_j)) = \nu_A(\rho_i.\mathfrak{e}) = \nu_A(\rho_i)$$

$$\text{and } \mathfrak{l}_A(\rho_j^{-1}.\rho_i.\rho_j) = \mathfrak{l}_A(\rho_j^{-1}.(\rho_i.\rho_j)) = \mathfrak{l}_A(\rho_i.(\rho_j^{-1}.\rho_j)) = \mathfrak{l}_A(\rho_i.\mathfrak{e}) = \mathfrak{l}_A(\rho_i).$$

If $(\Psi_A^N)_\mathcal{G}$ be PFNS normal subgroup of a group $\mathcal{G}$. Then it is obvious that PFNS left coset of $(\Psi_A^N)_\mathcal{G}$ and PFNS right coset of $(\Psi_A^N)_\mathcal{G}$ are equal. In this situation we can call PFNS coset instead of PFNS left coset or PFNS right coset.

Proposition 5..10 *Let $\mathcal{G}$ be a group. Then*
$(\Psi_A^N)_\mathcal{G} = \left\{ \left(e, \{\langle \rho, \mathfrak{l}_e(\rho), (\mu_e(\rho), \nu_e(\rho))\rangle\}\right), \rho \in \mathcal{G} : \mathfrak{l}_e(\rho) = \mathfrak{l}_e(\mathfrak{e}), \mu_e(\rho) = \mu_e(\mathfrak{e}), \nu_e(\rho) = \nu_e(\mathfrak{e})\right\}$ *is PFNS normal subgroup of $\mathcal{G}$.*

Proof: It is obvious.

Proposition 5..11 *Let $(\Psi_A^N)_\mathcal{G}$ be PFNSG of a group $\mathcal{G}$. Then $\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$ is a subgroup of group $\mathcal{G}$, where $\mu_A(\mathfrak{e}) \geq \alpha, \nu_A(\mathfrak{e}) \leq \beta, \mathfrak{l}_A(\mathfrak{e}) \geq \gamma$ and $\mathfrak{e}$ is the identity of $\mathcal{G}$.*
Proof: Let $\rho_i, \rho_j \in \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$ be any two elements. Then
$\mu_A(\rho_i) \geq \alpha, \nu_A(\rho_i) \leq \beta$ and $\mathfrak{l}_A(\rho_i) \geq \gamma$ and $\mu_A(\rho_j) \geq \alpha, \nu_A(\rho_j) \leq \beta$ and $\mathfrak{l}_A(\rho_j) \geq \gamma, \Rightarrow \mu_A(\rho_i) \wedge \mu_A(\rho_j) \geq \alpha, \nu_A(\rho_i) \vee \nu_A(\rho_j) \leq \beta$ and $\mathfrak{l}_A(\rho_i) \wedge \mathfrak{l}_A(\rho_j) \geq \gamma$. As $(\Psi_A^N)_\mathcal{G}$ be PFNSG of a group $\mathcal{G}$. Therefore, $\mu_A(\rho_i.\rho_j^{-1}) \geq \mu_A(\rho_i) \wedge \mu_A(\rho_j) \geq \alpha, \ \nu_A(\rho_i.\rho_j^{-1}) \leq \nu_A(\rho_i) \vee \nu_A(\rho_j) \leq \beta$ and $\mathfrak{l}_A(\rho_i.\rho_j^{-1}) \geq \mathfrak{l}_A(\rho_i) \wedge \mathfrak{l}_A(\rho_j) \geq \gamma$. Thus $\rho_i.\rho_j^{-1} \in \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$. Hence $\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$ is a subgroup of $\mathcal{G}$.

Proposition 5..12 *Let $(\Psi_A^N)_\mathcal{G}$ be PFNS normal subgroup of a group $\mathcal{G}$. Then $\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$ is a normal subgroup of group $\mathcal{G}$, where $\mu_A(\mathfrak{e}) \geq \alpha, \nu_A(\mathfrak{e}) \leq \beta, \mathfrak{l}_A(\mathfrak{e}) \geq \gamma$ and $\mathfrak{e}$ is the identity of $\mathcal{G}$.*
Proof: Let $\rho_i \in \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$ and $\rho_i \in \mathcal{G}$ be any element. Then
$\mu_A(\rho_i) \geq \beta, \nu_A(\rho_i) \leq \beta$ and $\mathfrak{l}_A(\rho_i) \geq \gamma$. *Also $(\Psi_A^N)_\mathcal{G}$ be PFNS normal subgroup of a group $\mathcal{G}$. Therefore, $\mu_A(\rho_j^{-1}.\rho_i.\rho_j) = \mu_A(\rho_i), \quad \nu_A(\rho_j^{-1}.\rho_i.\rho_j) = \nu_A(\rho_i)$ and $\mathfrak{l}_A(\rho_j^{-1}.\rho_i.\rho_j) = \mathfrak{l}_A(\rho_i)$. This implies that $\mu_A(\rho_j^{-1}.\rho_i.\rho_j) = \mu_A(\rho_i) \geq \alpha, \quad \nu_A(\rho_j^{-1}.\rho_i.\rho_j) = \nu_A(\rho_i) \leq \beta$ and $\mathfrak{l}_A(\rho_j^{-1}.\rho_i.\rho_j) = \mathfrak{l}_A(\rho_i^{-1}) \geq \gamma$. Thus $\rho_j^{-1}.\rho_i.\rho_j \in \mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$. Hence $\mathfrak{C}_{\alpha,\beta,\gamma}(\Psi_A^N)$ is a normal subgroup of $\mathcal{G}$.*

Example 5..13 *Let $S_3 \ = < \alpha, \beta| \ \alpha^3 = \beta^2 = (\alpha\beta)^2 = e >$ be a symmetric group w.r.t multiplication, $X = A = \{x_1, \ x_2, \ x_3, x_4, x_5, x_6\}$ and Ψ_A^7 be a* **PF7S** *subset H on X given below.*

$\mathfrak{C}_{0.1,\ 0.7,\ 1}(\Psi_A^7)$	e	α	α^2
x_1	$\langle 3, (0.2, 0.2) \rangle$	$\langle 3, (0.2, 0.4) \rangle$	$\langle 3, (0.2, 0.4) \rangle$
x_2	$\langle 5, (0.6, 0.3) \rangle$	$\langle 5, (0.6, 0.5) \rangle$	$\langle 5, (0.6, 0.5) \rangle$
x_3	$\langle 6, (0.9, 0.1) \rangle$	$\langle 6, (0.9, 0.3) \rangle$	$\langle 6, (0.9, 0.3) \rangle$
x_4	$\langle 4, (0.5, 0.4) \rangle$	$\langle 4, (0.5, 0.6) \rangle$	$\langle 4, (0.5, 0.6) \rangle$
x_5	$\langle 2, (0.2, 0.5) \rangle$	$\langle 2, (0.2, 0.5) \rangle$	$\langle 2, (0.2, 0.5) \rangle$
x_6	$\langle 1, (0.1, 0.7) \rangle$	$\langle 1, (0.1, 0.9) \rangle$	$\langle 1, (0.1, 0.9) $

which satisfied all the condition of PFNS normal subgroup $(\Psi_A^7)_H$. Hence $\mathfrak{C}_{0.1,\ 0.7,\ 1}(\Psi_A^7)$ is normal subgroup.

Proposition 5..14 *If $(\Psi_A^N)_\mathcal{G}$ and $(\Psi_B^N)_\mathcal{G}$ be two PFNSG's of a group $\mathcal{G}$, then $(\Psi_A^N)_\mathcal{G} \bigcap (\Psi_B^N)_\mathcal{G}$ is also a PFNSG of a group $\mathcal{G}$.*

6. CONCLUSION

In this article, we introduced novel concept of pythagorean fuzzy $N-$soft sets (PFNS sets). The purpose of this study is to lay the foundation of innovative theory of PFNS sets including basic algebraic operations like extended and restricted union, extended and restricted intersection and three types of complements (weak complement, top weak complement and bottom weak complement) on them and some results of primary importance by using these operations. A large number of illustrations are included with each definition to comprehend the notions effectively. Furthermore, we proposed $(\alpha, \beta, \gamma)-$cut PFNS sets, we elaborate some of its basic properties. We also initiated the notion of pythagorean fuzzy $N-$soft groups (PFNS-groups) and presented certain properties of PFNS-groups. PFNS-groups is the extension of soft groups, fuzzy soft groups, intuitionistic soft groups, pythagorean soft groups, N-soft groups. Later on, we intend to extend over research to the new operational laws and properties on PFNS-groups and plane to apply more ways to deal with different hybrid models including neutrosophic N-soft groups, bipolar N-soft groups and m-polar N-soft groups etc.

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