

Bi-Level Multi-criteria Multiple Constraint Level Optimization Models and Its Application

Lei Zhao^{*1}, Yihua Zhong², And Yilin Wang³

¹State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, Sichuan 610500, China

²School of Sciences, Southwest Petroleum University, Chengdu, Sichuan 610500, China

³CNOOC Shanghai Branch, Shanghai 200000, China

*Corresponding author, e-mail: zhaoleo927@foxmail.com, Zhongyh_65@126.com

Abstract

Because oil field development system is a large hierarchical and uncertain system, this paper uses the theory of bi-level programming and multi-criteria multiple constraint level (MC^2) to formulate a new oilfield measure structural optimization model which is bi-level multiple objectives and multiple constraint level nonlinear programming, and present a new method to solve the bi-level programming whose lower is multiple objectives nonlinear programming, whose upper is MC^2 linear programming. The result of this model not only may feed back to the comprehensive information of measures output distribution optimization to decision-makers as a whole, but also can provide decision makers oil field exploitation contingency planning to deal with changed resource constraint level. The case study shows that the result fitting calculation by the model is coincide with the historical data of oil field, the model is correct and effective. Moreover this research may provide a reliable new method for oil field development optimal decision-making.

Keyword: optimization model, oil field development programming, multi-criteria multiple constraint level, bi-level programming

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1. Introduction

Oil enterprise is a large hierarchical system which includes many oil production plants which includes many oil blocks. The upper system are leaders who control the overall situation and play a leading and coordinating role to minimize the cost of the entire system; the lower system are followers who make independent decisions to subject to the top decision-making and minimize the cost of the subsystem. Decision-making mechanism by bi-level programming method and theory are as follows: the upper make their selves decisions which may directly impact on the objective function and feasibility of the lower; the lower feed back to their optimal decision subjecting to the upper restriction to the upper, decisions of the lower also have effect on the objective function and feasibility of the upper that they make appropriate adjustments on their decisions until the top objective function is minimum. This decision-making mechanism make superior departments in charge of subordinate departments by making impact on the lower decision-making conditions to guide the lower indirectly to make favorable decision to the whole system, and it may avoid direct intervention and command of the upper. Therefore, bi-level programming method and theory which solves the problem of large-scale system programming could better solve the present measures component problem of oil enterprise [1], [2]. Moreover, most present planning studies are under the condition of given single fixed resource constraint level, but all the resource constraint levels such as total oil production, investment cost, workload, etc., are uncertain in actual programming, and even have multiple resource constraint levels in some planning. So, planning model not only should find the optimal program in fixed resource constraint level but also should find the optimal solutions which could deal with various resources constraint levels to take it as actual prorating plan. A new model combining multi-criteria multiple constraint level (MC^2) and bi-level programming of oilfield measures production component optimization was established by using bi-level programming and multiple constraint level linear programming framework in this paper [3], [4]. Compared with

the existing oilfield measures production component optimization model [5], this model can feed back the integrated measures production distribution optimization information to decision-makers as a whole, and also provide decision makers oil field exploitation contingency planning to deal with changed resource constraint level. Oil company only need to make overall planning programs, and all the oil production plants and production blocks implement the actual production planning.

2. Formulating of Oilfield Measures Structure Optimal Model Under Bi-Level Multi-Criteria Multiple Constraint Level

Eight main measures oilfield applying includes fracture, acid treat, heavy repair, perforations adding, electric pump, block off, transfer pumping and so on. Inputs of measures can vary with the corresponding costs, workload and develop dynamic planning laws, so it is essentially optimizing the corresponding costs, workload to optimize measures input so that make oil company and oil production plants complete production tasks, improve efficiency and meet the performance changes of development. According to the actual research about a domestic oilfield, factors affecting the production of various measures mainly include measures wells, measures cost, effective measures wells, water cut and reserve recovery, etc.

2.1. Decision Variable

Decision variables from the superior (Oilfield Company) are the measures production of oil production plants, their vector form is $x = (X_1, X_2, \dots, X_n)^T$, where X_i is measures production of the oil production plant, ($i = 1, 2, \dots, n$).

Decision variables from the subordinate (oil production plants) are the influencing factor of various sub-measures production, decision variables of the oil production plant are each measure production of the oil production plant, their vector form is $y_i = (y_i^1, y_i^2, \dots, y_i^8)^T$, ($i = 1, 2, \dots, n$), where y_i^k is the measure production of the oil production plant ($k = 1, 2, \dots, 8$).

2.2. Objective Function

The goals oil exploration system seeking are varied, such as the lowest production costs, the most economical investment, the highest production, the lowest water cut, the fastest production rate and the maximum ultimate recovery, etc.

2.2.1. The Objective Functions of the Top Decision-Making

The objective of the top decision-making (Oilfield Company) is the largest total measures production and the best benefits in this paper, their vector form is

$$F(x, y) = (F_1(X_i, y), F_2(X_i, y))^T \quad (1)$$

Where $F_1(x, y) = \sum_{i=1}^n X_i$, $F_2(x, y) = \sum_{i=1}^n [CX_i - (y_i^{12} + y_i^{22} + \dots + y_i^{82})]$

C —oil prices in planning year; y_i^{k2} —measures cost of measure of the oil production plant in planning year of the oil production plant ($k = 1, 2, \dots, 8$).

2.2.2. The Objective Function of the Underlying Decision-Making

The objective of the underlying decision-making (oil production plants) are the sum of various sub-measures production (post flush production, acidification production, overhaul production, perforations adding production, electric pump production, water shut-off production, transfer pumping production and other post treatment production) reaches the maximum and the best benefits, the objective function of the oil production plant is

$$f_i = (f_i^1, f_i^2), (i = 1, 2, \dots, n) \quad (2)$$

where $f_i^1 = \sum_{k=1}^8 y_i^k$, $f_i^2 = CX_i - (y_i^{12} + y_i^{22} + \dots + y_i^{82})$.

2.3. Constraint Conditions and Constraint Levels

2.3.1 Constraint Conditions and Constraint Levels of the Top Decision-Making

Constraint conditions of the top decision-making (Oilfield Company) are lower bound constraints about total measure production of oil production plants and measure production of each oil production plant.

In order to meet the national demand for crude oil, Group Company should have early warning project planning from the macro-and micro-management perspective, thus it is necessary to consider comprehensively the national demand for crude oil, reserves, oil exploration technology and development law, etc. to consult with the oil field about constraint levels for different measures productions. For simplicity, the extreme cases of constraints level of the total measure production and each measure production of oilfield company - low level and high level (other levels can be obtained by their equilibrium, are only considered, they are

$$\sum_{i=1}^n X_i \geq (d_1, d_2) \quad (3)$$

$$X_i \geq (q_{i1}, q_{i2}) \quad (4)$$

where the Eq. (3) means that total measure production of oilfield company is not less than the linear combination $(\gamma_1 d_1 + \gamma_2 d_2)$ of d_1 and d_2 , where $\gamma_1, \gamma_2 \in [0, 1]$ are the preference values of oilfield company decision-makers for low constraint level d_1 and high constraint level d_2 , and $\gamma_1 + \gamma_2 = 1$. The Eq.(4) shows that each measure production of oilfield company is not less than the linear combination $(\lambda_1 q_{i1} + \lambda_2 q_{i2})$ of q_{i1} and q_{i2} , where $\lambda_1, \lambda_2 \in [0, 1]$ are the preference values of oilfield company decision-makers for low constraint level q_{i1} and high constraint level q_{i2} , and $\lambda_1 + \lambda_2 = 1$.

2.3.2. Constraint Conditions and Constraint Levels of the Lower Decision-Making

The lower (all the oil production plants) constraint conditions are the minimum measure production, the maximum total cost of measure production, the minimum each measure production, the relationship between every measure production and their corresponding influence factors, upper and lower bound of corresponding influence factors of each measure production. Using neural network method mentioned in the literature to establish the correlation between every measure production and their corresponding influence factors [6]. Factors of the measure have m_k items mainly includes measures wells, measures cost, effective measures wells, water cut and reserve recovery, etc.

2.4. The Model Constructing

A new oilfield measure structure optimal model in planning year was obtained by the previous analysis, which is the nonlinear programming model under bi-level multiple objective and multiple constraint level.

The upper:

$$\begin{aligned} & \max \sum_{i=1}^n X_i \\ & \max \sum_{i=1}^n [CX_i - (y_i^{12} + y_i^{22} + \dots + y_i^{82})] \\ & s.t. \begin{cases} \sum_{i=1}^n X_i \geq (d_1, d_2) \\ X_i \geq (q_{i1}, q_{i2}) \quad (i=1, 2, \dots, n) \end{cases} \end{aligned} \quad (5)$$

The lower:

$$\begin{aligned}
 & \max \sum_{k=1}^8 y_i^k \\
 & \max [CX_i - (y_i^{12} + y_i^{22} + \dots + y_i^{82})] \\
 & \left\{ \begin{array}{l}
 \sum_{k=1}^8 y_i^k \geq X_i \\
 y_i^{12} + y_i^{22} + \dots + y_i^{82} \leq B \\
 y_i^1 = y_i^1(y_i^{11}, y_i^{12}, \dots, y_i^{1m_1}) \\
 y_i^2 = y_i^2(y_i^{21}, y_i^{22}, \dots, y_i^{2m_2}) \\
 \dots \\
 y_i^8 = y_i^8(y_i^{81}, y_i^{82}, \dots, y_i^{8m_8}) \\
 a_i^{km} \leq y_i^{km} \leq b_i^{km} \\
 y_i^k \geq s_i^k
 \end{array} \right. \quad (6)
 \end{aligned}$$

Where y_i^{km} —influencing factor of measure production of oil production plant, $m = 1, 2, \dots, m_k, i = 1, 2, \dots, n, k = 1, 2, \dots, 8$;

B —the maximum total cost of measure production;

s_i^k —the minimum measure production of the measure production of oil production plant;

a_i^{km} —the minimum of influencing factor of measure production of oil production plant;

b_i^{km} —the maximum of influencing factor of measure production of oil production plant;

3. Solution of Oilfield Measures Structure Optimal Model under Bi-Level Multi-Criteria Multiple Constraint Level

As the oilfield measure structure optimal model of bi-level multiple objectives and multiple constraint level was established, it is difficult to find an optimal solution for this model. In order to solve this problem which is bi-level multiple objectives and multiple constraint level nonlinear programming, this paper studied the theory and method of bi-level programming and MC^2 linear programming [7], [8], and presented a new method to solve the bi-level programming whose lower is multiple objectives nonlinear programming; whose upper is MC^2 linear programming. The method is to use neural network to establish the correlation between various sub-measures production of the lower and their corresponding influence factors, and then use the data obtained as a constraint to solve MC^2 linear programming of the upper. The specific steps of the solution are as follows [9], [10]:

Step 1: Build bi-level multiple criteria and multiple constraint level model according to the practical problems;

Step 2: Use neural network to establish the correlation between various sub-measures production of the lower and their corresponding influence factors, and solve multiple objectives nonlinear programming of lower to get the measures production of oil production plants in planning year $y_i^k, k = 1, 2, \dots, 8; i = 1, 2, \dots, n$;

Step 3: Use the data to determine the low constraint level d_1 and high constraint level d_2 of the upper and the MC^2 interior point method mentioned in the literature to solve the linear programming model of the upper to obtain production program under low and high level of constraint [11];

Step 4: For any given resource constraint level d which is the linear combination $(\gamma_1 d_1 + \gamma_2 d_2)$ of d_1 and d_2 , find out the linear coefficients γ_1, γ_2 and planning programs of measures production components [12];

Step 5: Introduce the results of step 4 to the upper management companies (Oilfield Company), process of decision-making is complete if they are satisfied with the results, otherwise, interact with the upper (Oilfield Company) and lower (oil production plants) respectively and modify the initial data in accordance with their preferences, then return to Step 2, until get the planning programs of measures production components which the upper (Oilfield Company) are satisfied.

4. Case Study

In order to validate the reliability of the model and the feasibility of the algorithm presented in the paper, this section made development planning of certain domestic oilfield in the middle and late development.

Eight sub-measures production of three oil production plants of certain domestic oilfield from 2004 to 2010 in the middle and late development and their corresponding influence factors historical data shown in Table 1-3. According to the dynamic variation of the development reflected by the historical data, measures production component of plants in 2011 the year of planning will be optimally distributed as follows. Oil production unit is ten thousand tons; cost unit is ten thousand Yuan.

Table 1. Plant I history data table

Parameters	2004	2005	2006	2007	2008	2009	2010
post flush production	6.348	7.211	3.203	2.186	3.012	4.386	8.711
fractured wells	55	60	44	41	33	53	91
fracturing costs	3366	4671	2144	1472	1968	3038	5364
acidification production	3.65	2.26	4.59	4.24	4.02	4.08	5.34
acidified wells	54	61	83	74	61	72	85
acidification costs	2847	1599	3374	3282	3039	3224	4013
perforations adding production	69.58	71.06	86.82	72.03	71.05	67.87	53.65
perforations adding wells	883	893	961	812	811	920	735
perforations adding costs	15277	15352	18647	15644	18311	11128	10977
transfer pumping production	4.23	4.12	3.83	2.47	2.03	2.57	2.39
transfer pumping wells	56	46	54	47	44	47	42
transfer pumping costs	2965	3050	2886	1522	1192	1720	1623
electric pump production	5.73	9.83	8.69	5.42	8.00	5.98	5.80
electric pump wells	120	147	144	116	129	114	87
electric pump costs	4975	5222	4582	2852	5209	3304	3145
water shut-off production	11.02	10.52	11.03	11.48	11.00	10.51	17.11
water shut-off wells	262	253	255	243	239	216	303
water shut-off costs	6958	6717	7046	7394	7182	5904	8760
overhaul production	4.97	5.63	5.58	2.08	3.00	2.27	1.73
overhaul wells	65	73	69	32	44	40	35
overhaul costs	3826	4036	3666	1325	2233	1454	1183
other post treatment production	8.85	10.13	6.23	5.73	4.00	8.29	7.24
other post treatment wells	176	184	132	119	107	187	146
other post treatment costs	7215	7559	4402	4658	2515	5877	5632

Establish an oilfield measures production component optimization model under bi-level multiple objective and multiple constraint level and solve it based on the method of this paper. It is that use neural network to establish the correlation between various sub-measures production of the lower and their corresponding influence factors, solve multiple objectives nonlinear programming of lower to get the measures production of oil production plants in planning year shown in Table 4, then use the data obtained to determine the low constraint level d_1 and high constraint level d_2 of the upper, and then use the MC^2 interior point method to solve the linear programming model of the upper to obtain production program under low and high level of constraint shown in Table 5.

Table 2. Plant II history data table

Parameters	2004	2005	2006	2007	2008	2009	2010
post flush production	7.513	8.445	2.664	3.012	3.557	6.224	9.437
fractured wells	59	60	41	45	40	62	89
fracturing costs	3467	4862	2044	1572	1868	3238	5664
acidification production	3.77	2.13	5.59	6.24	3.02	4.98	6.34
acidified wells	52	59	90	85	49	82	90
acidification costs	2733	1479	3884	4182	2079	3929	4433
perforations adding production	72.43	65.06	79.63	71.03	88.27	75.27	61.61
perforations adding wells	897	864	869	801	951	944	788
perforations adding costs	16437	14952	15537	15544	19111	17525	15627
transfer pumping production	3.66	5.51	4.23	3.47	2.28	3.57	2.39
transfer pumping wells	71	51	51	47	46	56	41
transfer pumping costs	3021	3499	3377	1522	1298	2320	1655
electric pump production	5.82	9.43	8.77	4.42	8.00	6.98	5.81
electric pump wells	123	141	152	99	128	135	87
electric pump costs	5012	5012	4642	2252	5201	3764	3155
water shut-off production	11.02	11.33	12.03	10.34	10.59	12.51	17.11
water shut-off wells	262	241	276	221	213	245	302
water shut-off costs	6958	6527	7235	7284	7081	6201	8750
overhaul production	4.96	6.63	6.22	2.67	3.24	2.88	2.73
overhaul wells	63	83	61	38	49	51	41
overhaul costs	3819	4216	3486	1325	2313	1634	1347
other post treatment production	7.85	10.12	5.19	5.73	3.67	7.29	7.28
other post treatment wells	156	183	121	119	99	167	148
other post treatment costs	7013	7560	4392	4658	2239	5653	5700

Table 3. Plant III history data table

Parameters	2004	2005	2006	2007	2008	2009	2010
post flush production	6.448	7.311	3.303	2.286	3.112	4.486	8.811
fractured wells	56	64	46	44	35	57	92
fracturing costs	3367	4672	2146	1473	1969	3040	5367
acidification production	3.67	2.28	4.60	4.26	4.07	4.11	5.38
acidified wells	52	60	82	73	59	70	82
acidification costs	2845	1597	3372	3280	3037	3222	4010
perforations adding production	69.60	71.09	86.85	72.07	71.06	67.88	53.67
perforations adding wells	880	891	960	812	810	918	732
perforations adding costs	15275	15350	18644	15641	18309	11126	10975
transfer pumping production	4.24	4.13	3.85	2.48	2.05	2.59	2.42
transfer pumping wells	55	45	53	46	43	46	41
transfer pumping costs	2967	3052	2887	1524	1193	1722	1624
electric pump production	5.75	9.84	8.71	5.43	8.03	6.02	5.84
electric pump wells	118	145	142	115	127	112	86
electric pump costs	4977	5225	4586	2854	5211	3307	3148
water shut-off production	11.03	10.54	11.05	11.49	11.03	10.53	17.12
water shut-off wells	261	252	254	241	237	214	302
water shut-off costs	6959	6719	7049	7397	7183	5906	8763
overhaul production	4.98	5.65	5.60	2.10	3.02	2.30	1.77
overhaul wells	64	71	67	30	42	37	33
overhaul costs	3828	4039	3668	1327	2235	1456	1184
other post treatment production	8.89	10.15	6.26	5.74	4.07	8.32	7.27
other post treatment wells	174	182	130	117	105	184	143
other post treatment costs	7216	7561	4405	4661	2518	5880	5637

Table 4. Single measures production of each plant in planning year

	Plant I	Plant II	Plant III
post flush production	6.69	7.64	6.91
acidification production	4.92	5.21	4.42
perforations adding production	61.33	73.25	59.88
transfer pumping production	2.48	3.07	3.42
electric pump production	6.24	6.61	5.97
water shut-off production	15.38	13.18	14.42
overhaul production	3.16	4.46	2.77
other post treatment production	7.74	6.55	5.91

Table 5. Measures production of each plant under the different constraint level in planning year

		under the condition of low constraint level	under the condition of high constraint level
Measures production of each plant (X^*)	Plant I	109.43	131.86
	Plant II	120.67	150.41
	Plant III	106.83	139.77

Measures production allocation optimization scheme is $\gamma_1 X_1^* + \gamma_2 X_2^*$, Where $0 < \gamma_1$, $\gamma_2 < 1$, $\gamma_1 + \gamma_2 = 1$.

For example, let $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, and then measures production of each plant in planning year was obtained shown in Table 6.

Table 6. Measures production of each plant in planning year

	Plant I	Plant II	Plant III
Measures production of each plant	120.645	135.54	123.3

This solution and functional simulation solution for 2011 was consistent, therefore, the model and algorithm is feasible.

5. Conclusion

A new model combining MC^2 and bi-level programming was proposed innovatively in this paper, whose upper level is a MC^2 linear programming, and lower level is multiple objectives nonlinear programming. With this special bi-level programming model, we used interior method of MC^2LP and the solving of multiple objectives nonlinear programming to present its solving method. The case study shows that the model to establish is the validity and the method to present in the paper may provide a reliable technical support for oil field development optimal decision-making. The further study will be the theories of MC^2 nonlinear programming and bi-level programming with MC^2 .

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