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A composite controller based on nonlinear H_∞ and nonlinear disturbance observer for attitude stabilization of a flying robot

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Article Info

ABSTRACT

In this article a novel composite control technique is introduced. We added a nonlinear disturbance observer to a nonlinear H_∞ control to form this composite controller. The quadrotor kinematics and dynamics is formulated using euler angles and parameters. After that, this nonlinear robust controller is developed for this flying robot attitude control for the outdoor conditions. Because under these conditions the flying robot, experiences both external disturbance and parametric uncertainty. Stability analysis is also presented to show the global asymptotical stability using a Lyapunov function. The simulation results showed that the suggested composite controller had a better performance in comparison with a nonlinear H_∞ control scheme.

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1. INTRODUCTION

In recent years, research on flying robots has been carried out by many researchers for nearly two decades. Flying robots have wide range of both civilian and military applications. Among the several kinds of flying robots, quadrotor has attracted particular attention due to its unique advantages: simple structure, low cost, vertical maneuvering ability, and becoming stationary in a special altitude [1]. Although, its characteristics, such as under-actuated property, inherent nonlinearity, and external disturbances related to the uncertain flying environment and, aerodynamics forces, make the flight control a challenging task.

Having a stable orientation of quadrotor is an important target [2]. So, designing an appropriate attitude controller is a significant work [3]. On the other hand, the presence of the inertia uncertainties and disturbances introduces a more complicated problem [4].

Recently, different attitude control schemes which consider external disturbance and model uncertainty have been developed [5-10]. Nonlinear disturbance observer-based control is a useful technique to enhance robustness [11, 12]. A nonlinear disturbance observer is developed to estimate and compensate disturbances with an effective feedback. Developing core strategy tends to attenuate the unmodeled dynamics [13].

Disturbance observer-based control methods have been extended to various applications since late 1980 [14]. Disturbance attenuation problem investigated for a class of multi-input multi-output (MIMO) nonlinear dynamics in the disturbance observer-based control technique [15]. A robust attitude following control strategy for a flying robot uing a nonlinear disturbance observer, which an effective design strategy is

developed [16]. By combining the disturbance observer and traditional control approaches, the disturbance can be attenuated, and good performances can be ensured. A robust controller is investigated for the attitude following and disturbance attenuation task for a solid spacecraft, which includes the sinusoidal disturbance. Hence, the problem is converted to a global stability analysis of a class of nonlinear system including disturbance as well as payload uncertainty [17].

Beyond the different nonlinear control approaches to overcome uncertainty for the flying robot different movements, the nonlinear disturbance observer-based control is shown as an important disturbance attenuation scheme, which can estimate disturbances and try to reject them through feedforward signals [18]. Terminal sliding mode control plus disturbance observer-based control are proposed for a MIMO nonlinear dynamics [19]. A hybrid control law is extended by combining feedback linearization and nonlinear disturbance observer to reject the effects of parameter variations and disturbances of a non-rigid spacecraft [20]. The nonlinear disturbance observer can introduce better disturbance rejection ability and improves its robustness and it is possible to use it with other classic control techniques as a combined strategy [20].

Nonlinear robust control methods are so attractive in the attitude control problem. Nonlinear H_∞ control method can attenuate the disturbance energy [21]. This approach has been implemented for the attitude control in [22, 23]. A nonlinear H_∞ method is solved analytically for altitude as well as attitude control [24].

In the present work, a novel attitude controller is introduced to enhance the disturbance rejection and robust stabilization of a quadrotor. A nonlinear disturbance observer, which can estimate disturbances, is designed based on which feedforward compensation is applied, which increases the disturbance attenuation and robustness of a quadrotor. The main contribution of this article can be expressed as follows. We proposed a composite control technique using combination of the nonlinear disturbance observer and the nonlinear H_∞ control to stabilize the attitude of a flying robot. This novel control approach can yield fast and precise performance and give the attitude and rotational velocity errors asymptotically stable with disturbances and uncertainty in the matrix of inertia.

The outline of the present article ia as follows. Section 2 shows the basics quadrotors attitude dynamics. Section 3 gives a review of nonlinear H_∞ control theory. Section 4 proposes a composite attitude control technique for a flying robot subject to external disturbances and inertia uncertainty. Simulations are brought to demonstrate the effectiveness of the suggested approach is presented in Section 5, and the conclusions are in Section 6.

2. PROBLEM FORMULATION

In spite of six degrees of freedom, different maneuvers of tha quadrotor are provided with four independent commands of motor-propeller systems that are located on a rigid X-shaped structure. So it is called under-actuated. The six degrees of freedom consist of rotational motion around three axes and translational motion in three directions. The diagonal rotors rotate clockwise and counterclockwise, respectively.

The attitude of the quadrotor is given by Euler angles, $\eta = [\psi \quad \theta \quad \phi]^T \in R^3$, where ψ, θ and ϕ denote angular yaw, pitch, and roll with respect to the inertia frame. The rotational dynamics of the quadrotor described by the following relations:

$$\dot{R} = S(\omega)R \tag{1}$$

$$J\dot{\omega} = -S(\omega)J\omega + \tau \tag{2}$$

where $\omega = [\omega_1 \quad \omega_2 \quad \omega_3]^T \in \mathbb{R}^3$ represents the rotational velocity of the quadrotor in the inertial frame defined in the body-fixed frame, $J = diag\{J_1, J_2, J_3\}$, is the matrix of inertia. The matrix $S(.)$ is defined in the following form:

$$S(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{3}$$

The matrix $R \in SO(3)$ which is defined in the following form:

$$R(\eta) = \begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - c\phi s\theta & c\phi c\psi s\theta + s\phi s\psi \\ c\theta s\psi & s\theta s\phi s\psi + c\phi c\psi & c\phi c\theta s\psi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

where s. $\triangleq \sin(.)$ and c. $\triangleq \cos(.)$. The vector $\tau = [\tau_1 \quad \tau_2 \quad \tau_3]^T \in \mathbb{R}^3$ represents the control input moment. Detailed information on mathematical modeling can be found in many papers such as [25, 26].

The equilibrium point for this task is equal to $R = I, \omega = 0$. At this time, we take into account a robust nonlinear controller which stabilizes the attitude of the flying robot. We considered $d = [d_1 \quad d_2 \quad d_3]^T$ as a model disturbance acting on the quadrotor. According to the system (1), (2) with disturbance d, the dynamic system is considered in the affine form [20] :

$$\dot{x} = f(x) + g(x)u + k(x)d \tag{4}$$

where $x = [\eta^T \quad \omega^T]^T$, and $f(x) = \begin{bmatrix} T(\eta) \\ J^{-1}S(\omega)J \end{bmatrix}, g(x) = \begin{bmatrix} 0_{3\times 3} \\ J^{-1} \end{bmatrix}, k(x) = \begin{bmatrix} 0_{3\times 3} \\ J^{-1} \end{bmatrix}$.
the g and k are generally different funcions. Although in (4), they are the same.

3.   NONLINEAR H_∞ CONTROL APPROACH

Suppose the following nonlinear dynamics:

$$\dot{x} = f(x) + g(x)u + k(x)d \, , f(x_0) = 0 \tag{5}$$

$$z = \begin{bmatrix} h(x) \\ u \end{bmatrix} , h(x_0) = 0$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $d \in \mathbb{R}^p$, and $z \in \mathbb{R}^q$ and f, g, h, and k are smooth functions. The objective of the nonlinear H_∞ approach is to design a control law, $u = l(x)$, such that for a known $\gamma > 1$, the following relation holds:

$$\int_0^T \|z(t)\|^2 dt \le \gamma^2 \int_0^T \|d(t)\|^2 dt, \ \forall \, d \in \mathcal{L}_2[0\,,T] \tag{6}$$

where $d \in \mathcal{L}_2[0\,,T]$ means that d is square-integrable on $[0\,,T]$ and $\|(.)\|$ shows the Euclidean norm. From [19], the nonlinear H_∞ control formula is obtained as:

$$u_r = -g^T(x)(\frac{\partial V}{\partial x})^T \tag{7}$$

Note that $V(x)$ is zero at the equilibrium point, and positive elsewhere, and

$$H_\gamma \left(x, \frac{\partial V}{\partial x} \right) = \frac{\partial V}{\partial x} f(x) + \frac{1}{2} \frac{\partial V}{\partial x} \left(\frac{1}{\gamma^2} k(x)k^T(x) - g(x)g^T(x) \right) \left(\frac{\partial V}{\partial x} \right)^T + \frac{1}{2} h(x)h^T(x) \le 0 \tag{8}$$

To calculate u_r in (7), we must solve (8) for $V(x)$. Relation (8) is called Hamilton-Jacobi-Isaac inequality.

Proposition1 [19]. If relation (5) is zero-state observable and there exists a proper solution $V(x) \ge 0$ to (8), then $V(x)$ is positive for $x(t) \ne x_0$ and global asymptotic stability is ensured for the closed-loop system (5), (7) with $d = 0$.

4.   THE PROPOSED COMPOSITE ATTITUDE STABILIZATION
4.1. The nonlinear H_∞ control for attitude stabilization

The nonlinear H_∞ controller was developed in [20] for this problem. The suggested $V(x)$, for the attitude stabilization model (8) is in the form [20] :

$$V(x) = \frac{1}{2} [Y^T \quad \omega^T] \begin{bmatrix} cI & -bJ \\ -bJ & aJ \end{bmatrix} \begin{bmatrix} Y \\ \omega \end{bmatrix} \tag{9}$$

where a, b, c are constants, I is the identity 3×3 matrix. Y is given by:

$$Y = \begin{bmatrix} s\psi s\theta c\phi - s\phi(c\psi + c\theta) \\ -s\theta(1 + c\psi c\phi) - s\psi s\phi \\ c\psi s\theta s\phi - s\psi(c\phi + c\theta) \end{bmatrix}$$

then the sufficient conditions guarantee that $V(x) \geq 0$ are $a > 0$, and $acI > b^2 J$.

According to (9), (11) the controller is:

$$u = -g^T(x)(\frac{\partial V}{\partial x})^T$$
$$= bY - a\omega \tag{10}$$

For more details, please see [20].

4.2. The nonlinear disturbance observer

This strategy, which provides a proper disturbance estimation for the attitude dynamics (4), is obtained by [14]:

$$\dot{y} = -L(x)k(x)y - L(x)[k(x)p(x) + f(x) + g(x)u]$$
$$\dot{\tilde{d}} = -L(x)g(x)\tilde{d} \tag{11}$$
$$\hat{d} = y + p(x)$$

where $\hat{d}$ and y are the estimations of the disturbance and the auxillary state respectively, $p(x)$ is a function that must be determined, $\tilde{d} = d - \hat{d}$, and $L(x)$ is calculated, as:

$$L(x) = \frac{\partial P(x)}{\partial x}$$

and the function $p(x)$ is considered as:

$$p(x) = \lambda J\omega \tag{12}$$

Hence, the composite control law (a combination of nonlinear H_∞ control and NDO), can be designed as:

$$u = u_r - \hat{d} \tag{13}$$

where $u = [u_1 \quad u_2 \quad u_3]^T$ and d is estimated by $\hat{d}$. Our suggested control scheme is depicted in Figure 1. Now, the following theorem summarizes the main result of the paper.
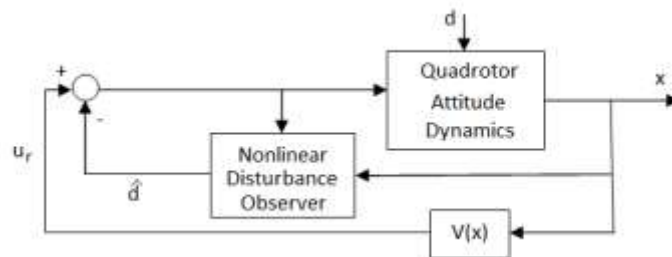

Figure 1. Structure of the suggested control scheme

Theorem 1. Suppose the attitude kinematics and dynamics systems (1), (2). The final control law consisting of nonlinear H_∞ control (10) and nonlinear disturbance observer (11) is given by:

$$u = bY - a\omega - (y + \lambda J\omega) \tag{14}$$

Then the attitude kinematics and dynamics systems (1), (2) are stable in global asymptotic manner, where $a > 0$, and $acI > b^2 J$ and $\lambda > 0$ in (14). The composite controller gives the attitude and rotational velocity errors asymptotically stable with inertia mismatching and external disturbance.

Proof: Suppose the candidate Lyapunov function as:

$$V(x, \hat{d}) = V_c + V_O = V_c(x) + \frac{1}{2}\mu \tilde{d}^T \tilde{d} \tag{15}$$

where μ is a positive value that is determined and V_c is considered as (9). Then, $\frac{dV}{dt}$ or $\dot{V}$ can be given by:

$$\dot{V} = \frac{\partial V_c(x)}{\partial x}(f(x) + g(x)u_r + g(x)\tilde{d}) + \dot{V}_o(\tilde{d}) \tag{16}$$
$$= \frac{\partial V_c(x)}{\partial x}(f(x) + g(x)u_r) + \frac{\partial V_c(x)}{\partial x}g(x)\tilde{d} + \dot{V}_o(\tilde{d})$$

calculating $\dot{V}_o(\tilde{d})$ and substituting that into (16), one can get:

$$\dot{V} = \frac{\partial V_c(x)}{\partial x}(f(x) + g(x)u_r) + \frac{\partial V_c(x)}{\partial x}g(x)\tilde{d} - \tilde{d}^T L(x)g(x)\tilde{d} \tag{17}$$

Now, (17) can be transformed to:

$$\dot{V} \leq -\delta_1\|x\|^2 + \delta_2\|x\|\|\tilde{d}\| - \delta_3\|\tilde{d}\|^2 \tag{18}$$
$$\leq -(\sqrt{\delta_1}\|x\| - \sqrt{\delta_3}\|\tilde{d}\|)^2 \leq 0$$

Now if $\delta_2 < 2\sqrt{\delta_1}\sqrt{\delta_3}$ in (18), we can conclude that $\dot{V} \leq 0$ is hold. This means that $V(t) \leq V(0)$. This concludes the proof.

5. SIMULATION RESULTS

Simulations show how this composite control method is effective in comparison with nonlinear H_∞ control. The numerical simulations are carried out in MATLAB. The numerical magnitudes of the model for simulation are extracted from [21]:

$$J = \begin{bmatrix} 0.004 & 0 & 0 \\ 0 & 0.004 & 0 \\ 0 & 0 & 0.0084 \end{bmatrix} Kg.m^2, \qquad m = 0.74\, kg, \qquad g = 9.81\, m.s^{-2}$$

The initial conditions are considered in the following form:

$$\eta_0 = [0.25 \quad 0.25 \quad 0.25]^T\, rad, \qquad \dot{\eta}_0 = [10.8982 \quad -1.6821 \quad 0.1]^T\, rad.s^{-1}$$

The nonlinear H_∞ controller gains and nonlinear disturbance observer have been adjusted with the following parameters:

$$\gamma = \sqrt{2}\;, a = 2.1708, b = 2, \lambda = 1$$

The following moment was used as a disturbance:

$$d(t) = [2 \quad 2 \quad 2.5]^T e^{-t}\, N.m$$

The results of simulation are depicted in Figures 2 to 4. Figure 2 presents the attitude of the flying robot with external disturbance and +40% inertia uncertainties. The control efforts are expressed in Figure 3. The angular velocities are shown in Figure 4. It is shown in Figure 2 that the composite control approach performing faster and better than the nonlinear H_∞ control with the existence of disturbances which are applied externally and uncertainty in the matrix of inertia.

The control efforts are illustrated in Figure 3. The control efforts of the proposed controller are faster than the nonlinear H_∞ control inputs. The angular velocity under the use of the composite control technique with disturbance and inertia uncertainty is depicted in Figure 4. The comparison between these two

controllers shows that the rotational velocities performance applying the composite control technique is reached faster to the stability conditions than the nonlinear H_∞ control approach.
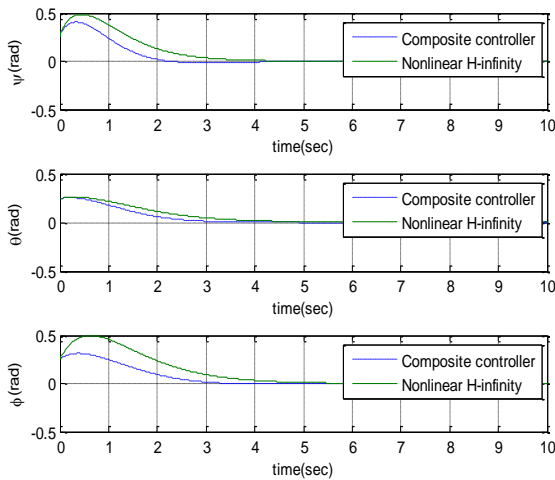

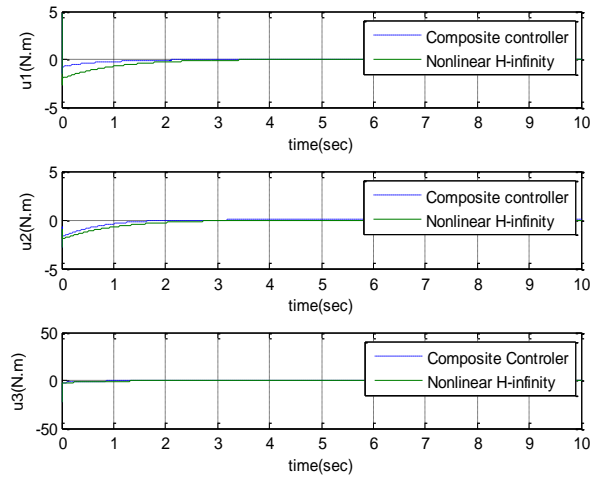Figure 2. Attitude with external disturbance and +40% moment of inertia increment
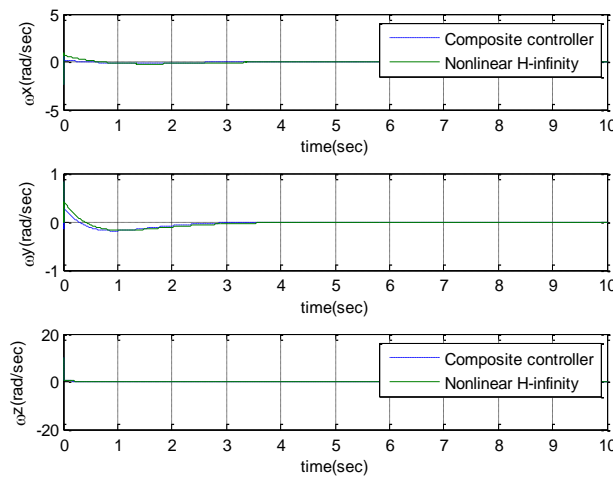
Figure 3. Control efforts

Figure 4. Angular velocities

6.   CONCLUSION

Both nonlinear H_∞ and nonlinear disturbance observer is famous for their robustness property. Nonlinear H_∞ control attenuates disturbance and parametric uncertainty. Nonlinear observer-based control has been used to attenuate the effect of partially known disturbances. In this research work, a combination of two techniques is introduced as a novel strategy. The proposed approach improves the nonlinear H_∞ control scheme. The Simulation results illustrate that with the suggested control approach, the improved performance of the system can be reached with the external disturbance and the parametric uncertainty. Implementing this proposed composite controller on a real-time experimental setup is suggested for the future work.

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