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Electromagnetic Vector Sensor array parameter estimation method

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Abstract

Joint estimation method for DOA and polarization parameters of single electromagnetic vector sensor is mostly concerned in radar signal processing now. This paper presents a new algorithm which uses the cyclostationary property of signal. It gets the estimation of DOA and polarization parameters from the noise subspace with minimum norm method. This method can reduce the estimate error of noise subspace vector. At the same time, it is immune to the additional stationary noise of any distribution and interference signals with different cyclic frequencies. Method has been tested by computer simulation and the test result is show in the paper.

Keywords: electromagnetic vector sensor, the minimum norm, DOA, polarization parameters, cyclostationary.

1. Introduction

Most radar and communication signals have the cyclostationary property [1, 2]. Cycle stationary signals with the same cycle frequency could presents circular correlation, while signals with different cycle frequency would generate a zero related function. Using this character, signals can inhibits any stable noises and interfering signals with different cycle frequencies. Therefore, it is especially suited to be used to process radar and communication signals.

Cardner and others first introduced method based on the cyclostationary property of signals into DOA estimates; they proposed the Cyclic-MUSIC method [3]. Pascal and others introduced the extension method to Cyclic-MUSIC [4]. Xu Guanghan and Kailath proposed spectrum correlation and subspace fitting (SC-SSF) method for DOA estimation, which can do the estimation with broadband signal. [5] All these method uses cyclostationay property of the source signal. They use cyclic correlation or other operator to get the estimation of DOA or other parameters. All these algorithms can restrain the noise and interference of all stable distribution or with different cyclic frequency. So these methods can expand the aperture of sensor array, they even can estimate DOA parameters when the destines number is more than sensors number [6].

All the methods mentioned above are based on MUSIC method. MUSIC is the globally recognized as one of the best estimating method for array signal processing [6]. However, there are two points that restrict the method to be used in actual work. First, MUSIC uses all the singular vectors in the noise subspace while searching for parameters, which lead to a large amount of calculations [7, 8]. Furthermore, because the affect of noise and interference, the vectors extracted from the covariance matrix are not accurately orthogonal with the direction vectors, which would lead to errors in the estimation of singular vector [9]. Scholars have done large number of researches to simplify the calculation required for MUSIC and search for ways to limit estimation errors [10]. There have been some solutions, such as Root-MUSIC, Least Squares MUSIC, multi-dimensional MUSIC and etc [11].

Targeting on this issue, this paper introduces a joint estimation algorithm for DOA and polarization parameters of single electromagnetic vector sensors based on cycle correlation function and minimum norm method. Because the array matrix of polarized angle-domain orthogonal with noise subspace, all of the singular vectors in the space compose a set of

orthonormal basis [13, 14]. Using the orthonormal basis set, any vector generated with the minimum norm will also be orthogonal with the direction vector matrix. With this orthogonality and minimum norm criterion, we can get the estimation of the signal parameter.

2. Signal Model

Assume there are K complete polarization signals with unknown spectrum density projects into single electromagnetic vector sensor, with incident angle (θ_k, ϕ_k), and polarization parameter (γ_k, η_k), $k = 1, 2, \cdots, K$ .Thus the output of trimmed vector sensor can be expressed as following:

$$
\begin{aligned}
X(t) &= [x_1(t), x_2(t), \cdots, x_6(t)]^T \\
&= \sum_{k=1}^{K} a_k \, \mathrm{s}_k(t) + \mathbf{n}(t) \\
&= A\mathbf{s}(t) + \mathbf{n}(t)
\end{aligned}
\tag{1}
$$

where $\mathbf{s}(t) = [\mathrm{s}_1(t), \mathrm{s}_2(t), \cdots, \mathrm{s}_K(t)]^T$ is Signal vector matrix, $\mathrm{s}_i(t), i = 1, \cdots K$, $A = [a_1, a_2, \cdots, a_K] = [\Theta_1 g_1, \Theta_2 g_2, \cdots, \Theta_K g_K]$ is polarization angle field oriented vector matrix; $\mathbf{n}(t) = [n_1(t), n_2(t), \cdots, n_6(t)]^T$ is the colored noise vector matrix.

With respect to normality, the assumption is [5, 6]:

1. $\{\mathrm{S}_k(t)\}_{k=1}^{K}$, are K ($K \leq 5$) zero-mean cycle stationary processes with unknown spectral density, that shares the same cycling frequency(the frequency refers to the base frequency, does not include the multipliers of the base frequency), and all the $S_k(t)$ are independent circular statistics;

2. $\{\mathrm{n}_i(t)\}_{i=1}^{6}$ is the stationary random process with zero-mean arbitrary distribution and unknown spectral density, and the statistics between the noises and signals are circled independently;

3. Cycle frequency α is known.

3. Minimum Norm Method Based on the Cycle Correlation Function

Set the single electromagnetic vector sensor output as formula (1). Define the functional matrix of output signal $X(t)$ as :

$$
R_X^\alpha \; \square \; \mathbf{E}[X(t+\frac{\tau}{2})X^H(t-\frac{\tau}{2})e^{-j2\pi\alpha t}]
\tag{2}
$$

where α is the cycle frequency, which is known as previously stated; Upper case H represents hermitian Transpose.

In function (2), the（p, q）element in matrix R_X^α is:

$$
\begin{aligned}
r_{xpxq}^\alpha(\tau) &= \mathbf{E}[x_p(t+\frac{\tau}{2})x_q^H(t-\frac{\tau}{2})e^{-j2\pi\alpha t}] \\
&= \mathbf{E}\Big\{ \, [\sum_{k=1}^{K} a_{pk}Sk(t+\frac{\tau}{2}) + n_p(t+\frac{\tau}{2})][\sum_{h=1}^{K} a_{qh}Sh(t-\frac{\tau}{2}) + n_q(t-\frac{\tau}{2})]^H \, e^{-j2\pi\alpha t} \Big\} \\
&= \mathbf{E}\Big\{ \, \{ [\sum_{k=1}^{K} a_{pk}Sk(t+\frac{\tau}{2})][\sum_{h=1}^{K} a_{qh}S_h^H(t-\frac{\tau}{2})] + [\sum_{h=1}^{K} a_{qh}S_h^H(t-\frac{\tau}{2})n_p(t+\frac{\tau}{2})] \\
&\quad + [\sum_{k=1}^{K} a_{pk}Sk(t+\frac{\tau}{2})n_q^H(t-\frac{\tau}{2})] + n_p(t+\frac{\tau}{2})n_q^H(t-\frac{\tau}{2}) \} e^{-j2\pi\alpha t} \Big\}
\end{aligned}
\tag{3}
$$

From previous assumption, we can say:

$$r_{x_p x_q}^{\alpha}(\tau) = \sum_{k=1}^{K} a_{pk} r_{s_k s_k}^{\alpha}(\tau) a_{qh}^{*}$$

(4)

Sub equation (4) into (2), we have:

$$\boldsymbol{R}_X^{\alpha} = \sum_{k=1}^{K} a_k r_{s_k s_k}^{\alpha}(\tau) a_k^H = \mathbf{A} \mathbf{R}_s^{\alpha} \mathbf{A}^H$$

(5)

where $\boldsymbol{R}_s^{\alpha} = diag[a_{11}^2 r_{s1s1}^{\alpha}(\tau), \cdots, |a_{kk}|^2 r_{sksk}^{\alpha}(\tau)]$; matrix $\mathbf{A} = [a_1, a_2, \cdots, a_k]$ is called polarized angle guided vector matrix.

Theorem 1 [12]: Assume single electromagnetic vector sensor output cycle matrix is R_x^{α} as in (5), all vector satisfy

$$R_X^{\alpha} \ F \ = \ 0$$

(6)

will yield the orthogonal between F and R_x^{α} polarized angle guided vector matrix, or:

$$A^H F = 0$$

(7)

When there are K (K $\leq$ 5) non-Gaussian polarization cycle stationary signal projecting into single electromagnetic vector sensor, the sequence of $\boldsymbol{R}_X^{\alpha}$ is K. Perform singular value decomposition (SVD) on $\boldsymbol{R}_X^{\alpha}$, we have:

$$\boldsymbol{R}_X^{\alpha} = \boldsymbol{U} \boldsymbol{\Sigma} \boldsymbol{V}^H$$

(8)

In the equation, $\Sigma = diag[\sigma_1, \sigma_2, \cdots, \sigma_6]$ is the SVD matrix of $\boldsymbol{R}_X^{\alpha}$, and all SVDs are descend ordered, where:

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_6$$

(9)

U、V are the left and right SVD of matrix $\boldsymbol{R}_X^{\alpha}$.

Because the domain of $\boldsymbol{R}_X^{\alpha}$ is $K(K \leq 5)$, Based on SVD theory, $\boldsymbol{R}_X^{\alpha}$ only has K SVD values, being

$$\sigma_1 \geq \sigma_2 \cdots \geq \sigma_k > 0$$

(10)

The rest of P-K SVD values are zeros. Set:

$$\Sigma_1 = diag[\sigma_1, \sigma_2, \cdots \sigma_k]$$

(11)

Then (8) can be rewritten as:

$$\boldsymbol{R}_X^{\alpha} = [\boldsymbol{U}_1 \quad \boldsymbol{U}_2] \begin{bmatrix} \Sigma_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1^H \\ \boldsymbol{V}_2^H \end{bmatrix}$$

(12)

Expand equation (12), we have

$$\boldsymbol{R}_X^{\alpha} = \boldsymbol{U}_1 \Sigma_1 V_1^H$$

(13)

and

$$\boldsymbol{R}_X^\alpha \boldsymbol{V}_2 = 0 \tag{14}$$

Based on theorem 1, all SVD values of V_2 will quadrate with matrix A. Therefore, we have

$$A^H \boldsymbol{V}_2 = 0 \tag{15}$$

Within these column vectors, we are most interested in the minimum norm in equation (15), that is:

$$\min_{W \in F} w^H w \tag{16}$$

where: $F = \{w : \boldsymbol{R}_X^\alpha w = 0 \text{ ,and } w(0) = 1\}$.

We choose minimum norm method, because we can only get the estimation $\hat{\boldsymbol{R}}_X^\alpha$ of matrix $\boldsymbol{R}_X^\alpha$. Based on linear algebra theorem, the minimum norm of $\boldsymbol{R}_X^\alpha$ is the least sensitive to estimation errors of matrix $\boldsymbol{R}_X^\alpha$. From equation (14), the value w must fall into the noise vector space V_2. Because the SVD in noise vector space $V_i(i = k+1, \cdots, 6)$ forms a complete orthogonal basis, therefore

$$w = \sum_{i=k+1}^{6} \alpha_i v_i^H \tag{17}$$

From the orthogonality of $V_i(i = k+1, \cdots, 6)$, we have:

$$w^H w = \sum_{i=k+1}^{6} \alpha_i^2 \tag{18}$$

$w(0) = 1$ can be paraphrased as:

$$\sum_{i=k+1}^{6} \alpha_i v_i^*(0) = 1 \tag{19}$$

Therefore, to calculate the minimum of $w^H w$ with restriction $\boldsymbol{R}_X^\alpha w = 0$, $w(0) = 1$, is the same as calculating the unconditional extreme values of:

$$f(\alpha_i, \lambda) = \sum_{i=k+1}^{6} \alpha_i^2 + \lambda \left(\sum_{i=k+1}^{6} \alpha_i v_i^*(0) - 1 \right) \tag{20}$$

Let

$$\frac{\partial f}{\partial \alpha_i} = 0 \quad , i = k+1, \cdots, 6 \tag{21}$$

We have:

$$2\alpha_i + \lambda v_i^*(0) = 0 \quad , i = k+1, \cdots, 6 \tag{22}$$

Substitute (22) into (17), we have:

$$w = \beta \sum_{i=k+1}^{6} v_i^*(0) v_i^H \tag{23}$$

From $w(0) = 1$, we have:

$$\beta = [\sum_{i=k+1}^{6} \left| v_i(0) \right|^2]^{-1} \tag{24}$$

Therefore

$$w = \frac{\sum_{i=k+1}^{6} v_i^*(0) v_i^H}{\sum_{i=k+1}^{6} \left| v_i(0) \right|^2} \tag{25}$$

Because characteristic vector w is the linear combination of noise vector subspace $V_i (i = k+1, \cdots, 6)$, so w can be located within noise vector subspace V_2. As previously shown, there must be:

$$A^H w = 0 \tag{26}$$

Two-dimension DOA and polarization parameter of the projecting signal can be obtained from four-dimensional search method:

$$
\begin{aligned}
P(\theta,\phi,\gamma,\eta) &= \arg \underset{\theta,\phi,\gamma,\eta}{MAX} \frac{1}{\left| w A^H(\theta,\phi,\gamma,\eta) \right|^2} \\
&= \arg \underset{\theta,\phi,\gamma,\eta}{MAX} \frac{1}{\det[A^H(\theta,\phi,\gamma,\eta) w w^H A(\theta,\phi,\gamma,\eta)]} \\
&= \arg \underset{\theta,\phi,\gamma,\eta}{MAX} \frac{1}{\det[g^H \underbrace{\Theta^H w w^H \Theta}_{Z(\theta,\phi)} g]}
\end{aligned}
\tag{27}
$$

Where $\det[\ ]$ is the operator of the matrix. Note the independence between the 2×2 matrix $Z(\theta,\phi)$ in equation (27) and polarized parameter (γ,η). Therefore, the DOA of signal can be found with the following two dimensional search:

$$\left\{ \hat{\theta}, \hat{\phi} \right\} = \arg \underset{\theta,\phi}{MAX} \frac{1}{\det[Z(\theta,\phi)]} \tag{28}$$

After getting the estimation of DOA $\left\{ \hat{\theta}, \hat{\phi} \right\}$, subtract it into $Z(\hat{\theta},\hat{\phi})$, and perform the following search to get the estimate of polarized parameter $(\hat{\gamma}, \hat{\eta})$:

$$\left\{ \hat{\gamma}, \hat{\eta} \right\} = \arg \underset{\gamma,\eta}{MAX} \frac{1}{\det[g^H Z(\hat{\theta},\hat{\phi}) g]} \tag{29}$$

By efficiently convert the four dimensional search in (27) into two dimensional search in (28) and (29), the method significantly reduces the work load.

This is the single electromagnetic vector sensor DOA under complicated noise background and subspace estimate of polarized parameter- minimum norm method.

4. Result and Discussion

Experiment 1: Test performance and the resisting ability to interference of the minimum norm method based on cycle related subspace.

Environment: a AM signal S_1 with projecting angle（75°, 50°）is being tested onto the single electromagnetic vector sensor. Its carrier frequency is $fc_1 = 15MHz$, and polarization parameter is（45°, 45°）; another AM interfering signal S_2 with angle（102°,50°）is projected. Its carrier frequency is $fc_2 = 12MHz$ and polarization parameter is（80°, 90°）; added noise being the stable ACGN. It is generated with Gaussian white noise through a Four order bandpass filter with SNR=10dB, SIR=3dB. Sampling rate f_S is 100MHz, beat is1000;cycle frequency α is $4fc_1 = 60MHz$.

Method: use the minimum norm method and perform Monte Carlo experiment 100 times.

Results: Typical DOA test results of signal and estimate of polarization parameter are shown in figure 1 and 2:
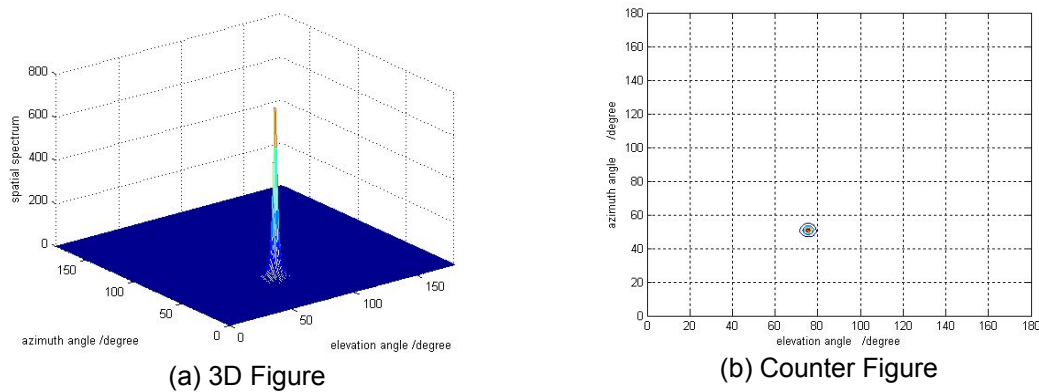

(a) 3D Figure   (b) Counter Figure

Figure 1. Estimate DOA of S_1 is SNR=10dB,SIR=3dB

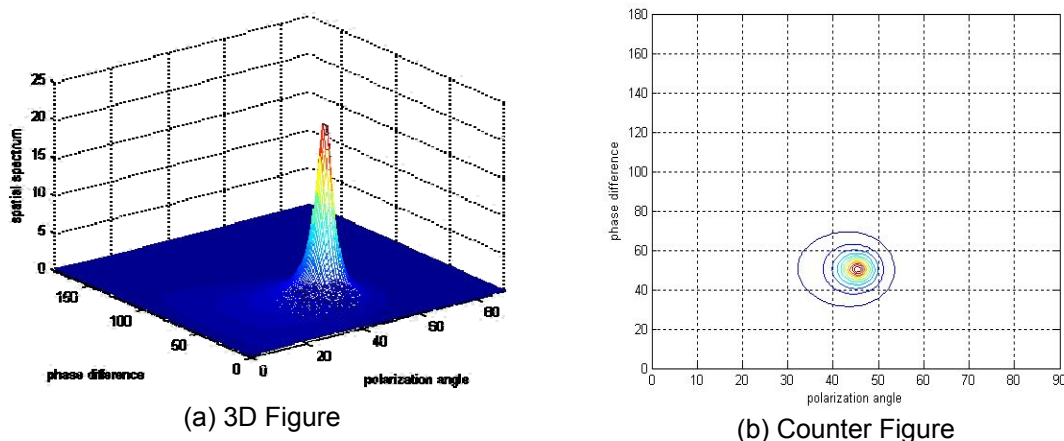(a) 3D Figure   (b) Counter Figure

Figure 2. Estimated polarization parameter of S_1 is (SNR=10dB,SIR=3dB)

From figure 1 and 2, when cycled stable interfering signals and color noise with different frequency coexist, the minimum normal method can effectively restrain the ACGN and

interfering signals with different cycling frequency. From figure 1 and 2, the accuracy of DOA and polarization parameter estimate is quite high as well.

        Experiment 2: Comparison between the minimum normal method and normal MUSIC method based on high order cumulants.
        Environment: Same as experiment 1 but do not use cycle frequency α.
        Method: Perform 100 times of Monte Carlo experiment using normal MUSIC method based on high order cumulants. Compare the results with experiment 1.
        Results: Estimates generated from MUSIC is shown in figure 3 and 4.
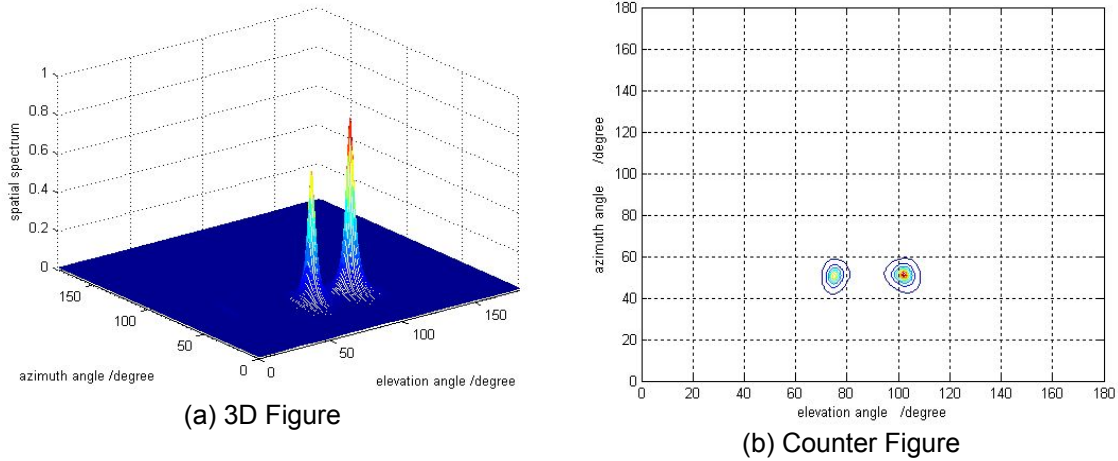

(a) 3D Figure

(b) Counter Figure

Figure 3. DOA estimate from the normal MUSIC method (SNR=10dB）

(a) 3D Figure

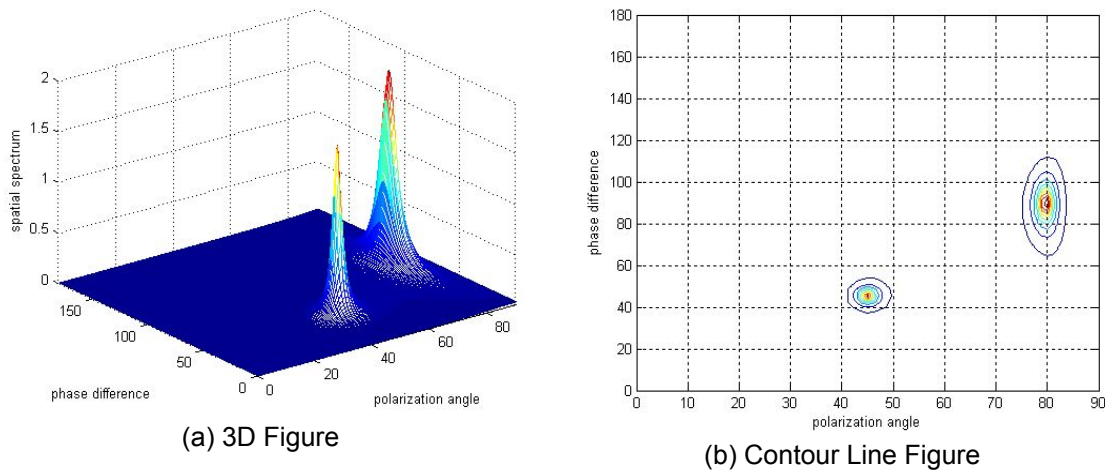(b) Contour Line Figure

Figure 4. Estimate of polarization parameter using normal MUSIC method, SNR=10dB）

        Compare the simulation results between figure 3 and figure 4, it is obvious, although the higher order cumulants MUSIC method can resist ACGN, but when interfering signal come in, false peaks appear significantly. This has implied the minimum norm method based on cycled subspace is very effective at isolating and resist interfering signals.
        Experiment 3:Test the estimation efficiency under various Signal to Noise Ratio(SNR) conditions.

Environment:Same as experiment 1 but use SNR between the range of 10dB and 40d, interval at 5dB.

Method:Perform Monte Carlo experiment on each SND 100 times. Using root mean square error (RMSE) to test these data. The RMSE of polarization parameters are calculated as following:

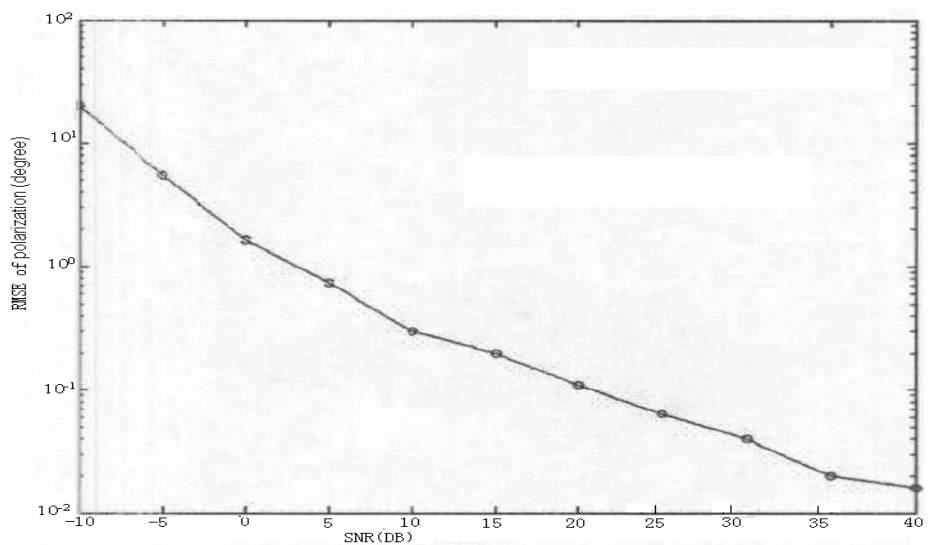$$RMSE_{(DOA)} = \sqrt{E[(\hat{\theta} - \theta)^2 + (\hat{\phi} - \phi)^2]}$$ (30)

$$RMSE_{(polarization)} = \sqrt{E[(\hat{\gamma} - \gamma)^2 + (\hat{\eta} - \eta)^2]}$$ (31)

In (30) and (31) , $\hat{\theta}$、$\hat{\phi}$、$\hat{\gamma}$、$\hat{\eta}$ are the estimates of each parameter, θ、ϕ、γ、η are the actual values.

Result:The RMSE curve of DOA and polarization parameters are shown in Figure 5:
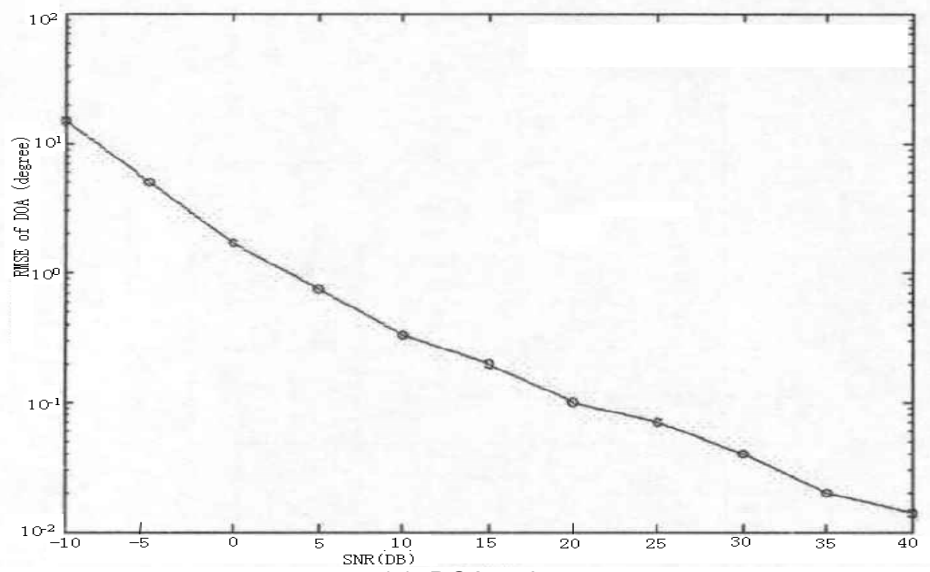
(a) DOA estimates

(b) polarization parameter estimates

Figure 5. RMSE of DOA and polarization parameter estimates

From figure 5(a) and figure 5(b), as the SNR increases, errors of RMSE on parameter estimates is decreasing; even when the SNR is very low, the estimate is still quite accurate. This has implied the minimum norm method based on cycled subspace can effectively resist random stationary colored noise and cycle frequency from alien interference signal.

5. Conclusion

This paper introduces a joint estimation algorithm of DOA and polarization parameters which is based on single cycle-related electromagnetic vector sensors. This method uses the circular stability of source signals and minimum norm method to get the estimation of DOA and polarization parameters. It can effectively inhibit stationary random noise and cycle stationary interference with different cycle frequency. Therefore, it has great signal estimation efficiency. This estimation method calculates the minimum norm of subspace noise, it assemble the singular vectors into an eigenvector. Not only it can suppress noise singular vector estimation error, it can also reduce the calculation amount required by traditional MUSIC method, therefore, represents a pragmatic value in engineering industry. This method can use the difference of polarization domain to distinguish signal sources which are close in space. Stimulation results verified the efficiency of the parameter estimation and its suppression ability to noises.

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