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Quantum State Sharing of an Arbitrary Three-qubit State Using Two Four-qubit Cluster States

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Abstract

A scheme of quantum state sharing (QSTS) an arbitrary three-qubit state is presented using two particular four-qubit cluster state as the quantum channel. With four Bell pairs state measurements and the local unitary operation, any one of the two agents has the access to reconstruct the original if he/she collaborates with the other one.

Keywords: *Four-qubit cluster states, quantum state sharing, Bell-state measurements, arbitrary three-qubit state.*

1. Introduction

Since Hillery et al. [1] demonstrated that a three-particle GHZ state can be used for quantum state sharing (QSTS), QSTS has been attracted a great deal of attention in recent several years [2-15]. In a QSTS scheme, the quantum information to be shared is an arbitrary unknown quantum state in the sender's site, if and only if all agents collaborate, the unknown state can be fully reconstructed by one final receiver. QSTS of an arbitrary two-qubit state was realized by using four Bell pairs [4], the five-cluster states [16]. QSTS of an arbitrary three-qubit state was realized by using four sets of W-class states [17].

It is known that the cluster state has some interesting entanglement properties. So far, many schemes to prepare two-dimensional cluster states have been proposed [18-22] and a lot of applications of cluster states have been realized [23-26]. In this paper, by utilizing two four-qubit cluster states as a quantum channel, we will propose a scheme for sharing an arbitrary unknown three-qubit state among three parties.

2. Quantum State Sharing of an Arbitrary Three-Qubit State

Suppose there are three legitimate parties, Alice, Bob and Charlie. Alice is the sender of quantum information. Bob and Charlie are two agents. We suppose Alice has an arbitrary three-qubit state, which can be described as

$$\begin{aligned}|\chi\rangle_{a_1a_2a_3} &= (x_0|000\rangle + x_1|001\rangle + x_2|010\rangle + x_3|011\rangle \\ &+ x_4|100\rangle + x_5|101\rangle + x_6|110\rangle + x_7|111\rangle)_{a_1a_2a_3}\end{aligned} \tag{1}$$

where x_0, x_1, $\cdots$ and x_7 are arbitrary complex numbers, and it is assumed that the wave function satisfies the normalization condition $\sum_{i=0}^{7}|x_i|^2=1$. The quantum channel is two four-qubit cluster state [24]

$$|\varphi\rangle_{A_1A_2B_1C_1} = \frac{1}{2}(|0000\rangle + |0110\rangle + |1001\rangle - |1111\rangle)_{A_1A_2B_1C_1}$$

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$$|\varphi\rangle_{B_2B_3A_3C_2} = \frac{1}{2}(|0000\rangle + |0110\rangle + |1001\rangle - |1111\rangle)_{B_2B_3A_3C_2} \quad (2)$$

where particles $a_1, a_2, a_3, A_1, A_2, A_3$ belong to Alice, qubits B_1, B_2, B_3 belong to Bob and qubits C_1, C_2 belong to Charlie.

Here, we assume that Alice wants to transmit the state $|\chi\rangle_{a_1a_2a_3}$ to Bob who is assigned to reconstruct the original state in his own qubits (i.e., qubits B_1, B_2, B_3) with the help of Charlie.

Thus the total state of system can be expressed as:

$$|\psi\rangle_s = |\chi\rangle_{a_1a_2a_3} \otimes |\varphi\rangle_{A_1A_2B_1C_1} \otimes |\varphi\rangle_{B_2B_3A_3C_2} \quad (3)$$

To split the original state $|\chi\rangle_{a_1a_2a_3}$ and send the quantum information to Bob, first, Alice performs three Bell state measurements on qubits a_1, A_1, a_2, A_2, and a_3, A_3. Then Alice informs Charlie and Bob of her measured results via a classical channel. To help Bob reconstruct the original state, Charlie needs to make a two-qubit Bell state measurements on qubits C_1, C_2, and then tells Bob his measured result via a classical channel. According to Alice and Charlie's classical messages, Bob can obtain the original state in his own qubits (i.e.,qubits B_1, B_2, B_3) by performing an appropriate unitary transformation operation.

For convenience, the Bell state can be written [27-29]

$$\begin{aligned}
|\varphi^1\rangle_{a_mA_m} &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{a_mA_m} \\
|\varphi^2\rangle_{a_mA_m} &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{a_mA_m} \qquad m = 1,2,3 \\
|\varphi^3\rangle_{a_mA_m} &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{a_mA_m} \\
|\varphi^4\rangle_{a_mA_m} &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{a_mA_m}
\end{aligned} \quad (4)$$

Without loss of generality, let us suppose the results of Alice's announcement are $|\varphi^1\rangle_{a_1A_1}$, $|\varphi^1\rangle_{a_2A_2}$, $|\varphi^1\rangle_{a_3A_3}$ respectively. As a result, the state of particles $(B_1, B_2, B_3, C_1, C_2)$ collapse into

$$\begin{aligned}
|\psi\rangle_{B_1B_2B_3C_1C_2} = \frac{1}{8\sqrt{2}}(&x_0|00000\rangle + x_1|01000\rangle + x_4|10000\rangle + x_5|11000\rangle \\
&+ x_0|00101\rangle - x_1|01101\rangle + x_4|10101\rangle - x_5|11101\rangle \\
&+ x_2|00010\rangle + x_3|01010\rangle - x_6|10010\rangle - x_7|11010\rangle \\
&+ x_2|00111\rangle - x_3|01111\rangle - x_6|10111\rangle + x_7|11111\rangle)_{B_1B_2B_3C_1C_2}
\end{aligned} \quad (5)$$

After that if Alice communicates to Charlie of her actual measurement outcome via a classical channel, then Charlie can make a Bell state measurements on qubits (C_1, C_2). If

Charlie's measurement result is $\left|\varphi^{1}\right\rangle_{C_1C_2}$, then the particle pair (B_1, B_2, B_3) will collapse into

$$\left|\psi\right\rangle_{B_1B_2B_3} = \frac{1}{16}(x_0\left|000\right\rangle + x_1\left|010\right\rangle + x_2\left|001\right\rangle - x_3\left|011\right\rangle + x_4\left|100\right\rangle + x_5\left|110\right\rangle - x_6\left|101\right\rangle + x_7\left|111\right\rangle)_{B_1B_2B_3} \quad (6)$$

Then Bob will be able to apply the following unitary operation

$$U_{B_1B_2B_3} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

on particles (B_1, B_2, B_3). The resulting state Bob's particles will be can the original unknown three-qubit state.

Actually, as we known, $\left|\psi\right\rangle_s$ can be represented in the following form [27-29]:

$$\begin{aligned}\left|\psi\right\rangle_s &= \left|\chi\right\rangle_{a_1a_2a_3} \otimes \left|\varphi\right\rangle_{A_1A_2B_1C_1} \otimes \left|\varphi\right\rangle_{B_2B_3A_3C_2} \\ &= \frac{1}{16}\sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{4}\sum_{l=1}^{4}\left|\varphi^{i}\right\rangle_{a_1A_1}\left|\varphi^{j}\right\rangle_{a_2A_2}\left|\varphi^{k}\right\rangle_{a_3A_3}\left|\varphi^{l}\right\rangle_{C_1C_2}\sigma^{ijk,l}_{B_1B_2B_3}\left|\chi\right\rangle_{B_1B_2B_3}\end{aligned} \quad (8)$$

where $\left|\varphi^{i}\right\rangle_{a_1A_1}, \left|\varphi^{j}\right\rangle_{a_2A_2}, \left|\varphi^{k}\right\rangle_{a_3A_3}, \left|\varphi^{l}\right\rangle_{C_1C_2}$ are Bell states, and

$$\begin{aligned}\left|\chi\right\rangle_{B_1B_2B_3} &= (x_0\left|000\right\rangle + x_1\left|001\right\rangle + x_2\left|010\right\rangle + x_3\left|011\right\rangle \\ &\quad + x_4\left|100\right\rangle + x_5\left|101\right\rangle + x_6\left|110\right\rangle + x_7\left|111\right\rangle)_{B_1B_2B_3}\end{aligned} \quad (9)$$

After four Bell –state measurements, the corresponding collapsed state of particle B_1, B_2, B_3 will be $\frac{1}{16}\sigma^{ijk,l}_{B_1B_2B_3}\left|\chi\right\rangle_{B_1B_2B_3}$. The operator $\sigma^{ijk,l}_{B_1B_2B_3}$ here is called the collapse operator [30]. If $\sigma^{ijk,l}_{B_1B_2B_3}$ is a unitary operator, according to the outcomes received, Bob can successfully reconstruct the original unknown three-qubit state exactly by the inverse of the collapse operator $(\sigma^{ijk,l}_{B_1B_2B_3})^{-1}$.

By using Eqs. (2-8), the collapse operator can be obtained

$$\sigma_{B_1B_2B_3}^{111,1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{10a}$$

$$\sigma_{B_1B_2B_3}^{111,2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{10b}$$

$$\sigma_{B_1B_2B_3}^{111,3} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix} \tag{10c}$$

$$\sigma_{B_1B_2B_3}^{111,4} = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix} \tag{10d}$$

Other collapse operators $\sigma_{B_1B_2B_3}^{ijk,l}$ is given by

$$\sigma_{B_1B_2B_3}^{ijk,1} = \sigma_{B_1B_2B_3}^{111,1}\left(\sigma_{B_1}^{i} \otimes \sigma_{B_2}^{j} \otimes \sigma_{B_3}^{k}\right),$$

$$\sigma_{B_1B_2B_3}^{ijk,2}=\sigma_{B_1B_2B_3}^{111,2}\left(\sigma_{B_1}^{i}\otimes\sigma_{B_2}^{j}\otimes\sigma_{B_3}^{k}\right),$$
$$\sigma_{B_1B_2B_3}^{ijk,3}=\sigma_{B_1B_2B_3}^{111,3}\left(\sigma_{B_1}^{i}\otimes\sigma_{B_2}^{j}\otimes\sigma_{B_3}^{k}\right)$$
$$\sigma_{B_1B_2B_3}^{ijk,4}=\sigma_{B_1B_2B_3}^{111,4}\left(\sigma_{B_1}^{i}\otimes\sigma_{B_2}^{j}\otimes\sigma_{B_3}^{k}\right) \tag{11}$$

where $\hat{\sigma}_m^1=I_m$, $\hat{\sigma}_m^2=\sigma_{mz}$, $\hat{\sigma}_m^3=\sigma_{mx}$, $\hat{\sigma}_m^4=-i\sigma_{my}$, $m=B_1,B_2,B_3$ I_m is the two-dimensional identity operator and σ_{mz}, σ_{mx}, σ_{my} are the Pauli operator. Therefore, if Charlie's and Alice's measurement result are $\left|\varphi^1\right\rangle_{C_1C_2}$ $\left|\varphi^1\right\rangle_{a_1A_1}$, $\left|\varphi^1\right\rangle_{a_2A_2}$, $\left|\varphi^1\right\rangle_{a_3A_3}$ respectively, then the particles $\left(B_1,B_2,B_3\right)$ will collapse into

$$\begin{aligned}\left|\psi^{111,1}\right\rangle_{B_1B_2B_3} &=\frac{1}{16}\sigma_{B_1B_2B_3}^{111,1}\left|\chi\right\rangle_{B_1B_2B_3}\\ &=\frac{1}{16}\sigma_{B_1B_2B_3}^{111,1}(x_0\left|000\right\rangle+x_1\left|001\right\rangle+x_2\left|010\right\rangle+x_3\left|011\right\rangle\\ &\quad+x_4\left|100\right\rangle+x_5\left|101\right\rangle+x_6\left|110\right\rangle+x_7\left|111\right\rangle)_{B_1B_2B_3}\\ &=\frac{1}{16}(x_0\left|000\right\rangle+x_1\left|010\right\rangle+x_2\left|001\right\rangle-x_3\left|011\right\rangle\\ &\quad+x_4\left|100\right\rangle+x_5\left|110\right\rangle-x_6\left|101\right\rangle+x_7\left|111\right\rangle)_{B_1B_2B_3}\end{aligned} \tag{12}$$

It is just that of Eq. (6).

For other measurement results, similarly, Bob should perform operations

$$\left(\sigma_{B_1B_2B_3}^{ijk,l}\right)^{-1}=\left(\sigma_{B_1}^{i}\otimes\sigma_{B_2}^{j}\otimes\sigma_{B_3}^{k}\right)^{-1}\left(\sigma_{B_1B_2B_3}^{111,l}\right)^{-1}=\left(\sigma_{B_1}^{i}\otimes\sigma_{B_2}^{j}\otimes\sigma_{B_3}^{k}\right)\left(\sigma_{B_1B_2B_3}^{111,l}\right) \tag{13}$$

on particles $\left(B_1,B_2,B_3\right)$. Bob can successfully reconstruct the original unknown three-qubit state.

3. Conclusion

In this paper, by employing two four-qubit cluster state as the quantum channel, we have proposed a scheme for sharing an arbitrary unknown three-qubit state among three parties. Alice is the sender of quantum information. Bob and Charlie are two agents. According to Alice and Charlie's classical messages, Bob can obtain the original state in his own qubits by performing an appropriate unitary transformation operation. We hope that such an arbitrary three-qubit QSTS scheme can be realized experimentally with photons.

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