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T/4 Fractionally Spaced Decision Feedback Blind Equalization with RLS Algorithm

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Abstract

This work proposed a new recursive least squares (RLS) T/4 fractionally spaced decision feedback blind equalization algorithm. The cost function of CMA is simplified to meet second norm formand then RLS algorithm can be used to update the forward filter of equalizer with T/4 fractionally-spaceddirectly.The feedback filter of equalizer with symbol-spaced still updates by gradient descent algorithm.This method can improve the performance of blind equalization based on T/4 fractionally spaced decision feedback equalizerwhich adopt gradient descent algorithm. Computer simulation results show that the algorithm proposed in this work has faster convergence rate and lower mean square error compare with blind equalization based on symbol-spaced and T/4 fractionally-spaced decision feedback equalizer based on gradient descend algorithm.

Keywords*: blind equalization, RLS algorithm, decision feedback equalizer, constant modulus algorithm, fractionally-spaced equalizer*

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1. Introduction

In digital communication system, multipath propagation leads to inter-symbol interference (ISI) at the receiver seriously affects the quality of communication. Adaptive equalization technique is one of the effective techniques to eliminate ISI [1, 2]. Blind equalization can achieve compensation and tracking of the channel needs no training sequence to eliminate the ISI at the receiver. For interception of military information and multi-user broadcast communications, there is no training sequence can be exploited, under these conditions, blind equalization has important practical value [3]. For blind equalization restores the send signal only relies on the statistical properties of the received signal, thus it can maintain the stability of the communication system to avoid the equalizer unlocking [4]. Linear equalizer blind equalization algorithm performance is poor for nonlinear distortion communication channel, otherwise, decision feedback equalizer (DFE) can use feedback filter to compensate channel nonlinear characteristics that can eliminate the residual ISI of forward equalizer [5, 6]. Furthermore, if the channel frequency response has depth spectrum zero, the symbol-spaced equalizer would cause severe noise amplification which results in equalization performance degradation, meanwhile, when the channel frequency response is in deep fading, to guarantee robustness of the equalization algorithm needs larger number of equalizer taps which would slow the convergence rate and increase the computational complexity [7].

Fractionally-spaced equalizer (FSE) oversamples the output of the channel to provide sufficient diversity, and then it can compensate the channel distortion which has zero point near the unit circle [8]. Furthermore, FSE is not sensitive for system timing error, so it is less affected by system noise. As a result, FSE has better performance than symbol-spaced equalizer [9]. But fractionally-spaced blind equalization adopting gradient descent algorithm still has slow convergence rate and large steady-state error [10]. Therefore, this paper proposes a *T*/4 fractionally-spaced DFE blind equalization algorithm, in which the forward filter uses *T*/4 fractionally-spaced and the feedback filter uses symbol-spaced. For RLS is a supervised algorithm and cannot be used in blind equalization, in this work, The cost function of CMA is simplified to meet second norm form, and then RLS algorithm can be used to update the forward filter weights directly, the feedback filter still update by gradient descent algorithm. This

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method takes advantage over FSE and DFE, meanwhile, RLS algorithm can further improve the blind equalization performance for convergence rate and tracking ability. The simulation results show that the algorithm proposed in this work has faster convergence rate and lower mean square error compare with blind equalization based on symbol-spaced and *T*/4 fractionally-spaced DFE based on gradient descend algorithm.

2. Proposed RLS *T*/4 FSE DFE Blind Equalization Algorithm

DFE performs better than the linear equalizer for it can compensate the amplitude distortion with minimum noise gain, furthermore, DFE can completely equalize the channel that does not exceed the length of itself [11]. The block diagram of symbol-spaced DFE is shown as Figure 1. The basic idea of DFE is that once an information symbol is detected, the ISI caused by this symbol will be estimated and subtracted in advance [12]. From Figure 1 we can see that DFE includes three parts: the forward filter ***f***(*n*), the feedback filter ***b***(*n*) and the quantified decision device ***Q***(.).

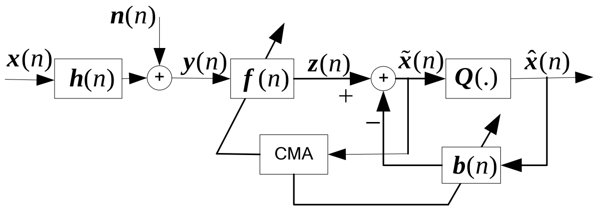

Figure 1. Block Diagram of Symbol-spaced DFE Blind Equalization

The input signal $\boldsymbol{x}(n)$ transmits through the unknown channel $\boldsymbol{h}(n)$ adding Gauss white noise $\boldsymbol{n}(n)$, and then we can get the observation signal $\boldsymbol{y}(n)$ before the blind equalizer. $\tilde{\boldsymbol{x}}(n)$ is the output signal of the blind equalizer and $\hat{\boldsymbol{x}}(n)$ is the decision symbol of $\tilde{\boldsymbol{x}}(n)$. Blind equalization can recover the input signal $\boldsymbol{x}(n)$ only relies on the observation signal $\boldsymbol{y}(n)$ without the information of $\boldsymbol{x}(n)$ and $\boldsymbol{h}(n)$. If let the length of $\boldsymbol{f}(n)$ is N_f and the length of $\boldsymbol{b}(n)$ is N_b, then the symbol representation can be given by:

$$\boldsymbol{y}(n)=\left[y(n),y(n-1),\cdots,y(n-N_f+1)\right] \quad (1)$$

$$\boldsymbol{f}(n)=\left[f(0),f(1),\cdots,f(N_f+1)\right]^{\mathrm{T}} \quad (2)$$

$$\boldsymbol{b}(n)=\left[b(0),b(1),\cdots,b(N_b)\right]^{\mathrm{T}} \quad (3)$$

$$\hat{\boldsymbol{x}}(n)=\left[\hat{x}(n-1),\hat{x}(n-2),\cdots,\hat{x}(n-N_b)\right]^{T} \quad (4)$$

Where symbol "*T*" denotes transpose operation. The output signal $\tilde{\boldsymbol{x}}(n)$ can be given by;

$$\tilde{x}(n)=\boldsymbol{f}^{\mathrm{H}}(n)\boldsymbol{y}(n)-\boldsymbol{b}^{\mathrm{H}}(n)\hat{\boldsymbol{x}}(n) \quad (5)$$

Traditional DFE blind equalization algorithm updates the equalizer weights based on CMA algorithm.The cost function of CMA is [13]:

$$J=\frac{1}{2}\left[R-\left|\tilde{x}(n)\right|^2\right]^2 \quad (6)$$

where*R* is the constant modulus can be computed by:

$$R = \mathrm{E}\left\{\left|\boldsymbol{x}(n)\right|^4\right\} \Big/ \mathrm{E}\left\{\left|\boldsymbol{x}(n)\right|^2\right\} \tag{7}$$

where symbol E{.} denotes mathematical expectation operation. According to the stochastic gradient descent algorithm, decision feedback blind equalization updates weights based on CMA is given by [14]:

$$\boldsymbol{f}(n) = \boldsymbol{f}(n-1) + \mu\tilde{x}(n)\left[R - \left|\tilde{x}(n)\right|^2\right]\boldsymbol{y}^*(n) \tag{8}$$

$$\boldsymbol{b}(n) = \boldsymbol{b}(n-1) + \mu\tilde{x}(n)\left[R - \left|\tilde{x}(n)\right|^2\right]\hat{\boldsymbol{x}}^*(n) \tag{9}$$

where μ is the step size and the symbol "*" denotes conjugate operation.

CMA blind equalization updates the equalizer weights based on stochastic gradient descent (SGD) algorithm which convergence rate is slow and has big steady state error after convergence. Compared with SGD algorithm, RLS algorithm has faster convergence rate and better tracking performance [15]. From (6) can see that the cost function of CMA does not meet second norm form and then RLS algorithm cannot be used directly. According to (5) we can get

$$\boldsymbol{u}_b(n) = \hat{\boldsymbol{x}}(n)\left[\boldsymbol{f}^H(n)\boldsymbol{y}(n) - \boldsymbol{b}^H(n)\hat{\boldsymbol{x}}(n)\right]^* \tag{10}$$

After expansion of (10) we can get:

$$\left|\tilde{x}\right|^2 = \boldsymbol{f}^H(n)\boldsymbol{y}(n)\left[\boldsymbol{f}^H(n)\boldsymbol{y}(n) - \boldsymbol{b}^H(n)\hat{\boldsymbol{x}}(n)\right]^* - \boldsymbol{b}^H(n)\hat{\boldsymbol{x}}(n)\left[\boldsymbol{f}^H(n)\boldsymbol{y}(n) - \boldsymbol{b}^H(n)\hat{\boldsymbol{x}}(n)\right]^* \tag{11}$$

Obviously, if let $\boldsymbol{u}_f(n)$ and $\boldsymbol{u}_b(n)$ as follows:

$$\boldsymbol{u}_f(n) = \boldsymbol{y}(n)\left[\boldsymbol{f}^H(n)\boldsymbol{y}(n) - \boldsymbol{b}^H(n)\hat{\boldsymbol{x}}(n)\right]^* \tag{12}$$

$$\boldsymbol{u}_b(n) = \hat{\boldsymbol{x}}(n)\left[\boldsymbol{f}^H(n)\boldsymbol{y}(n) - \boldsymbol{b}^H(n)\hat{\boldsymbol{x}}(n)\right]^* \tag{13}$$

Then the cost function of CMA can be rewritten as follow:

$$J = \frac{1}{2}\left[R - \left[\boldsymbol{f}^{\mathrm{H}}(n)\boldsymbol{u}_f(n) - \boldsymbol{b}^{\mathrm{H}}(n)\boldsymbol{u}_b(n)\right]\right]^2 \tag{14}$$

The cost function with the form of (14) meets the second norm form, but $\boldsymbol{u}_f(n)$ and $\boldsymbol{u}_b(n)$ include the equalization weights $\boldsymbol{f}(n)$ and $\boldsymbol{b}(n)$ explicit, then RLS algorithm still cannot be applied to update the weights of the equalizer. Here we assume that the equalizer can obtain convergence, and then the weights vector would have relations as follows

$$\lim_{n\to\infty}\left\|\boldsymbol{f}(n) - \boldsymbol{f}(n-1)\right\| \to 0 \tag{15}$$

$$\lim_{n\to\infty}\left\|\boldsymbol{b}(n) - \boldsymbol{b}(n-1)\right\| \to 0 \tag{16}$$

According to (15) and (16) defined $\tilde{\boldsymbol{u}}_f(n)$ and $\tilde{\boldsymbol{u}}_b(n)$ as follows:

$$\tilde{\boldsymbol{u}}_f(n) = \boldsymbol{y}(n)\left[\boldsymbol{f}^{\mathrm{H}}(n-1)\boldsymbol{y}(n) - \boldsymbol{b}^{\mathrm{H}}(n-1)\hat{\boldsymbol{x}}(n)\right] \tag{17}$$

$$\tilde{\boldsymbol{u}}_b(n) = \hat{\boldsymbol{x}}(n)\left[\boldsymbol{f}^{\mathrm{H}}(n-1)\boldsymbol{y}(n) - \boldsymbol{b}^{\mathrm{H}}(n-1)\hat{\boldsymbol{x}}(n)\right]^* \tag{18}$$

If use $\tilde{\boldsymbol{u}}_f(n)$ and $\tilde{\boldsymbol{u}}_b(n)$ instead of $\boldsymbol{u}_f(n)$ and $\boldsymbol{u}_b(n)$ in the cost function expressed by (14), the cost function can be written as:

$$J = \frac{1}{2}\left[R - \left[\boldsymbol{f}^{\mathrm{H}}(n)\tilde{\boldsymbol{u}}_f(n) - \boldsymbol{b}^{\mathrm{H}}(n)\tilde{\boldsymbol{u}}_b(n)\right]\right]^2 \tag{19}$$

Let $\boldsymbol{C}^{\mathrm{H}}(n) = \left[\boldsymbol{f}^{\mathrm{H}}(n); \boldsymbol{b}^{\mathrm{H}}(n)\right]$ and $\boldsymbol{U}(n) = \left[\tilde{\boldsymbol{u}}_f(n); \tilde{\boldsymbol{u}}_b(n)\right]$, the cost function of CMA can be simplified as:

$$J = \frac{1}{2}\left[R - \boldsymbol{C}^{\mathrm{H}}(n)\boldsymbol{U}(n)\right]^2 \tag{20}$$

Then the CMA cost function is simplified to meet second norm form as (20) and RLS algorithm can be used for DFE blind equalization directly.Compared with symbol-spaced equalizer, FSE has better performance. FSE oversamples the output signal of the channel greater than symbol rate *T*, and then the sampling frequency is larger than Nyquist frequency, so it can avoid spectrum aliasing caused by under sampling and provide more detailed channel information for equalization [16]. To implement compensation of channel distortion, FSE only needs setting the length of the equalizer equal or a little than the length of channel. The length of the equalizer determines the computational complexity, so FSE usually has lower computational complexity compared with symbol-spaced equalizer to implement channel equalization.Meanwhile, it can obtain faster convergence rate and lower steady state residual error. In this work, *T*/4 fractionally-spaced DFE blind equalizer which the forward filter is T/4 fractionally-spaced and the feedback filter is symbol-spaced is adopted. According to the principle of fractionally-spaced, the system can be equivalent to a single input multiple output (SIMO) model, as shown in Figure 2.

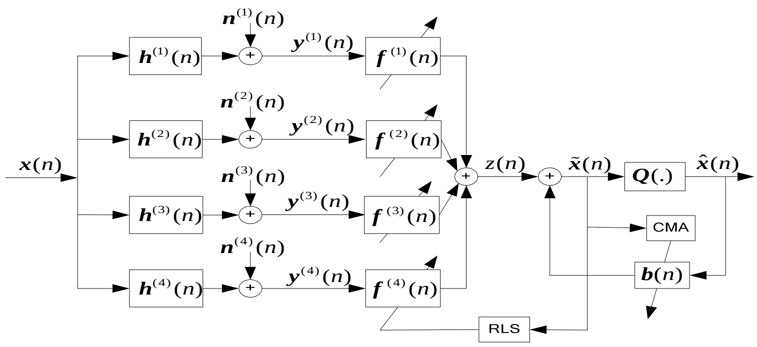

Figure 2. Block Diagram of T/4 Fractionally-spaced DFE Blind Equalization

The output of each sub-channel is:

$$\boldsymbol{y}^{(p)}(n) = \sum_{i=0}^{N}\boldsymbol{h}^{(p)}(n)\boldsymbol{x}(n-i) + \boldsymbol{n}^{(p)}(n) \tag{21}$$

Where $p = 1, 2, \cdots, 4$, and *N* is the length of discrete channel impulse response of $\boldsymbol{h}^{(p)}(n)$,set the weight length of each sub-equalizer $\boldsymbol{f}^{(p)}(n)$ is *L*, the input signal is $\boldsymbol{x}(n)$, noise vector $\boldsymbol{n}^{(p)}(n)$ is $(L+N-1)\times 1$ vector, (21) can be rewritten as matrix product.

$$\boldsymbol{y}^{(p)}(n) = \boldsymbol{h}^{(p)}(n)\boldsymbol{x}(n) + \boldsymbol{n}^{(p)}(n) \tag{22}$$

Where:

$$\left.\begin{aligned} \boldsymbol{x}(n) &= \left[x(n), x(n-1), \cdots, x(n-L-N+1)\right]^{\mathrm{T}} \\ \boldsymbol{y}^{(p)}(n) &= \left[y^{(p)}(n), y^{(p)}(n-1), \cdots, y^{(p)}(n-L+1)\right]^{\mathrm{T}} \\ \boldsymbol{n}^{(p)}(n) &= \left[n^{(p)}(n), n^{(p)}(n-1), \cdots, n^{(p)}(n-L+1)\right]^{\mathrm{T}} \end{aligned}\right\} \tag{23}$$

$\boldsymbol{h}^{(p)}(n)$ is a block Toeplitz matrix with $L\times(L+N-1)$ dimension can be given by:

$$\boldsymbol{h}^{(p)}(n)=\begin{bmatrix} h^{(p)}(0) & \cdots & h^{(p)}(N-1) & \cdots & 0 \\ 0 & h^{(p)}(0) & \cdots & h^{(p)}(N-1) & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & h^{(p)}(0) & \cdots & h^{(p)}(N-1) \end{bmatrix} \tag{24}$$

According to (23) and (24), (22) can be rewritten as:

$$\boldsymbol{y}(n)=\boldsymbol{h}(n)\boldsymbol{x}(n)+\boldsymbol{n}(n) \tag{25}$$

where the channel matrix $\boldsymbol{h}(n)$, received signal vector $\boldsymbol{y}(n)$ and the noise $\boldsymbol{n}(n)$ are $(N-1)\times 1$ vector.

$$\left.\begin{aligned} \boldsymbol{h}(n) &= \left[\boldsymbol{h}^{(1)}(n), \boldsymbol{h}^{(2)}(n), \cdots, \boldsymbol{h}^{(P)}(n)\right] \\ \boldsymbol{y}(n) &= \left[\boldsymbol{y}^{(1)}(n), \boldsymbol{y}^{(2)}(n), \cdots, \boldsymbol{y}^{(P)}(n)\right] \\ \boldsymbol{n}(n) &= \left[\boldsymbol{n}^{(1)}(n), \boldsymbol{n}^{(2)}(n), \cdots, \boldsymbol{n}^{(P)}(n)\right] \end{aligned}\right\} \tag{26}$$

The output of the forward filter is:

$$z(n)=\boldsymbol{f}^{\mathrm{T}}(n)\boldsymbol{y}(n) \tag{27}$$

$$\boldsymbol{f}(n)=\left[\boldsymbol{f}^{(1)}(n), \boldsymbol{f}^{(2)}(n), \cdots, \boldsymbol{f}^{(P)}(n)\right]^{\mathrm{T}} \tag{28}$$

$$\boldsymbol{f}^{(p)}(n)=\left[f^{(p)}(0), f^{(p)}(1), \cdots, f^{(p)}(L-1)\right] \tag{29}$$

If we drop part of the forward filter output and only keep the odd sample, then the output of forward filter can be given by:

$$z(n)=\sum_{p=1}^{4}\sum_{k=0}^{L-1} f^{(p)}(k)y^{(p)}(n-k) \tag{30}$$

The output of the feedback filter is:

$$\tilde{x}(n)=z(n)+\boldsymbol{b}^{H}(n)\hat{\boldsymbol{x}}(n) \tag{31}$$

From analysis above we can obtain that *T*/4 fractionally-spaced DFE blind equalization based on RLS algorithm can be summarized as follows.

begin

Step1: Parameters set, λ =forgetting factor δ =value to initialize $\boldsymbol{P}^{(p)}(0)$;

Step2: Initialization: $\boldsymbol{P}^{(p)}(0)$ in addition to the central tap is 1, $p=1,2,3,4$. $\boldsymbol{b}(0)=\boldsymbol{0}$, $\boldsymbol{P}^{(p)}(0)=\delta^{-1}\boldsymbol{I}$, where $\boldsymbol{I}$ is a $L\times L$ unit matrix and L is the length of sub-equalizer;

Step3: Computation for *n*=1,2,… $p=1,2,3,4$

$$\tilde{\boldsymbol{u}}_f^{(p)}(n)=\boldsymbol{y}(n)\left[\boldsymbol{f}^{\mathrm{H}(p)}(n-1)\boldsymbol{y}^{(p)}(n)-\boldsymbol{b}^{\mathrm{H}}(n-1)\hat{\boldsymbol{x}}(n)\right]$$

$$\tilde{\boldsymbol{u}}_b(n)=\hat{\boldsymbol{x}}(n)\left[\boldsymbol{f}^{\mathrm{H}(p)}(n-1)\boldsymbol{y}^{(p)}(n)-\boldsymbol{b}^{\mathrm{H}}(n-1)\hat{\boldsymbol{x}}(n)\right]^*$$

$$z(n)=\sum_{p=1}^{4}\sum_{k=0}^{L-1}f^{(p)}(k)y^{(p)}(n-k)$$

$$\tilde{x}(n)=z(n)+\boldsymbol{b}^H(n)\hat{\boldsymbol{x}}(n)$$

$$e_f(n)=R-\tilde{x}(n)$$

$$\boldsymbol{k}^{(p)}(n)=\frac{\boldsymbol{P}^{(p)}(n-1)\tilde{\boldsymbol{u}}_f^{(p)}(n)}{\lambda+\tilde{\boldsymbol{u}}_f^{\mathrm{H}(p)}(n)\boldsymbol{P}^{(p)}(n-1)\tilde{\boldsymbol{u}}_f^{(p)}(n)}$$

$$\boldsymbol{P}^{(p)}(n)=\frac{1}{\lambda}\left[\boldsymbol{P}^{(p)}(n-1)-\boldsymbol{k}^{(p)}(n)\tilde{\boldsymbol{u}}_f^{\mathrm{H}(p)}(n)\boldsymbol{P}^{(p)}(n-1)\right]$$

$$\boldsymbol{f}^{(p)}(n)=\boldsymbol{f}^{(p)}(n-1)+\boldsymbol{k}^{(p)}(n)e_f^*(n)$$

$$e_b(n)=R-\left|\tilde{x}(n)\right|^2$$

$$\boldsymbol{b}(n)=\boldsymbol{b}(n-1)-\mu\tilde{x}(n)[R_{CM}-\left|\tilde{x}(n)\right|^2]\hat{\boldsymbol{x}}^*(n)$$

end

From the iterative process of *T*/4 fractionally-spaced DFE blind equalization based on RLS algorithm we can see that the matrix $\boldsymbol{P}^{(p)}(n)$ needs to be set and recursive calculated.The computational complexity of $\boldsymbol{P}^{(p)}(n)$ is $O(L^2)$ which mainly depends on the length of the sub-equalizer. As a result of fractionally spaced equalizer needs only to meet the length and zero condition to implement the channel equalization.For the channel with spectrum zero point, symbol-spaced equalizer needs quite longer weight length to obtain convergence. Obviously, if the symbol-spaced DFE weight length is L_b, and if $4\times L^2<L_b^2$, $L<L_b/2$, then fractionally-spaced DFE blind equalization based on RLS algorithm has a lower computational complexity than the symbol-spaced DFE blind equalization based on RLS algorithm.

3. Research Method

In order to verify T/4 fractionally-spaced DFE blind equalization based on RLS algorithm (RLS-FS-DFE) performance, symbol-spaced DFE blind equalization stochastic gradient descent algorithm (SG-BS-DFE), T/4 fractionally-spaced DFE blind equalization based on stochastic gradient descent algorithm (SG-FS-DFE) and symbol-spaced DFE blind equalization based on RLS algorithm (RLS-BS-DFE) weredone in the simulation for comparison. The comparison is in terms of mean square error (*MSE*), which is defined as [17].

Experiment 1: The channel impulse response is *h1*= [1 0.6 -0.55 -0.1] which has 1 zero point close to the unit circle as shown in Figure 3, then it has a deep spectrum zero in frequency response, thus it is difficult to be equalized by the linear equalizer. QPSK signal is used in the simulation as the input signal. In symbol-spaced DFE blind equalization algorithm, the length of forward filter is 16 with central tap is initialized to 1and the length of feedback filter is 8. Step size

$\mu = 0.001$ in SGD algorithm and the forgetting factor $\lambda = 0.998$ in RLS algorithm. For fractionally-spaced DFE blind equalization, the length of each sub-equalizer of the forward filter is 4 and the second tap is initialized to 1, the length of the feedback filter is 4. 500 times independent Monte Carlo simulations under the condition of SNR=22dB obtained the *MSE* curve is shown as Figure 4.

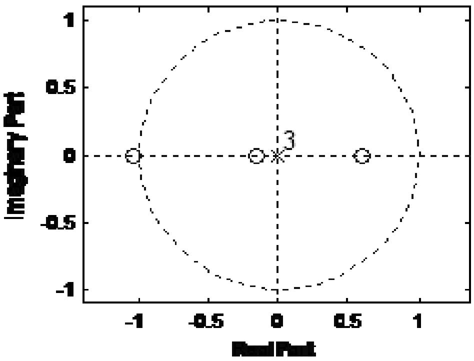

Figure 3. Zero-pole Point Distribution of *h1*

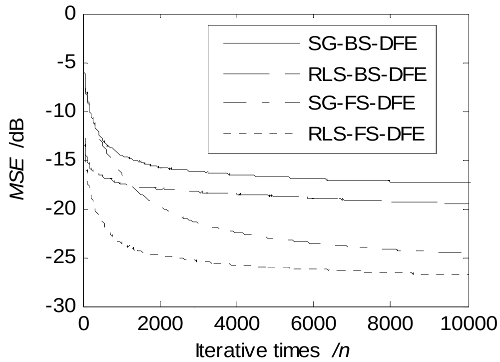

Figure 4. *MSE* Convergence Curve of *h1*

Experiment 2: The channel impulse response is *h2*= [1 0.6 -0.55 -0.1] which has 2 zero point close to the unit circle as shown in Figure 5, so it is more difficult to be equalized. QPSK signal is used in the simulation as the input signal. In symbol-spaced DFE blind equalization algorithm, the length of forward filter is 32 with central tap is initialized to 1 and the length of feedback filter is 12. Step size $\mu = 0.0006$ in SGD algorithm and the forgetting factor $\lambda = 0.998$ in RLS algorithm. For fractionally-spaced DFE blind equalization, the length of each sub-equalizer of the forward filter is 8 and the fourth tap is initialized to 1, the length of the feedback filter is 6. 500 times independent Monte Carlo simulations under the condition of SNR=24dB obtained the *MSE* curve is shown as Figure 6.

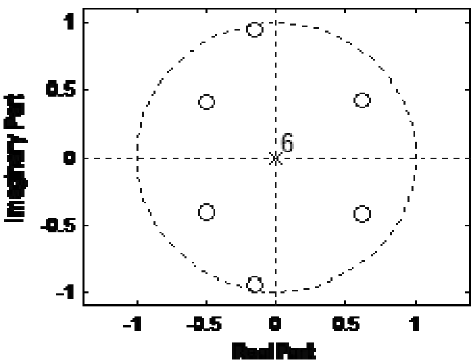

Figure 5. Zero-pole point distribution of *h2*

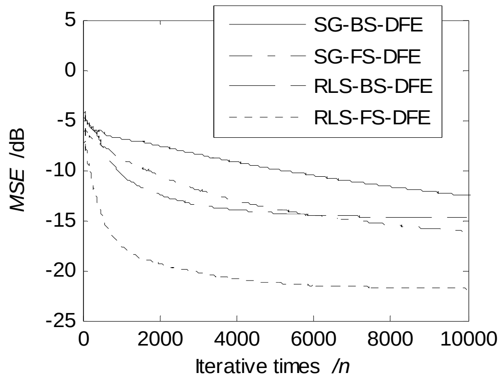

Figure 6. *MSE* convergence curve of *h2*

From simulation results we can see thatRLS-FS-DFE we proposed has fastest convergence rate and lowest *MSE* in the four blind equalizaitonalgorithms. It overcomes the shortcomes of CMA blind equalization such as slow convergence rate and big steady state residual error. For taking advantage over FSE and DFE, it can attain better performance even the channel has deep spectrum zero, also RLS algorithm leads a fast tracking ability for blind equalization.

4. Conclusion

In this study, the cost function of CMA is simplified to meet second norm form and the RLS blind equalization algorithm is obtained, and then extends it to the *T*/4 fractionally-spaced DFE blind equalization. The new algorithm takes advantages over FSE and DFE effectively to improve the blind equalization performance. Although RLS algorithm has a higher computational complexity compared with gradient descent algorithm, it has strong engineering practicability for the complex communication channel which needs fast tracking. Symbol-spaced DFE blind equalization requires considerable equalizer order when the channel has depth spectrum zero point, but fractionally–spaced equalizer needs only to meet the length and zero condition to realize the channel equalization, therefore, under certain conditions, RLS fractionally-spaced DFE has lower computational complexity compared with RLS symbol-spaced DFE.

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