Certain properties of ω-Q-fuzzy subrings

Dilshad Alghazzawi[1], Wafaa H. Hanoon[2], Muhammad Gulzar[3], Ghazanfar Abbas[4], Nasreen Kausar[5]
[1]Department of mathematics, King Abdulaziz University (Rabigh), Jeddah, Saudi Arabia
[2]Department of Computer Science, College of Education for Girls, University of Kufa, Kufa, Iraq
[3]Department of Mathematics, Government College University Faisalabad, Pakistan
[4]Department of Mathematics and Statistics, Institute of Southern Punjab, Multan, Pakistan
[5]Department of mathematics, University of Agriculture Faisalabad, Faisalabad, Pakistan

Article Info

Article history:

Received Jun 23, 2020
Revised Aug 11, 2020
Accepted Aug 21, 2020

Keywords:

Q-fuzzy set
Q-fuzzy subring
ω-Q-fuzzy ideal
ω-Q-fuzzy set
ω-Q-fuzzy subring

ABSTRACT

In this paper, we define the ω-Q-fuzzy subring and discussed various fundamental aspects of ω-Q-fuzzy subrings. We introduce the concept of ω-Q-level subset of this new fuzzy set and prove that ω-Q-level subset of ω-Q-fuzzy subring form a ring. We define ω-Q-fuzzy ideal and show that set of all ω-Q-fuzzy cosets form a ring. Moreover, we investigate the properties of homomorphic image of ω-Q-fuzzy subring.

Corresponding Author:

Muhammad Gulzar
Department of Mathematics
Government College University Faisalabad, 38000, Pakistan
Email:98kohly@gmail.com

1. INTRODUCTION

In mathematics, ring theory is one of the most important part of abstract algebra. In algebra, ring theory studies the algebraic structures of rings. Rings algebraic structure is a framework in which addition and multiplication are well defined with some more properties.

The concept of fuzzy sets was introduced by Zadeh [1] in 1965. Many mathematician have applied various hybrid models of fuzzy sets and intuitionistic fuzzy sets to several algebraic structure such as group theory [2, 3], non-associative ring [4, 5], time series [6, 7] and decision making [8]. Rosenfeld [9] commenced the idea of fuzzy subgroups in 1971. The fuzzy subrings were initiated by Liu [10] Dixit et al. [11] described the notion of level subgroup in 1990. The idea of anti-fuzzy subgroups was invented by Biswas [12]. Gupta [13] defined many classical t-operators in 1991. Solairaju and Nagarajan [14] explored a new structure and construction of Q-fuzzy groups in 2009. Muthuraj et al. [15] proposed the study of lower level subsets of anti-QFS in 2010. The concept of Q-fuzzy normal subgroups and Q-fuzzy normalizer were established by Priya et al. [16] in 2013. Sither Selvam et al. [17] used the Biswas work to modify the idea of anti-QFNS in 2014. Alsarahead and Ahmed [18-20] commenced new concept of complex fuzzy subring, complex fuzzy subgroup and complex fuzzy soft subgroups in 2017. Makaba and Murali [21] discussed fuzzy subgroups of finite groups. Rasuli [22] discussed Q-fuzzy subring with respect to t-norm in 2018. The Q-fuzzy subgroup in algebra was discussed in [23]. More development about fuzzy subgroup may be viewed in [24]. Shafei et al [25] studied the fuzzy logic control systems for demand management in airports and energy efficiency by using 3D simulator.

This paper is organized as the section 2 contains the elementary definition of Q-fuzzy subrings and related results which are thoroughly crucial to understand the novelty of this article. In section 3, we define the ω-Q-fuzzy subring and prove that the level subset of ω-Q-fuzzy subrings is a subring. We also define ω-Q-fuzzy ideal and discuss its properties. In section 5, we use the classical ring homomorphism to investigate the behavior of homomorphic image (inverse-image) of ω-Q-fuzzy subring.

2. PRELIMINARIES

We recall first the elementary notion of fuzzy sets which play a key role for our further analysis.

Definition 2.1. [1]: A fuzzy set A of a nonempty set M is a function,

$$A : P \to [0, 1]$$

Definition 2.2. [10]: Let A be fuzzy subset of a ring R. Then A is said to a fuzzy subring if

i. $A(u-v) \geq \min\{A(u), A(v)\}$
ii. $A(uv) \geq \min\{A(u), A(v)\}$, for all $u, v \in R$.

Definition 2.3. [14]: Let M and Q be two nonempty sets. A Q-fuzzy subset A of set M is a function $A: X \times Q \to [0,1]$ for all $u, v \in M$ and $q \in Q$.

Definition 2.4. [14]: A function $A : R \times Q \longrightarrow [0,1]$ is a QFSRR of a ring R if

i. $A(u-v,q) \geq \min\{A(u,q), A(v,q)\}$
ii. $A(uv,q) \geq \min\{A(u,q), A(v,q)\}$, for all $u, v \in R$ and $q \in Q$.

Definition 2.5. Let the mapping $f: R_1 \to R_2$ be a homomorphism. Let A and B be ω-QFSRs of R_1 and R_2 respectively, then $f(A)$ and $f^{-1}(B)$ are image of A and the inverse image of B respectively, defined as

i. $f(A)(v,q) = \begin{cases} \sup\{A(u,q): u \in f^{-1}(v)\}, \text{if } f^{-1}(v) \neq \emptyset \\ 0, \text{if } f^{-1}(v) = \emptyset \end{cases}$, for every $v \in R_2$ and $q \in Q$
ii. $f^{-1}(B)(u,q) = B(f(u),q)$, for every $u \in R_1$ and $q \in Q$

Definition 2.6. [13]: Let $t_p: [0,1] \times [0,1] \to [0,1]$ be the algebraic product t-norm on $[0,1]$ and is described as $t_p\{a,b\} = ab, 0 \leq a \leq 1, 0 \leq b \leq 1$

3. PROPERTIES OF ω-Q-FUZZY SUBRINGS

Definition 3.1. Let M and Q be any two nonempty sets and A be a Q-fuzzy subset of a set P, any $\omega \in [0,1]$. Then fuzzy set A^ω of M is said to be ω-Q-fuzzy subset of M (w.r.t Q-fuzzy set A) and defined by:

$$A^\omega(m,q) = t_p\{A(m,q), \omega\}, \text{for all } m \in M \text{ and } q \in Q$$

Remark 3.2. Clearly, $A^1(m,q) = A(m,q)$ and $A^0(m,q) = 0$.

Remark 3.3. If A and B be two Q-fuzzy sets of M. Then $(A \cap B)^\omega = A^\omega \cap B^\omega$.

Definition 3.4. A Q-fuzzy subset of a ring R is called ω-QFSR, and $\omega \in [0,1]$, if

i $A^\omega(m-n,q) \geq \min\{A^\omega(m,q), A^\omega(n,q)\}$, for all $m, n \in R$ and $q \in Q$.

Theorem 3.5. If A is a ω-QFSR of a ring R, then
$A^\omega(m,q) \leq A^\omega(0,q)$, for all $m \in R$ and $q \in Q$ where 0 is the additive identity of R.
Proof: Consider $A^\omega(0,q) = A^\omega(m-m,q) \geq \min\{A^\omega(m,q), A^\omega(m^{-1},q)\}$
$= \min\{A^\omega(m,q), A^\omega(m,q)\} = A^\omega(m,q)$
Hence, $A^\omega(0,q) \geq A^\omega(m,q)$, for all $m \in R$

Theorem 3.6. If A is QFSR of a ring R, then A is an ω-QFSR of R.
Proof: Assume that A is a QFSR of a ring R and $\forall\, a, b \in R$ and $q \in Q$.

Assume that, $A^{\omega}(a-b,q)=t_p\{A(a-b,q),\mu\}\geq t_p\{\min\{A(a,q),A(b,q)\},\omega\}$
$=\min\left\{t_p\{A(a,q),\omega\},t_p\{A(b,q),\omega\}\right\}=\min\{A^{\omega}(a,q),A^{\omega}(b,q)\}$
$A^{\omega}(a-b,q)\geq\min\{A^{\omega}(a,q),A^{\omega}(b,q)\}$
Further $A^{\omega}(ab,q)=t_p\{A(ab,q),\mu\}\geq t_p\{\min\{A(a,q),A(b,q)\},\omega\}$
$=\min\left\{t_p\{A(a,q),\omega\},t_p\{A(b,q),\omega\}\right\}=\min\{A^{\omega}(a,q),A^{\omega}(b,q)\}$

$$A^{\omega}(ab,q)\geq\min\{A^{\omega}(a,q),A^{\omega}(b,q)\}$$

Consequently, A is ω-QFSR of R. In general, the converse may not be true.
Note 3.7. we take $Q=\{q\}$ in all the examples

Example 3.8 Let $R=\{0,1,2,3\}$, be a ring and $Q=\{q\}$. Let the Q-fuzzy set A of R described by:

$$A(a,q)=\begin{cases}0.3, \text{if } a=0\\ 0.5, \text{if } a=1 \text{ or } 3\\ 0.4, \text{if } a=2\end{cases}$$

Take $\omega=0$ then,

$$A^{\omega}(a,q)=t_p\{A(a,q),\omega\}=t_p\{A(a,q),0\}=0, \text{for all } a\in R$$

$$\Rightarrow A^{\omega}(a-b,q)\geq\min\{A^{\omega}(a,q),A^{\omega}(b,q)\}$$

Further, we have $A^{\omega}(ab,q)\geq\min\{A^{\omega}(a,q),A^{\omega}(b,q)\}$
Consequently A is ω-QFSR of R and A is not QFSR of R.

Definition 3.9. Let A be ω-Q-fuzzy set of universe set M. For $t,\omega\in[0,1]$ the level subset A_t^{ω} of ω-Q-fuzzy set is given by:

$$A_t^{\omega}=\{m\in M: A^{\omega}(m,q)\geq t\}$$

Theorem 3.10. Let A is ω-Q-fuzzy subring of R then A_t^{ω} is subring of R for all $t\leq A(0,q)$.
Proof: It is quite obvious that A^{ω} is non-empty. Since A be ω-Q-fuzzy subring of a ring R, which implies that $A^{\omega}(m,q)\leq A^{\omega}(0,q)$, for all $m\in R$ and $q\in Q$. Let $m,n\in A_t^{\omega}$ then $A^{\omega}(m,q)\geq t\;\; A^{\omega}(n,q)\geq t$.
Now,

$$A^{\omega}(m-n,q)\geq\min\{A^{\omega}(m,q),A^{\omega}(n,q)\}\geq\min\{t,t\}=t$$

$$A^{\omega}(mn,q)\geq\min\{A^{\omega}(m,q),A^{\omega}(n,q)\}\geq\min\{t,t\}=t$$

This implies that $-n,mn\in A_t^{\omega}$. Hence, A_t^{ω} is subring of R.

Definition 3.11. Let A be a Q-fuzzy subset of a ring R and $\omega\in[0,1]$. Then A^{ω} is ω-Q-fuzzy left ideal (ω-QFLI) of R if,

i $A^{\omega}(m-n,q)\geq\min\{A^{\omega}(m,q),A^{\omega}(n,q)\}$
ii $A^{\omega}(mn,q)\geq A^{\omega}(n,q)$, for all $m,n\in R$ and $q\in Q$

Definition 3.12. Let A be a Q-fuzzy subset of a ring R and $\omega\in[0,1]$. Then A^{ω} is ω-Q-fuzzy right ideal (ω-QFRI) of R if,

i. $A^{\omega}(m-n,q)\geq\min\{A^{\omega}(m,q),A^{\omega}(n,q)\}$
ii. $A^{\omega}(mn,q)\geq A^{\omega}(m,q)$, for all $m,n\in R$ and $q\in Q$

Definition 3.13. Let A be a Q-fuzzy subset of a ring R and $\omega\in[0,1]$. Then A^{ω} is ω-QFI of R if,

i. $A^{\omega}(m-n,q)\geq\min\{A^{\omega}(m,q),A^{\omega}(n,q)\}$
ii. $A^{\omega}(mn,q)\geq\max\{A^{\omega}(m,q),A^{\omega}(n,q)\}$, for all $m,n\in R$ and $q\in Q$

Definition 3.14. Let A be a ω-QFSR of a ring R and $\omega\in[0,1]$. For any $m\in R$ and $q\in Q$, The ω-Q-fuzzy coset of A in R is represented by $m+A^{\omega}$ as defined as,

$(m + A^{\omega})(h, q) = t_p\{A(h - m, q), \omega\} = A^{\omega}(h - m)$, for all $m, h \in R$ and $q \in Q$

Theorem 3.15. Let A be ω-QFI of ring R. Then the set
$A_0^{\omega} = \{ m \in R: A^{\omega}(m, q) = A^{\omega}(0, q)\}$ is an ideal of ring R.

Proof: Obviously $A_0^{\omega} \neq \varphi$ because $0 \in R$. Let $m, n \in A_0^{\omega}$ be any elements.
Consider

$$A^{\omega}(m - n, q) \geq \min\{A^{\omega}(m, q), A^{\omega}(n, q)\} = \min\{A^{\omega}(0, q), A^{\omega}(0, q)\}$$

Implies that $A^{\omega}(m - n, q) \geq A^{\omega}(0, q)$. But $A^{\omega}(m - n, q) \leq A^{\omega}(0, q)$
Therefore, $A^{\omega}(m - n, q) = A^{\omega}(0)$
Implies that $m - n \in A_0^{\omega}$.
Further, let $m \in A_0^{\omega}$ and $\in R$. Consider
$A^{\omega}(mn, q) \geq \max\{A^{\omega}(m, q), A^{\omega}(n, q)\} = \max\{A^{\omega}(0, q), A^{\omega}(n, q)\}$,
Implies that $A^{\omega}(mn, q) \geq A^{\omega}(0, q)$. But $A^{\omega}(mn, q) \leq A^{\omega}(0, q)$
Therefore, $A^{\omega}(mn, q) = A^{\omega}(0, q)$.
Similarly, $A^{\omega}(nm, q) = A^{\omega}(0, q)$
Implies that $mn, nm \in A_0^{\omega}$.
Implies that A_0^{ω} is an ideal.

Theorem 3.16. Let A_0^{ω} be an ω-QFI of ring R, $m, n \in R$ *and* $q \in Q$. Then,

$$m + A^{\omega} = n + A^{\omega} \text{ if and if only } m - n \in A_0^{\omega}.$$

Proof: For any $m, n \in S$, we have $m + A^{\omega} = n + A^{\omega}$.
Consider,

$$A^{\omega}(m - n, q) = (n + A^{\omega})(m, q) = (m + A^{\omega})(m, q) = A^{\omega}(0, q)$$

Therefore, $m - n \in A_0^{\omega}$.
Conversely, let $m - n \in A_0^{\omega}$
Implies that $A^{\omega}(m - n, q) = A^{\omega}(0, q)$
Consider, $(m + A^{\omega})(h, q) = A^{\omega}(h - m, q) = A^{\omega}\big((h - n) - (m - n), q\big)$
$\geq \min\big\{A^{\omega}\big((h - n), q\big), A^{\omega}\big((m - n), q\big)\big\}$
$= \min\big\{A^{\omega}\big((h - n), q\big), A^{\omega}(0, q)\big\} = A^{\omega}\big((h - n), q\big) = (n + A^{\omega})(h, q)$
Interchange the role of p and q we get $(n + A^{\omega})(h, q) \geq (m + A^{\omega})(h, q)$
Therefore, $(m + A^{\omega})(h, q) = (n + A^{\omega})(h, q)$, for all $h \in R$

Definition 3.17. Let A be a ω-QFI of a ring R. The set of all ω-Q-fuzzy cosets of A denoted by $R/_{A^{\omega}}$ form a ring with respect to binary operation * defined by
$(m + A^{\omega}) + (n + A^{\omega}) = (m + n) + A^{\omega}, where\ m + A^{\omega}, n + A^{\omega} \in R/_{A^{\omega}}, m, n \in R.$
$(m + A^{\omega}) * (n + A^{\omega}) = (m * n) + A^{\omega}, where\ m + A^{\omega}, n + A^{\omega} \in R/_{A^{\omega}}, m, n \in R$. The ring $R/_{A^{\omega}}$ is called the factor ring of R with respect to ω-QFI A^{ω}.

Theorem 3.18. The set $R/_{A^{\omega}}$ forms a ring with respect to the above stated binary operation.

Proof: Let $m_1 + A^{\omega} = m_2 + A^{\omega}$ *and* $n_1 + A^{\omega} = n_2 + A^{\omega}$ for some $m_1, m_2, n_1, n_2 \in R$. Let $g \in R$ be any element of R and $q \in Q$.

$$(m_2 + n_2 + A^{\omega})(g, q) = A^{\omega}(g - (m_2 + n_2), q)$$

$$= A^{\omega}((g - m_2 - n_2), q) = n_2 + A^{\omega}((g - m_2), q) = n_1 + A^{\omega}((g - m_2), q)$$

$$= A^{\omega}((g - m_2 - n_1), q) = m_2 + A^{\omega}((g - n_1), q) = m_1 + A^{\omega}((g - n_1), q)$$

$$= A^{\omega}((g - m_1 - n_1), q) = A^{\omega}(g - (m_1 + n_1), q) = (m_1 + n_1 + A^{\omega})(g, q)$$

Moreover,

$$(m_2 n_2 + A^{\omega})(g, q) = A^{\omega}(g - m_1 n_1 - (m_2 n_2 - m_1 n_1), q)$$

$$\geq \min\{A^{\omega}(g - m_1 n_1), A^{\omega}((m_2 n_2 - m_1 n_1), q)\}$$

But we have, $A^{\omega}\big((m_2 n_2 - m_1 n_1), q\big) = A^{\omega}((m_1 n_1 - m_2 n_1 + m_2 n_1 - m_2 n_2), q)$

$$= A^{\omega}\big((m_1 - m_2)n_1 + m_2(n_1 - n_2), q\big) \geq \min\{A^{\omega}(m_1 - m_2)n_1, q), A^{\omega}(m_2(n_1 - n_2), q)\}$$

$$= \min\{A^{\omega}((m_1 - m_2), q), A^{\omega}((n_1 - n_2), q)\}$$

$$= \min\{A^{\omega}(0, q), A^{\omega}(0, q)\}$$

$$A^{\omega}\big((m_2 n_2 - m_1 n_1), q\big) \geq A^{\omega}(0, q)$$

$$(m_2 n_2 + A^{\omega})(g, q) \geq A^{\omega}(g - m_1 n_1), q)$$

$$= (m_1 n_1 + A^{\omega})(g, q)$$

Similarly, we can prove that $(m_2 n_2 + A^{\omega})(g, q) \leq (m_1 n_1 + A^{\omega})(g, q)$
Consequently, $(m_2 n_2 + A^{\omega})(g, q) = (m_1 n_1 + A^{\omega})(g, q)$.

Therefore $*$ is well defined. Now we prove that the following axioms of ring, for any $m, n \in R$.

1) $(m + A^{\omega}) + (n + A^{\omega}) = m + n + A^{\omega}$
2) $(m + A^{\omega}) + [(n + A^{\omega}) + (r + A^{\omega})] = m + A^{\omega} + [n + r + A^{\omega}] = (m + n) + r + A^{\omega} = [m + n + A^{\omega}] + r + A^{\omega} = [(m + A^{\omega}) + (n + A^{\omega})] + (r + A^{\omega})$
3) $(m + A^{\omega}) + (n + A^{\omega}) = m + n + A^{\omega} = n + m + A^{\omega} = (n + A^{\omega}) + (m + A^{\omega})$
4) $(0 + A^{\omega}) + (n + A^{\omega}) = (n + A^{\omega})$
5) $(m + A^{\omega}) + (-m + A^{\omega}) = A^{\omega}$
6) $(m + A^{\omega})(n + A^{\omega}) = mn + A^{\omega}$
7) $(m + A^{\omega})[(n + A^{\omega})(r + A^{\omega})] = m + A^{\omega} + [nr + A^{\omega}] = mnr + A^{\omega} = [mn + A^{\omega}] + r + A^{\omega} = [(m + A^{\omega})(n + A^{\omega})](r + A^{\omega})$
8) $(m + A^{\omega})[(n + A^{\omega}) + (r + A^{\omega})] = (m + A^{\omega})[(n + r) + A^{\omega}] = m(n + r) + A^{\omega} = (mn + mr) + A^{\omega} = (mn + A^{\omega}) + (mr + A^{\omega}) = [(m + A^{\omega})(n + A^{\omega}) + (m + A^{\omega})(r + A^{\omega})]$,
9) $[(n + A^{\omega}) + (r + A^{\omega})](m + A^{\omega}) = [(n + r) + A^{\omega}](m + A^{\omega}) = (n + r)m + A^{\omega} = (nm + rm) + A^{\omega} = (nm + A^{\omega}) + (rm + A^{\omega}) = [(n + A^{\omega})(m + A^{\omega}) + (r + A^{\omega})(m + A^{\omega})]$

Consequently, $\left(R/_{A^{\omega}}, +, *\right)$ is a ring.

4. HOMOMORPHISM OF ω-Q-FUZZY SUBRINGS

In this section, we investigate the behavior of homomorphic image and inverse image of ω-QFSR.

Lemma 4.1. Let $f: M \to N$ be a mapping and A and B be two fuzzy subsets of M and N respectively, then

i. $f^{-1}(B^{\omega})(m, q) = \big(f^{-1}(B)\big)^{\omega}(m, q)$, for all $m \in M$ and $q \in Q$
ii. $f(A^{\omega})(n, q) = \big(f(A)\big)^{\omega}(n, q)$, for all $n \in N$ and $q \in Q$

Proof:
(i) $f^{-1}(B^{\omega})(m) = B^{\omega}\big(f(m)\big) = t_p\{B\big(f(m)\big), \omega\} = t_p\{f^{-1}(B)(m), \omega\}$

$$f^{-1}(B^{\omega})(m) = \big(f^{-1}(B)\big)^{\omega}(m), \text{for all } m \in M$$

(ii) $f(A^{\omega})(n, q) = \sup\{A^{\omega}(m, q): f(m) = y\} = \sup\{t_p\{A(m, q), \omega\}: f(m) = n\}$
$= t_p\left\{\sup\{\{A(m, q): f(m) = n\}, \omega\}\right\} = t_p\{f(A)(n, q), \omega\} = \big(f(A)\big)^{\omega}(n, q)$, for all $n \in N$
Hence, $f(A^{\omega})(n, q) = \big(f(A)\big)^{\omega}(n, q)$

Theorem 4.2. Let$f: R_1 \longrightarrow R_2$ be a homomorphism from a ring R_1 to a ring R_2 and A be a ω-QFSR of ring R_1 . Then $f(A)$ is a ω-QFSR of ring R_2.
Proof: Let A be a ω-QFSR of ring R_1. Let $n_1, n_2 \in R_2$ be any element. Then there exists unique elements $m_1, m_2 \in R_1$ such that $f(m_1) = n_1$ and $f(m_2) = n_2$ and for $q \in Q$.
Consider,

$\left(f(A)\right)^{\omega}(n_1-n_2,q)=t_p\{f(A)(n_1-n_2,q),\omega\}=t_p\{f(A)(f(m_1)-f(m_2),q),\omega\}$
$=t_p\{f(A)(f(m_1-m_2),q),\omega\}=t_p\{A(m_1-m_2,q),\omega\}=A^{\omega}(m_1-m_2,q)$

$\geq \min\{A^{\omega}(m_1,q),A^{\omega}(m_2,q)\}$, for all $m_1,m_2\in H_1$ such that $f(m_1)=n_1$ and $f(m_2)=n_2\}$

$\geq \min\{\sup\{A^{\omega}(m_1,q)\ :f(m_1)=n_1\},\sup\{A^{\omega}\ (m_2,q)\ :f(m_2)=n_2\}\}$

$=\min\{f(A^{\omega})(n_1,q),f(A^{\omega})(n_2,q)\}=\min\{\left(f(A)\right)^{\omega}(n_1,q),\left(f(A)\right)^{\omega}(n_2,q)\}$

Thus, $\left(f(A)\right)^{\omega}(n_1n_2,q)\geq \min\{\left(f(A)\right)^{\omega}(n_1,q),\left(f(A)\right)^{\omega}(n_2,q)\}$

Further, $\left(f(A)\right)^{\omega}(n_1n_2,q)=t_p\{f(A)(n_1n_2,q),\omega\}=t_p\{f(A)(f(m_1)f(m_2),q),\omega\}$

$=t_p\{f(A)(f(m_1m_2),q),\omega\}=t_p\{A(m_1m_2,q),\omega\}=A^{\omega}(m_1m_2,q)$

$\geq \min\{A^{\omega}(m_1,q),A^{\omega}(m_2,q)\}$, for all $m_1,m_2\in H_1$ such that $f(m_1)=n_1$ and $f(m_2)=n_2\}$

$\geq \min\{\sup\{A^{\omega}(m_1,q)\ :f(m_1)=n_1\},\sup\{A^{\omega}\ (m_2,q)\ :f(m_2)=n_2\}\}$

$=\min\{f(A^{\omega})(n_1,q),f(A^{\omega})(n_2,q)\}=\min\{\left(f(A)\right)^{\omega}(n_1,q),\left(f(A)\right)^{\omega}(n_2,q)\}$

Thus, $\left(f(A)\right)^{\omega}(n_1n_2,q)\geq \min\{\left(f(A)\right)^{\omega}(n_1,q),\left(f(A)\right)^{\omega}(n_2,q)\}$

Consequently, $f(A)$ is ω-QFSR of R_2.

Theorem 4.3. Let $f:R_1\longrightarrow R_2$ be a homomorphism from ring R_1 into a ring R_2 and B be a ω-QFSR of ring R_2. Then $f^{-1}(B)$ is ω-QFSR of ring R_1.

Proof: Let B be ω-QFSR of ring R_2. Let $m_1,m_2\in R_1$ be any elements, then $\left(f^{-1}(B)\right)^{\omega}(m_1-m_2,q)=f^{-1}(B^{\omega})(m_1-m_2,q)=B^{\omega}(f(m_1-m_2),q)$

$=B^{\omega}(f(m_1)-f(m_2),q)$

$\geq \min\{B^{\omega}(f(m_1),q),B^{\omega}(f(m_2),q)\}=\min\{f^{-1}(B^{\omega})(m_1,q),f^{-1}(B^{\omega})(m_2,q)\}$

$=\min\{\left(f^{-1}(B)\right)^{\omega}(m_1,q),\left(f^{-1}(B)\right)^{\omega}(m_2,q)\}$

Thus, $\left(f^{-1}(B)\right)^{\omega}(m_1m_2,q)\geq \min\{\left(f^{-1}(B)\right)^{\omega}(m_1,q),\left(f^{-1}(B)\right)^{\omega}(m_2,q)\}$
Further, $\left(f^{-1}(B)\right)^{\omega}(m_1m_2,q)=f^{-1}(B^{\omega})(m_1m_2,q)=B^{\omega}(f(m_1m_2),q)=B^{\omega}(f(m_1)f(m_2),q)$
$\geq \min\{B^{\omega}(f(m_1),q),B^{\omega}(f(m_2),q)\}=\min\{f^{-1}(B^{\omega})(m_1,q),f^{-1}(B^{\omega})(m_2,q)\}$
$=\min\{\left(f^{-1}(B)\right)^{\omega}(m_1,q),\left(f^{-1}(B)\right)^{\omega}(m_2,q)\}$
Thus, $\left(f^{-1}(B)\right)^{\omega}(m_1m_2,q)\geq \min\{\left(f^{-1}(B)\right)^{\omega}(m_1,q),\left(f^{-1}(B)\right)^{\omega}(m_2,q)\}$

Consequently, $f^{-1}(B)$ is ω-QFSR of a ring $R_{1.}$

5. CONCLUSION

In paper, we have proved the level subset of two ω-Q-fuzzy subrings is a subring. In addition, we have extended the study of this ideology to investigate the effect of image and inverse image of ω-QFSR under ring homomorphism.

ACKNOWLEDGEMENT

This work was funded by the Deanship of Scientific Research (DSR) at King Abdulaziz University, Jeddah, Saudi Arabia. The authors, therefore, acknowledge with thanks DSR for technical and financial support.

Conflict of interest

All authors declare no conflict of interest in this paper

REFERENCE

[1] L.A. Zadeh, "Fuzzy sets, Information and Control," vol. 8 no. 3, pp. 338-353, 1965.
[2] U. Shuaib, M. Shaheryar and W. Asghar, "On some characterization of o-fuzzy subgroups," *Internationa journal of Mathematics and Computer Science*, vol. 13, no. 2, pp. 119-131, 2018.
[3] M. Gulzar, G. Abbas, F. Dilawar, "Algebraic properties of ω-Q-fuzzy subgroup", *International journal of Mathematics and Computer Science*, vol. 15, pp. 265-274, 2020.
[4] N. Kausar, B. U. Islam, M. Y. Javid, S. A. Ahmed, and U. Ijaz, "Characterization of non-associative rings by the properties of their fuzzy ideals", *Journal of Taibah University for Science*, vol. 13, no. 1, pp. 820-833, 2019.
[5] N. Kausar, "Direct product of finite intuitionistic fuzzy normal subrings over non-associative rings", *European Journal of Pure and Applied Mathematics*, vol. 12, no. 2, pp. 622-648, 2019.
[6] D. Shan, W. Lu and J. Yang, "The data-driven fuzzy cognitive map model and its application to prediction of time series, "*International Journal of Innovative Computing, Information and Control*, vol. 14, no. 5, pp. 1583-1602, 2018.
[7] M. N. Alemu, "Fuzzy model for chaotic time series prediction", *International Journal of Innovative Computing, Information and Control*, vol. 14, no. 5, pp. 1767-1786, 2018, 2018.
[8] Y. Qin, Y. Liu, Z. Hong and H. Jia, "Hesitant fuzzy muirhead mean operations and its application to multiple attributed decision making", *International Journal of Innovative Computing, Information and Control*, vol. 14, no. 4, pp. 1223-1238, 2018.
[9] A. Rosenfeld, "Fuzzy groups," *Journal of Mathematical Analysis and Application*, vol. 35, no. 3 pp. 512-517, 1971.
[10] W. J. Liu, "Fuzzy invariant subgroups and fuzzy ideals," *Fuzzy Sets and Systems*, vol. 8, no. 2, pp. 133-139, 1982.
[11] V. N. Dixit, R. Kumar, N. Ajmal, "Level subgroups and union of fuzzy subgroup,". *Fuzzy Sets and Systems*, vol. 37, no. 3, pp. 359-371, 1990.
[12] R. Biswas, "Fuzzy subgroups and anti-fuzzy subgroups," *Fuzzy Sets and Systems*, vol. 35, no. 1, pp. 121-124, 1990.
[13] M. M. Gupta, J. Qi, "Theory of T-norms and fuzzy inference methods," *Fuzzy Sets and Systems*, vol. 40, no. 3, pp. 431-450, 1991.
[14] A. Solairaju, and R. Nagarajan, "A new structure and construction of Q- fuzzy groups" *Advances in Fuzzy Mathematics*, vol. 4, no. 1, pp. 23-29, 2009.
[15] Dr. R. Muthuraj, P. M. Sitharselvam, M. S. Muthuraman, "Anti Q-Fuzzy Group and Its Lower Level Subgroups," *International Journal of Computer Application*, vol. 3, no. 3, pp. 16-20, 2010.
[16] Priya, T. Ramachandran, K. T. Nagalakshmi, "On Q-fuzzy Normal Subgroups" *International Journal of computer and Organization Trends*, vol. 3, no. 6, pp. 39-42, 2013.
[17] P. M. Sithar selvam, T. Priya, K. T. Nagalakshmi and T. Ramachandran, "On Some properties of anti -Q-fuzzy Normal Subgroups" *General Mathematics Notes*, vol. 22, no. 1, pp. 1-10, 2014.
[18] M. O. Alsarahead, A. G. Ahmad, "Complex fuzzy subgroups," *Applied Mathematical Sciences*, vol. 11, no. 41, pp. 2011-2021, 2017.
[19] M. O. Alsarahead, A. G. Ahmad, "Complex fuzzy subrings," *International Journal of Pure and Applied Mathematics*, vol. 117, no. 4, pp. 563-577, 2017.
[20] M. O. Alsarahead, A. G. Ahmad, "Complex fuzzy soft subgroups, *Journal of Quality Measurement and Analysis,"* vol. 13, no. 2, pp. 17-28, 2017.
[21] B. B. Makamba, V. Murali, "A class of fuzzy subgroups of finite reflection groups," *Journal of Intelligent and Fuzzy Systems*, vol. 33, pp. 979-983, 2017.
[22] R. Rasuli, "Characterization of Q-fuzzy subrings (Anti Q-fuzzy subrings) with respect to a T-norm (T-conorm)," *Journal of Information and Optimization Sciences*, vol. 39, pp. 827-837, 2018.
[23] Dr. R. Jahir Hussain, "A Review On Q-fuzzy subgroup in Algebra," *International Journal of Applied Engineering Research*, vol. 14, pp. 60-63, 2019.
[24] S. A. Trevijano, M. J. Chasco and J. Elorza, "The annihilator of fuzzy subgroups," *Fuzzy Sets and Systems*, vol. 369, pp. 122-131, 2019.
[25] M. A. R. Shafei, M. A. Tawfik and D. K. Ibrahim, "Fuzzy control scheme for energy efficiency and demand management in airports using 3D simulator," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 20, no. 2, pp. 583-592, 2020.