

Optimized Passive Coupling Control for Biped Robot

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Abstract

A popular hypothesis regarding legged locomotion is that humans and other large animals walk and run in a manner that minimizes the metabolic energy expenditure for locomotion. Here, we just consider the walking gait patterns. And we presented a hybrid model for a passive 2D walker with knees and point feet. The dynamics of this model were fully derived analytically. We have also proposed optimized virtual passive and virtual coupling control laws. This is also a simple and effective gait-generation method based on this kneed walker model, which imitates the energy behavior in every walking cycle. The control strategy is formed by taking into account the features of mechanical energy dissipation and restoration. Following the proposed method, we use computer optimization to find which gaits are indeed energetically optimal for this model. And we also prove some walking rules maybe true by the results of simulations.

Keywords: biped robot, virtual passive, optimization, energy

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1. Introduction

Why do people not walk or even run with a smooth level gait [1], like a waiter holding two cups brim-full of boiling coffee? Why do people select walking and running from the other possibilities? We address such questions by modeling a person as a machine describable with the equations of newtonian mechanics. We wish to find how a person can get from one place to another with the least muscle work.

Passive dynamic walkers exhibit a stable gait [2] when placed on a downward slope with no actuation. These systems demonstrate how the inherent dynamics of walkers can be exploited to achieve natural and energy-efficient gaits. Our main goals are the following aspects.

- 1) To present a mathematical model for a simple two-dimensional planar kneed walker with point feet and knees, making it both a logical extension of the compass gait model and a physically realizable model
- 2) Realization of safe virtual passive dynamic control against human being and outside environment.
- 3) According to the control strategy, energy optimization will be carried out. We seek an explanation of gait choice with no essential dependence on elastic energy storage.

2. Model of a Kneed Biped

This section addresses the walking robot model. We deal with a planer biped model which has knee joints. Figure 1 shows the model of a kneed biped walking robot and Table 1 lists its notations and numerical settings for simulations.

At the start of each step, the stance leg is modeled as a single link of length L , while the swing leg is modeled as two links connected by a frictionless joint. The system is governed by its unlocked knee dynamics until the swing leg straightens out. When the leg is fully extended,

the kneestrike occurs. At this point, the velocities change instantly due to the collision, and immediately afterwards, we switch to a two-link system in its locked knee dynamics phase.

Table 1. Notations and Numerical Settings

Notations	Name	Number &unit
m_t	Thigh mass	0.5 kg
m_s	Shank mass	0.05 kg
m_H	Hip mass	0.5 kg
a_1	Shank length (below point mass)	0.375 m
b_1	Shank length (above point mass)	0.125 m
a_2	Thigh length (below point mass)	0.175 m
b_2	Thigh length (above point mass)	0.325 m
ϕ	Virtual slope	Rad
q_1	Stance leg angle w.r.t. vertical	Rad
q_2	Thigh leg angle w.r.t. vertical	Rad
q_3	Shake leg angle w.r.t. vertical	Rad
u_1	Ankle torque	N-m
u_2	Hip torque	N-m
u_3	Knee torque	N-m

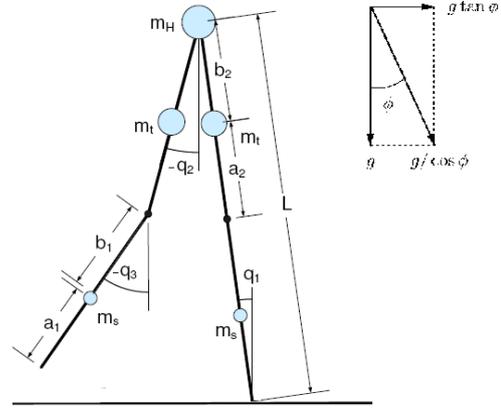


Figure 1. Four-link Kneaded Biped Model

2.1. Dynamic Equations

1) Unlocked Knee Dynamics. During the unlocked swing phase, the system is a three-link pendulum [3, 4]. The dynamics [5] are shown in the form of planar manipulator dynamics in Equation (1).

$$H(q)\ddot{q} + B(q, \dot{q})\dot{q} + G(q) = -J^T \lambda + \tau + \tau_c \quad (1)$$

Where, $J^T \lambda$ is the constraint force at knee-joints. τ is the control input and τ_c is the vector due to the environmental forces of the robot. The specific inertia, velocity-dependent and gravitational matrices are given in Equation (2).

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12} & H_{22} & H_{23} \\ H_{13} & H_{23} & H_{33} \end{bmatrix} \quad B = \begin{bmatrix} 0 & h_{122}\dot{q}_2 & h_{133}\dot{q}_3 \\ h_{211}\dot{q}_1 & 0 & h_{233}\dot{q}_3 \\ h_{311}\dot{q}_1 & h_{322}\dot{q}_2 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} -(m_s a_1 + m_t (l_s + a_2) + (m_h + m_s + m_t) L) g \sin(q_1) \\ (m_t b_2 + m_s l_t) g \sin(q_2) \\ m_s b_1 g \sin(q_3) \end{bmatrix} \quad (2)$$

2) Locked Knee Dynamics. After the kneestrike, the knee remains locked and the system switch to double-link pendulum dynamics. The remainder of the swing phase occurs with straight legs. The dynamics for the newly-locked system are exactly those of the compass gait dynamics but with a different mass configuration. The matrices of dynamics are shown in Equation (3) for completeness.

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix} \quad B = \begin{bmatrix} 0 & h\dot{q}_2 \\ -h\dot{q}_1 & 0 \end{bmatrix} \quad G = \begin{bmatrix} -(m_s a_1 + m_t (l_s + a_2) + (m_h + m_s + m_t) L) g \sin(q_1) \\ (m_t b_2 + m_s (l_t + b_1)) g \sin(q_2) \end{bmatrix} \quad (3)$$

After the swing foot touches the ground, the system switch the stance and swing leg. This completes a full step.

2.2. Collision Event

1) Kneestrike Dynamics. We model the kneestrike as a discrete collision event in a three-link chain and switch to the compass gait model afterwards. Since the only external force on this system is at the stance foot, angular momentum is preserved for the entire system about the stance foot and for the swing leg about the hip. When looking at the lower link of the swing leg, however, the kneestrike acts as an external impulse. Therefore, angular momentum is not conserved about the knee. But the knee joint angle corresponding to the knee is locked after the collision. Therefore, its post-collision velocity will be that of the second link. We express the change in velocities as:

$$Q^+ \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = Q^- \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \quad \dot{q}_3^+ = \dot{q}_2^+ \quad (4)$$

2) Heelstrike Dynamics. The heelstrike is modeled as an inelastic collision about the colliding foot. This heelstrike event is, again, identical to the heelstrike for the compass gait. Angular momentum is then conserved for the entire system about the colliding foot and for the swing leg after impact about the hip. Right after the event, the model switches both legs and the impact foot becomes the new stance foot. The model also switches back to the unlocked three-link dynamics to start a new step cycle. The third joint angle starts with the same angular position and velocity as the second one. This collision event is expressed in Equation (5).

$$q^+ = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} q^- \quad Q^+ \dot{q}^+ = Q^- \dot{q}^- \quad \dot{q}_3^+ = \dot{q}_2^+ \quad (5)$$

3. Optimized Virtual Passive Dynamic Walking

Passive dynamic walkers exhibit a stable, natural and energy-efficient gait. However, the passive walker cannot walk on the level ground without any external energy sources, so we will introduce “optimized virtual gravity” for biped robots to create the walking pattern automatically with the least muscle work and without loss of properties of passive dynamic walk on the floor.

3.1. Virtual Passive Dynamic Walking

1) Virtual Gravity and Active Walking. A virtual gravity toward the horizontal direction is as a driving force to walk forward [6]. We can transform the virtual gravity effect to the actuator's torque and it can be expressed as follows: ϕ is the virtual slope angle.

$$\tau = \begin{bmatrix} -(m_h L + m_s a_1 + m_t (l_s + a_2) + m_t L + m_s L) \cos(q_1) \\ (m_t b_2 + m_s l_t) \cos(q_2) \\ m_s b_1 \cos(q_3) \end{bmatrix} g \tan \phi \quad (6)$$

The transformation of control inputs are:

$$\tau = S \cdot u = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (7)$$

From Equation (6) and (7), we get:

$$u_1 = \left[(m_h L + m_s a_1 + m_t (l_s + a_2) + m_s L + m_t L) \cos(q_1) - (m_t b_2 + m_s l_t) \cos(q_2) - m_s b_1 \cos(q_3) \right] \cdot g \cdot \tan(\phi)$$

$$u_2 = \left[(m_t b_2 + m_s l_t) \cos(q_2) + m_s b_1 \cos(q_3) \right] \cdot g \cdot \tan(\phi) \quad u_3 = \left[m_s b_1 \cos(q_3) \right] \cdot g \cdot \tan(\phi)$$

2) Multi Virtual Gravity. The dynamics of kneed biped robots is more complex than that of compass-gait ones. So the steady gait of a kneed walker with single virtual gravity cannot be obtained easily without suitable parameter choice. At the same time, we also want to optimize the total energy that the biped robot has consumed on the actuators during the walking. Based on the observation, we propose “Multi Virtual Gravity” for the kneed biped robot in order to generate steady walking patterns and optimize the walking energy without loss of virtual passivity (Figure 2.). The transformed torque of Multi Virtual Gravity effect is given by:

$$\tau = R_c(q) \cdot \Gamma \cdot \tan \phi \cdot g \cdot \alpha_{nks} \quad (8)$$

Where,

$$\tan(\phi) = \begin{bmatrix} \tan \phi_H \\ \tan \phi_1 \\ \tan \phi_2 \\ \tan \phi_3 \\ \tan \phi_4 \end{bmatrix} R_c(q) = \begin{bmatrix} \cos(q_1) & & & & \\ & \cos(q_2) & & & \\ & & \cos(q_3) & & \\ & & & & \\ & & & & \end{bmatrix} \Gamma = \begin{bmatrix} -m_h \cdot L & -m_s \cdot a_1 & -m_t (l_s + a_2) & -m_t \cdot L & -m_s L \\ & & & m_t b_2 & m_s l_t \\ & & & & m_s b_1 \end{bmatrix}$$

And α_{nks} is the sensitivity function of control inputs (virtual gravity scale).

3) Virtual Energy. The passivity of virtual passive walker is also able to be shown using “virtual energy”, that is:

$$E_v = \frac{1}{2} \dot{q}^T M(q) \dot{q} + P_v(q, \phi) \quad (9)$$

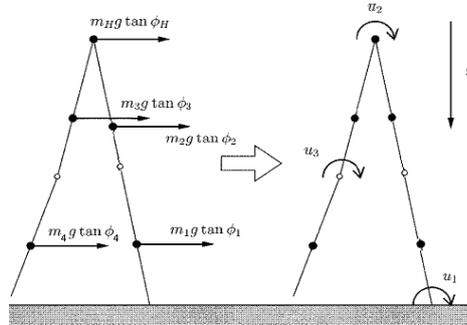


Figure 2. Optimizing Multi Virtual Gravity Fields

The virtual potential energy is given by:

$$P_v(q, \phi) = \frac{J_H \cdot g}{\cos(\phi_H)} + \sum_{i=1}^4 \frac{J_i \cdot g}{\cos(\phi_i)} \quad (10)$$

3.2. Optimized Multi Virtual Gravity

Why do humans and other animals move the way they do? An ancient hypothesis, dating back at least to a contemporary of Galileo and Newton (Borelli [7]), is that animals move in a manner that minimizes effort, perhaps as quantified by metabolic cost per distance travelled [8-10].

In order to optimizing the total actuator's torque energy and keep steady walking pattern without loss of virtual passivity, we use the 'SNOPT' software to calculating the suitable control parameter of the biped robot.

1) Optimizing Target. The optimizing target is the total actuator's torque energy. During the Unlocked Knee Stage the optimizing target is setting as follows:

$$\int_0^t (\tau(1)^2 + \tau(2)^2 + \tau(3)^2) dt \quad (11)$$

2) Optimizing Condition. A gait is characterized by the position and velocity of the body at the start of a stance phase relative to the stance foot, by the step period, and by τ .

The optimal solutions have cost arbitrarily close to zero unless the optimization is further constrained. Firstly, the cost P can be reduced nearly to zero by taking small steps [11-13]. So, we optimize for various fixed values of step length d . Secondly, the cost P has a non-anthropomorphic lower bound (corresponding to standing on one leg for an infinite time mid-step), approached as the average speed v goes to zero, so we constrain v .

Because we just want to find the walking gait with the least muscle work W , we choose low speed $v=0.185$, and short step length $d=0.1$. These can guarantee that the robot operates a classic inverted-pendulum walking gait but not an impulsive running gait.

We can find a steady fixed point for the Table 1 parameters when the virtual slope angle ϕ is 0.0504 rad. We use the fixed point and ϕ as the optimizing initial condition.

3) Optimized Numerical Simulation Result. By using the 'SNOPT' software and according to the optimizing target and constraint condition, we can get the result which makes the total actuator's torque energy minimal. At the same time, the result has steady walking patterns and can walk without loss of virtual passivity. A limit cycle for the upper link of one leg starting from this fixed point is shown in Figure 3.

The instantaneous velocity changes from the heelstrike and kneestrike events can be observed in this limit cycle as straight lines where the cycle jumps with the instantaneous velocity changes while the positions remain the same. In contrast with the compass gait, however, in addition to the two heel-strikes, there are two more instantaneous velocity changes produced by the kneestrikes.

The Angle trajectory of the virtual passive dynamic walking after energy optimization is shown on the left hand of Figure 4. An energy plot, showing potential energy, as well as total mechanical energy is shown on the right hand of Figure 4.

In this plot, there are step increases in the potential energy on each foot transfer, since we write the energy of the system relative to the stance point. These increases are shown to exactly balance out the kinetic energy lost throughout one step. We can see that the final mechanical energy is constant throughout.

And by using the optimized result to control the biped robot, we can get a batch stable Eigenvalues for the linearized map.

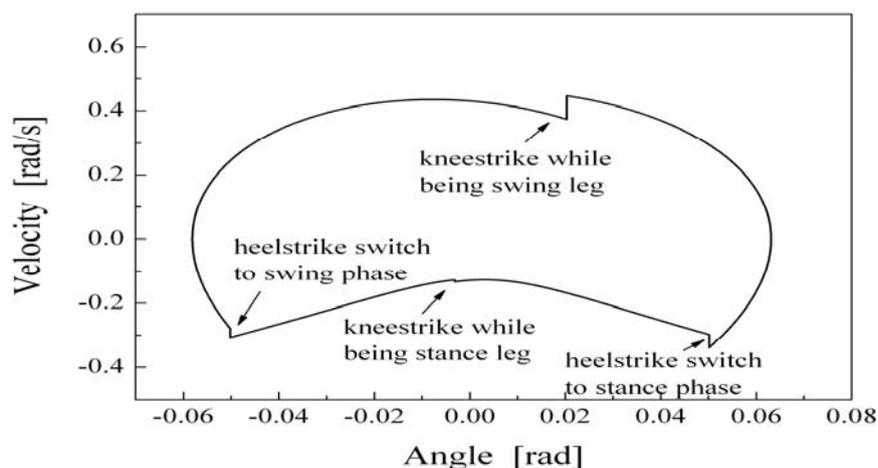


Figure 3. Limit Cycle Trajectory

So, this approach can achieve walking gait with the least muscle work at fixed speed and step length, and keep steady walking patterns without loss of virtual passivity.

This maybe means that the optimized multi virtual gravity passive walking pattern on the level ground is also another people's natural walking motion. And people can also use the similar walking pattern on the level ground as on the slope. That is why as we walk, especially downhill, we do no stop ourselves at every step. On the contrary, we let our body fall forward, only stopping ourselves at each footstep.

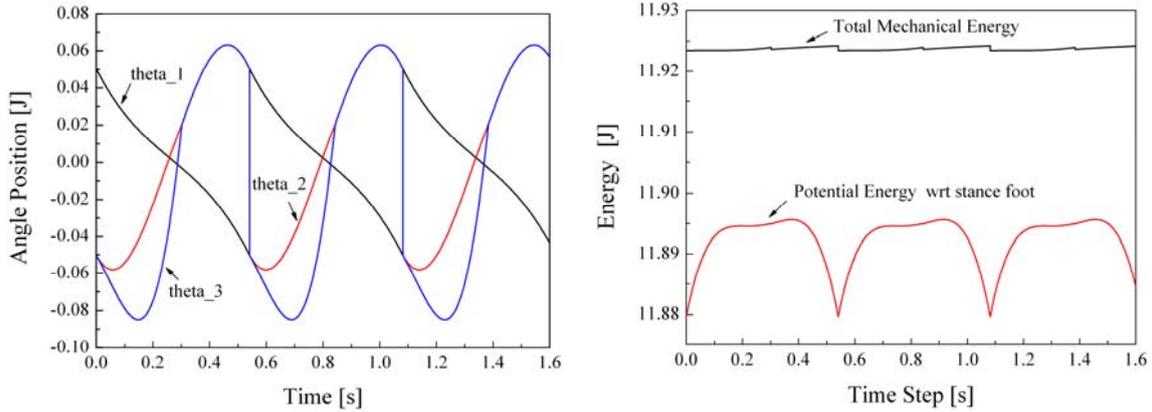


Figure 4. Virtual Passive Dynamic Walking with Energy Optimization

4. Optimized Virtual Coupling Control

In this section, we introduce “Optimizing Virtual Coupling Control” in order to generate variable walking pattern with respect to the energy levels without loss of properties of passive dynamic walk on the floor. The basic concept is to regard hybrid dynamical systems as impact-less (smooth) dynamical systems virtually and the energy information is only utilized for the control. In order to realize the robot system passive and smooth, we consider a virtual flywheel in computer. Then the biped robot combined with flywheel exhibits passive and smooth as an augmented mechanical system. After that, we try to find the walking gait with the least muscle work by using this control method.

4.1. Augmented Mechanical System

The dynamic equation of the biped robot is given by:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) = \tau + \tau_e \quad (12)$$

And that of flywheel:

$$M_f\ddot{\theta}_f = \tau_f \quad (13)$$

The augmented mechanical system is given by the following equation:

$$\begin{bmatrix} M(\theta) \\ M_f \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\theta}_f \end{bmatrix} + \begin{bmatrix} C(\theta, \dot{\theta}) \\ 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\theta}_f \end{bmatrix} + \begin{bmatrix} g(\theta) \\ 0 \end{bmatrix} = \begin{bmatrix} \tau \\ \tau_f \end{bmatrix} + \begin{bmatrix} \tau_e \\ 0 \end{bmatrix} \quad (14)$$

We denote (12) as:

$$M^a(q)\ddot{q} + C^a(q, \dot{q})\dot{q} + g^a(q) = \tau^a + \tau_e^a \quad (15)$$

4.2. Coupling Control Law

The control input τ^a for the system (15) is given by:

$$\tau^a = \begin{bmatrix} \tau \\ \tau_f \end{bmatrix} = \begin{bmatrix} R_c(\theta)\Gamma \tan \phi g \\ 0 \end{bmatrix} + \begin{bmatrix} \bar{\tau} \\ \tau_f \end{bmatrix} \quad (16)$$

The second term of the right hand is the coupling control input for the decoupled system which is defined as following:

$$\begin{bmatrix} \bar{\tau} \\ \tau_f \end{bmatrix} = P\dot{q} = \begin{bmatrix} P \\ -P^T \end{bmatrix} \dot{q} \quad (17)$$

At first, we will introduce augmented virtual energy which is the sum of virtual energy of the biped robot and kinetic energy of the flywheel:

$$E_v^a(q, \dot{q}, \phi) = E_v(\theta, \dot{\theta}, \phi) + K_f \quad (18)$$

The control formulation (17) can be explained from the following viewpoint. The target condition of the augmented mechanical energy is given by:

$$\frac{d}{dt}(E_v + K_f) = 0 \quad (19)$$

And then we get:

$$\frac{d}{dt}K_f = \frac{d}{dt}\left(\frac{1}{2}M_f\dot{\theta}_f^2\right) = M_f\dot{\theta}_f\ddot{\theta}_f = \dot{\theta}_f\tau_f = -\dot{\theta}^T\bar{\tau} \quad (20)$$

Hence, from (20) the following result is obtained.

$$\tau_f = -\dot{\theta}_f^{-1}\dot{\theta}^T\bar{\tau} \quad (21)$$

This is equivalent to (17) where $p = \dot{\theta}_f^{-1}\bar{\tau}$. Considering the structure of (10), let us set the vector p in (17) as the following form:

$$p = R_c(\theta)\Gamma \tan(\phi)\eta\Delta E_v^a\dot{\theta}_f^{-1} \quad (22)$$

Where, $\Delta E_v^a = E_v^* - E_v^a$ is the change of the augmented virtual energy and $\eta > 0$ is the feedback gain. E_v^* is the nominal value of the E_v^a .

4.3. Optimized Virtual Coupling Control

In this section, we will use virtual coupling controller to drive the biped robot, optimize the total actuator's torque energy, and get the walking gait with the least muscle work. Then we try to find some rules of human walking through this kind of walking control pattern that is without any reaction force against external forces.

In order to optimizing the total actuator's torque energy, we use the 'SNOPT' software to calculating the suitable control parameter of the biped robot.

1) Numerical Simulation Result. By using the 'SNOPT' software and according to the optimizing target and constraint condition, we can get the result which makes the total actuator's torque energy minimal. At the same time, the method can realize the robot system passive and smooth, the energy information is only utilized for the control.

An energy plot, showing real and virtual total mechanical energy and potential energy, as well as augmented mechanical energy is shown in Figure 5. The graph on the left hand is the

result with energy optimization, and the graph on the right hand is the result without energy optimization.

The actuator's torques of the virtual coupling control dynamic walking is shown in Figure 6. The left hand is the result with energy optimization, and the right hand is without energy optimization.

2) Result Analysis. From Figure 5 and Figure 6, we can see that the robot and virtual flywheel storage energy each other and as a result of it the total energy value of the augmented system is kept constant. Furthermore, from Figure 6 we can see that the constant-like torque is also succeeded. It means that these walking patterns are natural motion.

According to the simulation result, we can also know that after energy optimization, the total mechanical energy and augmented mechanical energy are both smaller than those without optimization.

From the comparison between the figures of actuator's torques, we can see that the amplitudes of the torques of hip joint and knee joint keep similar between after optimization and before optimization, and the value of ankle torque is much bigger before optimization. This maybe means another walking rule that the cost consumed by the torques on knee joint and hip joint is similar in different kinds of walking gaits, and the biggest different muscle work mainly come from the cost of ankle torque. So, if human want to raise the total input walking energy to increase the walking speed and enlarge step length, the most practical and effective way is to increase the push-off impulses force, even though people use the muscle of thigh or shank to enforce.

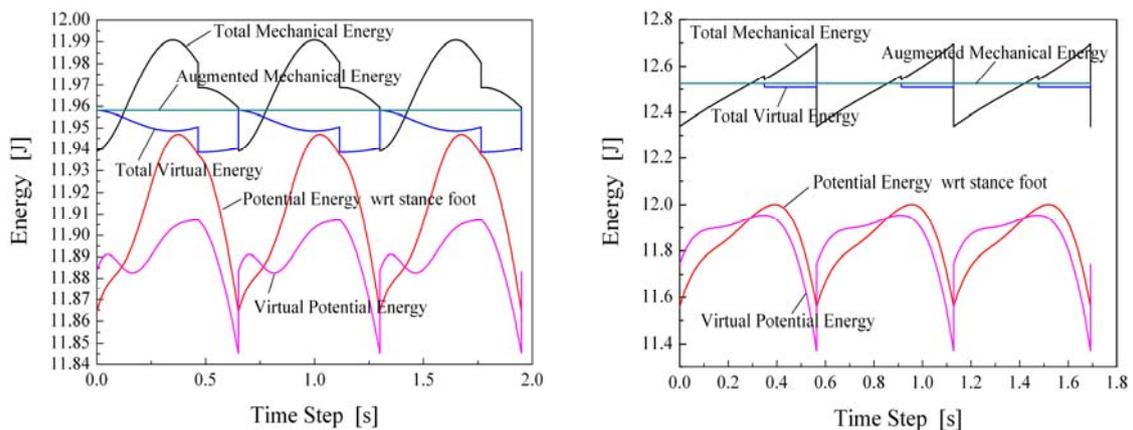


Figure 5. Energy Plot

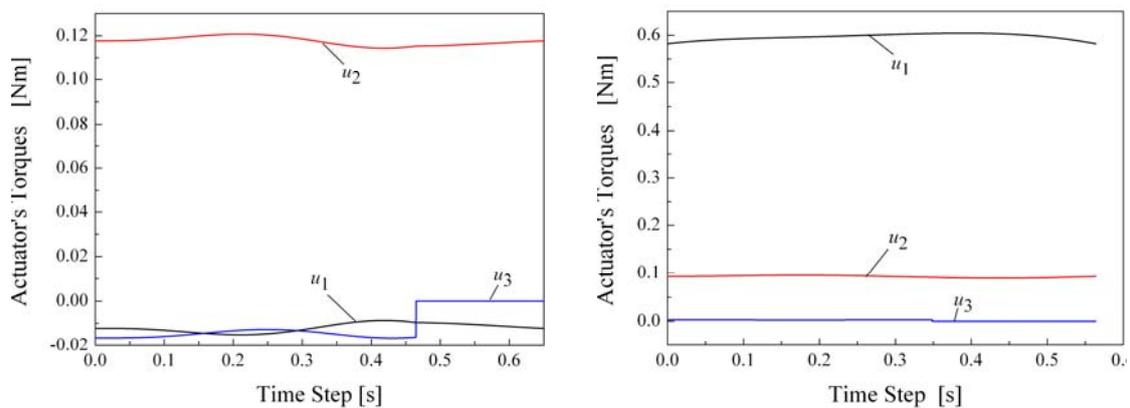


Figure 6. Actuator's Torques

5. Conclusion

Here, using numerical optimization and supporting analytical arguments, we realize the safe passive dynamic control against human being and outside environment and obtain the energy minimizing gaits by using optimized virtual passive and virtual coupling control strategy. At low speeds the optimization discovers the inverted-pendulum walk with the least muscle work. We compare the results of simulations, and we discover some walking rules maybe true.

- (1) The the optimized multi virtual gravity passive walking pattern is also another people's natural walking motion on the level ground.
- (2) The biggest different muscle work mainly come from the cost of ankle torque in different kinds of walking gaits. So, if human want to change the walking pattern, the most practical and effective way is to change the push-off impulses force.

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