

## Internal pilot insertion for polar codes

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### ABSTRACT

Two internal pilot insertion methods are proposed for polar codes to improve their error correction performance. The presented methods are based on a study of the weight distribution of the given polar code. The insertion of pilot bits provided a new way to control the coding rate of the modified polar code on the basis of the Hamming weight properties without sacrificing the code construction and the related channel condition. Rate control is highly demanded by 5G channel coding schemes. Two short-length polar codes were considered in the work with successive cancellation list decoding. The results showed that advantages in the range of 0.1 to 0.75 dB were obtained in the relative tolerance of the modified coded signal to the additive white Gaussian noise and fading channels at a bit error rate of  $10^{-4}$ . The simulation results also revealed that the performance improvements were possible with a careful insertion of the pilots. The modified polar code with pilot insertion provided performance improvement and offered the control of the coding rate without any added complexity at both the encoder and the decoder.

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## 1. INTRODUCTION

Polar code, introduced by Arıkan in [1], is a linear block code that realises the symmetric capacity of a binary memoryless channel by exploiting channel polarisation. Polar codes split  $N$  independent channels into  $K$  reliable bit-channels, which are used to send the information bits and the  $(N-K)$  unreliable bit-channels to carry the frozen known bits. The promising bit error rate (BER) performance and low-complexity recursive structure decoding are the useful features that let polar codes be chosen as the candidate channel codes for 5G systems [2], [3]. Successive cancellation (SC) decoding algorithm is used for polar codes. Because of the mediocre performance of SC decoding at short and moderate block lengths, SC with list (SCL) decoding was proposed as a polar code decoder [4]. Additional improvement of the SCL performance is achieved when using the cyclic redundant check (CRC) code to select the correct codewords at the end of the decoding process. Such performance improvement in decoding results in increased complexity and latency [4], [5]. Polar codes with SCL decoding are approved by the 3<sup>rd</sup> generation partnership project (3GPP) to encode the control information for the enhanced mobile broadband (eMBB) use case. Polar codes are also candidate for use with other 5G use cases [5].

The construction of polar codes includes the process of finding the best  $K$  bit-channels out of  $N$  total bit-channels on the basis of a design signal-to-noise power ratio given by  $R E_b/N_0$ , where  $R=K/N$  is the coding rate,  $E_b$  is the average energy per bit, and  $N_0$  is the single-sided power spectral density of the additive white Gaussian noise (AWGN) [6]. The reliability order for the bit-channels of the polar codes is non-universal because it depends on the codeword length and the channel condition. This leads to practical

problems in constructing polar codes, particularly when flexibility in the coding rate is required [7]. Different works have attempted to achieve polar code rate flexibility, and among them, the construction of a rate-compatible polar code was proposed in [8]. A flexible, highly parallel, and low-complexity systematic algorithm for polar encoding was introduced in [9]. Different studies on the reliability order of bit-channels based on the polarisation effect have been reported [10]-[12]. These studies introduced opportunities to produce universal reliability sequences irrespective of the channel condition. A universal and unique sequence for bit-channel reliability was proposed for coding lengths smaller than or equal to 1024. This sequence is considered to be a base to find the reliability order for the bit-channels and to specify the frozen bits in constructing polar codes for 5G systems [7]. In addition, other works were concerned with increasing the minimum distance of the polar codes either by using other super code, as in [13], or by using multikernal polar codes, as in [14].

Pilot signals are usually used in wireless transmission systems to estimate the impulse response of the transmission channel. The number and the locations of the pilots have a significant effect on the wireless system performance [15], [16]. Combining pilot symbols with channel coding has attracted the attention of several researchers. The pilot symbols can be inserted within the encoded bits after the encoding process in an arrangement known as ‘external pilot insertion’ [17]. This approach has been utilised in many works to improve the system performance. Low-density parity check (LDPC) codes were combined with pilot symbols in [18], [19] to reduce both the experienced error floor and the effect of error propagation. In addition, the problems of phase offset, frequency offset, and burst error in coherent reception for wireless communication systems were solved. In [20], [21], pilot symbols were combined with the turbo code to reduce the problem of error propagation and enhance the spectral efficiency for multiple-input multiple-output systems and multiple access sparse code systems. A pilot-assisted transmission scheme using polar codes was presented in [22]-[24] to improve the system performance and to reduce the overhead in channel estimation.

The pilot symbols (bits in this case) can be inserted among the data bits prior to the encoding process. This approach is called ‘internal pilot insertion’ and was utilised in the present work. A method was suggested in [25] for combining internal pilot insertion with the turbo code. The results showed an improvement in the BER performance, and thus, a wide range of coding rates could be provided. In the present work, internal pilot insertion methods were used with polar codes to enhance the code distance properties and thereby improve the BER performance of the code. These insertion methods introduced a new mechanism for data rate control on the basis of the distance properties of the code. The proposed methods avoid code reconstruction and provide a universal rate control mechanism for the given polar code independent of the channel condition. The remainder of this paper is organised as follows: The proposed methods for inserting pilot bits are described in section 2. The system model is presented in section 3. Results and the features of the proposed methods are given in section 4. Finally, the concluding remarks of the paper are presented in section 5.

## 2. PILOT INSERTION METHODS

Two methods for pilot insertion are introduced here. The pilots are inserted before the encoding process. The locations of the pilot bits are determined by these methods, and then, the pilot data bit (binary ‘1’) is inserted. This can be considered to be a type of pre-encoding process. The intention here is to improve the error performance for a given polar code by changing the distance properties of the code. This is achieved by either removing the codewords that have the least Hamming weights or reducing their multiplicity order. The two methods, named Method-1 and Method-2, share the common preliminary steps, which are described here first.

Consider a polar code PC ( $N$ ,  $K$ ) having a code length of  $N$  bits and a data block size of  $K$  bits. Initially, all  $2^K$  possible input data sequences  $\{X_i\}$  are generated, and their corresponding codewords  $\{C_i\}$  are determined together with their Hamming weights  $H_w$  as follows:

$$H_w = [h_{min}, h_1, h_2, \dots, h_{max}] \quad (1)$$

where  $h_{min}$  and  $h_{max}$  are the minimum and the maximum Hamming weight for the code, respectively. The weight distribution vector  $A$  of the code, with  $\{A_{hi}\}$  as its components, can be expressed as follows:

$$A = [A_{hmin}, A_{h1}, A_{h2}, \dots, A_{hmax}] \quad (2)$$

where  $A_{hi}$  represents the number of codewords having Hamming weight  $h_i$ . The first and the last components of  $A$  ( $A_{hmin}$  and  $A_{hmax}$ ) represent the number of codewords having the minimum and the maximum weights, respectively, while other  $\{A_{hi}\}$  hold weights between the minimum and the maximum. The components of  $A$

for polar codes are symmetric about their centre weight ( $h_{cent}$ ). This weight also has the largest component in  $A$ . Moreover,  $h_{cent}$  can be expressed as follows [26]:

$$h_{cent} = N / 2 \tag{3}$$

To insert  $N_p$  pilots within the input data sequences  $\{X_i\}$ , we need to determine  $N_p$  pilot positions  $\{s_v\}$  for  $v = 1, 2, \dots, N_p$  among the  $K$  bits of the input data sequence where  $N_p < K$ . For weight  $h_i, h_i \in H_w$  and  $min. \leq i \leq max.$ , define the matrix  $B_{hi}$  with dimensionality  $A_{hi} \times K$ , where  $A_{hi} \in A$ . The matrix  $B_{hi}$  holds the subset of input data sequences  $\{X_i^{hi}\}, \{X_i^{hi}\} \subset \{X_i\}$ , producing codewords  $\{C_i^{hi}\}, \{C_i^{hi}\} \subset \{C_i\}$ , having a Hamming weight of  $h_i$ . For the successful operation of each method, the zeros at each position of the input data sequences,  $\{X_i^{hi}\}$ , are determined first. There are  $K$  such positions possible. The redundant number of zeros at each position is simply determined by the number of zeros in each column of  $B_{hi}$ . The resulting number of zeros is stored in the  $K$ -component row vector  $R$ .

$$R = [r_1, r_2, r_3, \dots, r_k] \tag{4}$$

Thus,  $R$  holds the number of repeated zeros at each data bit position of the input subsets  $\{X_i^{hi}\}$ . For example, the first component  $r_1$  is determined by the number of zeros occurring in the first position of all the input sequences  $\{X_i^{hi}\}$ , and the second component  $r_2$  is determined by the number of zeros occurring in the second position. The best pilot position  $s_l$  is then determined by selecting the subscript  $i$  of the maximum  $r_i$  for the given matrix  $B_{hi}$  as the first pilot position. After inserting the first pilot, a new data sequence  $\{X_i\}$  and the corresponding codewords  $\{C_i\}$  are generated. A new weight vector  $H_w$  and a weight distributed vector  $A$  are then determined, and the process just described is used to determine the second pilot position. This is also repeated for all the required number of pilots  $N_p$ .

**2.1. Method-1**

In this method, the matrix  $B_{hi}$  is constructed on the basis of the minimum Hamming weight ( $h_{min}$ ), i.e.  $B_{hmin}$ . The dimension of the matrix  $B_{hmin}$  is  $A_{hmin} \times K$ . From the  $K$ -element vector  $R$ , the index with the maximum value is considered the best position for inserting the first pilot  $s_l$ . The pilot bit value is set to binary ‘1’ at position  $s_l$  for all the input sequences  $\{X_i\}$ . Inserting such pilot values produces new data sequences  $\{X_i\}$ , which are used to generate the corresponding codewords  $\{C_i\}$ . The vectors  $H_w$  and  $A$  are then determined for the new  $\{C_i\}$  and the corresponding matrix  $B_{hi}$  is constructed on the basis of the new minimum weight  $h_{min}$ . The procedure is then repeated to determine the second pilot position ( $s_2$ ). This process is then repeated until all the  $N_p$  pilots are inserted. Note that the value of the minimum weight may increase during this procedure.

In the case where  $q$  ( $q > 1$ ) components of the vector  $R$  sharing the maximum value of  $r_i$ , an ambiguity about the best position for the pilot will occur. To solve such a problem, the Hamming weight ( $h_l$ ) is selected to construct the matrix  $B_{hi}$  (i.e.  $B_{hl}$  in this case) with the dimension of  $A_{hl} \times K$ . Create the vector  $R'$  with  $q$  components to hold the number of repeated zeros in the columns of  $B_{hl}$  only for the  $q$  positions that share the same maximum value in the vector  $R$ . The index of the component with the maximum value in  $R'$  will be selected as the pilot position. This approach is used here to make the selection process more accurate and to solve the problem of having more than one component in  $R$  sharing the same maximum number of repeated zeros.

**2.2. Method-2**

This method follows the same steps as Method-1, but here, the construction of the matrix  $B_{hi}$  is based on the centre Hamming weight  $h_{cent}$ , given in (3), rather than on  $h_{min}$  as in Method-1. Therefore, the matrix  $B_{hcent}$  with the dimension of  $A_{hcent} \times K$  is used to determine the vector  $R$ . The index of the maximum component of  $R$  is used as the position  $s_1$  to insert the first pilot. A new data sequence  $\{X_i\}$  is then created, and the corresponding codewords  $\{C_i\}$ , their Hamming weight vector  $H_w$ , and weight distribution vector  $A$  are determined to be ready to construct new  $B_{hcent}$ . The position of the second pilot bit is then determined, and the process is continued in this manner until all the  $N_p$  pilots are inserted.

When two or more components of  $R$  share the maximum value, Method-2 considers the minimum weight component  $h_{min}$  of the vector  $H_w$  instead of  $h_{cent}$  to construct the matrix  $B_{hi}$  ( $B_{hmin}$  in this case). The vector  $R'$  is constructed as described in Method-1. Therefore, the index of the component with the maximum value in  $R'$  is selected as the pilot position. The internal pilot insertion procedure for the polar code with the two considered methods is clarified by Algorithm-1.

### 3. SYSTEM MODEL

A fading channel model based on 5G parameters is used in this work to evaluate the BER performance of the proposed encoder. The main features of the system model are: the use of millimeter Wave (mmWave) band, orthogonal frequency division multiplexing (OFDM), quadratic phase shift keying (QPSK) modulation, and multi-input multi-output (MIMO) [27]-[29]. The channel model considered here was approved by 3GPP and defined over range of frequencies from 0.5 GHz to 100 GHz for maximum bandwidth of 2 GHz [27]. Model D (indoor office with short-delay profile) is considered using tapped delay line channel model. The system related parameters used in the simulation are those presented in Tables 1 and 2. The justification of the used parameters values and details of their settings can be found elsewhere [27]-[29].

#### Algorithm-1: Pilot insertion algorithm for polar codes

```

1 Specify N, K, and Np; // Np: number of
  inserting pilots
2 sv = { }; // sv
  : a set holds indexes for pilot positions
3 Generate {Xi}, {Ci}, Hw and A;
4 Select pilot insertion method (M);
5 For i= 1 to Np
6   If M=" Method-1" then
7     hi=hmin, where hmin ∈ Hw ;
8   Else
9     hi=hcent, where hcent ∈ Hw ;
10  End If
11  Bhi ← Call Constructing_matrix_B ({Xi}, {Ci}, hi) // Bhi: matrix hold data set Xihi
12  R ← Call Constructing_vector_R (Bhi, K) // R : a vector hold zeros
  multiplicity at Bhi columns
13  imax ← Max ( R ) // imax
  : an index of maximum element in R
14  If R hold single maximum element then
15    si = imax
16  Else
17    su = { } //
  su : a set holds indexes share max. value in R
18    su ← indexes of maximum elements in vector R
19    If M=" Method-1" then
20      hk= h1 , where h1 ∈ Hw // hk : a Hamming weight used if R hold
  multiple maximum element
21    Else
22      hk = hmin, where hmin ∈ Hw
23    End if
24    si←Call Finding_Best_Position (hk, {Xi}, {Ci}, K, su)
25  End if
26  sv ← si
27  ∀ p ≤ K, p ∈ sv, set {Xi} at p to 1 // p: an index
  for data block
28  generate {Ci}, Hw, and A
29 End for
30 Return sv

```

#### Subroutine-1: Constructing matrix B

```

1 Constructing_matrix_B ({Xi}, {Ci}, hi)
2 ∀ c ∈ {Ci} and weight(c)= hi, then c ∈ Cihi // Cihi : a set of codewords
  with Hamming weight hi
3 ∀ x ∈ {Xi} and encode(x)→c where c ∈ Cihi, then x ∈ Xihi // Xihi : a data set
  that produce Cihi
4 Bhi ← {Xihi}
5 Return Bhi

```

#### Subroutine-2: Constructing vector R

```

1 Constructing_vector_R (Bhi, K)
2 For i = 1 to K
3   R(i) ← zeros count at Bhi (:,i) // Counts the number of zeros at each column of Bhi
4 End for
5 Return R

```

**Subroutine-3: Finding Best Position**

```

1 Finding_Best_Position (hk, {Xi}, {Ci}, K, su)
2 Bhk ← Call Constructing_matrix_B ({Xi}, {Ci}, hk) //Bhk: a matrix holds
the data set Xihk
3 R' ← Call Constructing_vector_R' ( Bhk, K, su) // R': a vector holds zeros
multiplicity at Bhk columns
4 kmax ← Max (R') // kmax : an index for the maximum element in R'
5 Return kmax

```

**Subroutine-4: Construction vector R'**

```

1 Constructing_vector_R' ( Bhk, K, su)
2 For i = 1 to K
3 If i ∈ su then
4 R'(i) ← count zeros at Bhk(:,i)
5 End if
6 End for
7 Return R'

```

**Table 1. Simulation parameters of 5G transmission system**

The Parameter	Value or Type
Carrier Frequency	28 GHz
Bandwidth	400 MHz
Modulation	QPSK
No. of Subcarriers in OFDM	4096
Length of Cyclic Prefix	288
Sub-Carrier Spacing	120 kHz
OFDM Frame Duration	8.33 μs
MIMO Type	4x4 (STBC)

**Table 2. Simulation parameters of fading channel model**

The Parameter	Value
No. of multipaths	3
Path Gains	[ -0.2 -18.8 -21] dB
Normalized Path Delay	[ 0 0.035 0.612]
Delay Spread	16 ns
K-Factor	13.3 dB
Velocity	3 km/Hr
Max. Doppler Shift	77.784 Hz

**4. SIMULATION TESTS AND DISCUSSION**

Polar codes with short lengths and low coding rates were considered the best choice for 5G system among all the coding schemes [2]. The complexity of the proposed methods is mainly based on the value of K; therefore, relatively small values of N and K were used in the simulation tests. In fact, two short lengths polar codes were considered in the tests: the first was with N=32 and K=12 (i.e. PC(32,12)) and the second one with N=32 and K=16 (i.e. PC(32,16)). Up to four pilots were assumed in the simulation tests. An SCL decoder was used with different list sizes (L) of 8, 16, and 32. The simulation tests were carried out to study the effect of pilot insertion on the distance properties and the BER performance of the considered polar codes. The performance of both the original polar code (without pilot insertion) and the modified polar code (with pilot insertion) was compared according to the corresponding BER performance over AWGN and fading channels. The comparison was based on the SNR gain (in decibels) achieved by the modified polar code at a BER of 10<sup>-4</sup>. Test results with unclear improvement, because of the selection of the list size and the number of pilots, are not presented. Instead, remarks on such cases are given in the following subsections.

**4.1. Distance properties of modified polar codes**

The main intention of the proposed pilot insertion methods was to improve the distance properties of the given polar code, aiming to achieve better BER performance. The Hamming weight distribution for the codewords of the original and modified polar codes by using Method-1 and Method-2 is shown in Tables 3 and 4 for PC(32,12) and PC(32,16), respectively. As the pilot bits were fixed, the total number of possible output codewords was reduced from 2<sup>K</sup> to 2<sup>(K - N<sub>p</sub>)</sup> with the maximum number of codewords having the centre weight (h<sub>cent</sub>) as expected. The weight distribution after the pilot insertion maintained its symmetry around h<sub>cent</sub>. The above-mentioned tables also clarified that the minimum (non-zero) weight increased after the insertion of the pilot bits for both the methods in an indication for a possible BER performance improvement. Both the tables showed that when the number of inserted pilots increased, the

minimum distance of the code also increased or the number of codewords having the minimum weight decreased. This clarified the improvement in the BER performance for the modified polar code, as will be shown next.

Table 3. The number of codewords with given Hamming weight for the proposed methods using PC(32,12)

Hamming Weight	No. of Pilots( $N_p$ )									
	0	1		2		3		4		
	Original Code	M1	M2	M1	M2	M1	M2	M1	M2	
0	1	0	0	0	0	0	0	0	0	
8	15	4	7	1	6	0	2	0	1	
10	120	62	75	25	32	14	14	7	3	
12	440	218	171	109	87	54	49	29	25	
14	920	486	571	249	294	122	150	59	88	
16	1135	554	409	279	193	142	89	67	40	
18	872	410	553	211	276	106	138	57	59	
20	440	238	177	119	97	58	51	31	29	
22	136	66	81	27	38	14	18	5	10	
24	17	10	4	4	1	2	1	1	1	

Table 4. The number of codewords with given Hamming weight for the proposed methods using PC(32,16)

Hamming Weight	No. of Pilots( $N_p$ )									
	0	1		2		3		4		
	Original Code	M1	M2	M1	M2	M1	M2	M1	M2	
0	1	0	0	0	0	0	0	0	0	
6	17	0	0	0	0	0	0	0	0	
8	319	126	126	51	67	20	31	6	17	
10	2007	1032	1032	528	516	272	266	142	132	
12	6970	3576	3576	1840	1732	932	858	470	424	
14	14050	7160	7160	3496	3724	1728	1866	862	936	
16	18727	8980	8980	4486	4330	2236	2160	1106	1086	
18	14214	7160	7160	3648	3676	1840	1838	922	916	
20	6916	3576	3576	1752	1748	876	886	450	440	
22	1949	1032	1032	520	532	256	254	122	128	
24	345	126	126	63	59	32	33	16	17	
26	19	0	0	0	0	0	0	0	0	
28	2	0	0	0	0	0	0	0	0	

## 4.2. BER Performance of modified polar codes

### 4.2.1. AWGN channel

The BER performance of PC(32,12) for the original and modified polar codes with the list sizes of 8, 16, and 32 are presented in Figures 1, 2, and 3, respectively. These figures show the effect of different parameters and pilot insertion methods on the BER performance. As shown in Figure 1, Method-2 with one pilot (M2,  $N_p=1$ ) achieved an SNR gain of approximately 0.5 dB. Method-1 and Method-2 with four pilots (i.e. (M1,  $N_p=4$ ) and (M2,  $N_p=4$ )) achieved gains less than 0.35 dB. The performance of the different cases of PC(32,12) with  $L=16$  and 32 was almost identical, as shown in Figures 2 and 3, respectively. In general, the gains achieved with  $L=16$  and 32 were less than those with  $L=8$ . For (M2,  $N_p=1$ ), the obtained gain was about 0.35 dB for both the list sizes of 16 and 32. In addition, less gains ( $\leq 0.2$  dB) were obtained by both the methods with four pilots using  $L=16$  and 32.

Figures 4, 5, and 6 show the performance of PC(32,16) for the list sizes of 8, 16, and 32, respectively, with one to four pilots. Both the methods produced the same pilot position in the case of a single pilot; therefore, Method-1 with a single pilot was considered to represent both the cases in the results. In general, the SNR gains achieved by PC(32,16) were higher than those achieved by PC(32,12) with the same list size. For PC(32,16) with the list size of 8, the gain achieved by (M2,  $N_p=4$ ) was approximately 0.75 dB. Moreover, gains in the range of 0.4 to 0.5 dB were provided by Method-1 with 1, 3, and 4 pilots (i.e. (M1,  $N_p=1$ ), (M1,  $N_p=3$ ), and (M1,  $N_p=4$ )). As in PC(32,12), when larger list sizes ( $L=16$  and 32) were used, the resulting gains were less than those with  $L=8$ . Furthermore, the performances of PC(32,16) with  $L=16$  and 32 were close to each other. A gain of approximately 0.55 dB was obtained by (M2,  $N_p=4$ ), while the corresponding gains for (M1,  $N_p=1$ ), (M1,  $N_p=3$ ), and (M1,  $N_p=4$ ) were in the range of 0.25 to 0.3 dB. As final remarks for the BER performance, the modified polar codes achieved gains compared to the original polar codes in the SNR range from 0.2 to 0.5 dB for PC(32,12) and from 0.25 to 0.75 dB for PC(32,16) at a BER of  $10^{-4}$  for most of the considered cases.

**4.2.2. Fading channel**

The performance for PC(32,16) is evaluated over the considered fading channel with 5G environment using one to four pilots and with the list sizes of 8, 16, and 32. As the selection of pilot positions is independent on the channel and is based on the Hamming weight, the pilot positions with fading channel are identical to that with AWGN channel. The performance of PC(32,16) with list sizes of 8, 16, and 32 is shown by Figures 7, 8, and 9, respectively. For PC(32,16) with the list size of 8, the gain achieved by (M1,  $N_p=1$ ) was about 0.5 dB. Moreover, about 0.3 dB gain was provided by (M1,  $N_p=3$ ) and (M2,  $N_p=4$ ). The performance of the modified code with (M1,  $N_p=4$ ), and for all list sizes, is close to that for the original polar code with slight advantage at high SNR (0.1 dB). Furthermore, the performances of PC(32,16) with  $L=16$  and 32 were almost identical to each other. A gain of approximately 0.5 dB was obtained by (M1,  $N_p=3$ ), while the corresponding gains for (M1,  $N_p=1$ ), and (M2,  $N_p=4$ ) were about 0.3 dB. As final remarks for the BER performance, the modified polar codes achieved gains compared to the original polar codes in the SNR ranging from 0.1 to 0.5 dB for PC(32,16) at a BER of  $10^{-4}$  for most of the considered cases.

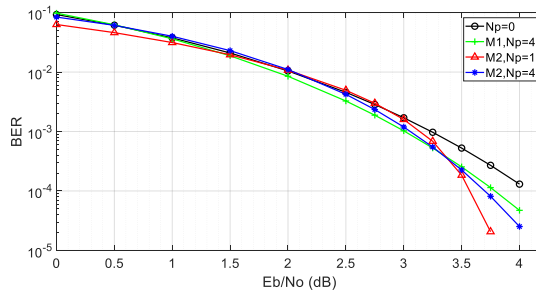


Figure 1. BER performance for PC(32,123) with  $L=8$  over AWGN channel

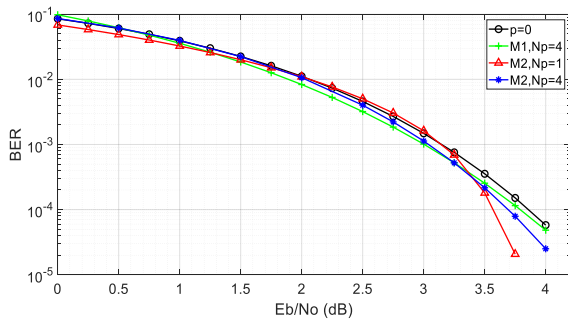


Figure 2. BER performance for PC(32,12) with  $L=16$  over AWGN channel

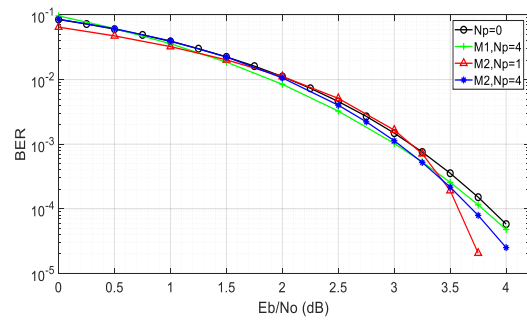


Figure 3. BER performance for PC(32,12) with  $L=32$  over AWGN channel

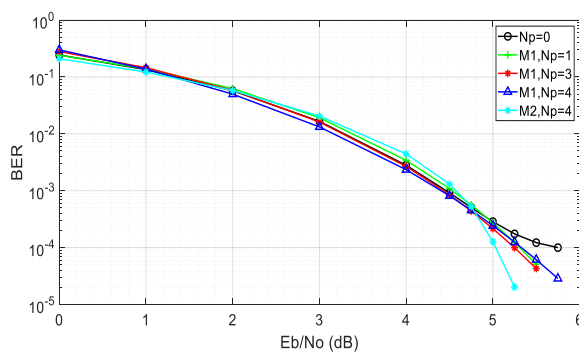


Figure 4. BER performance for PC(32,16) with  $L=8$  over AWGN channel

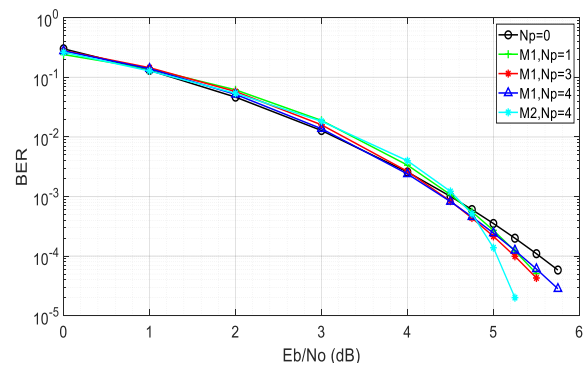


Figure 5. BER performance for PC(32,16) with  $L=16$  over AWGN channel

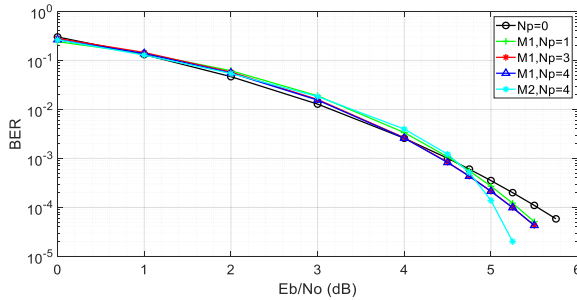


Figure 6. BER performance for PC(32,16) with  $L=32$  over AWGN channel

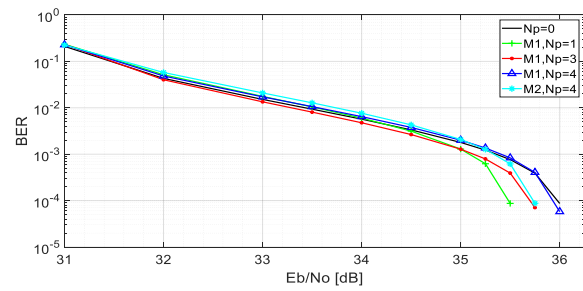


Figure 7. BER performance for PC(32,16) with  $L=32$  over fading channel

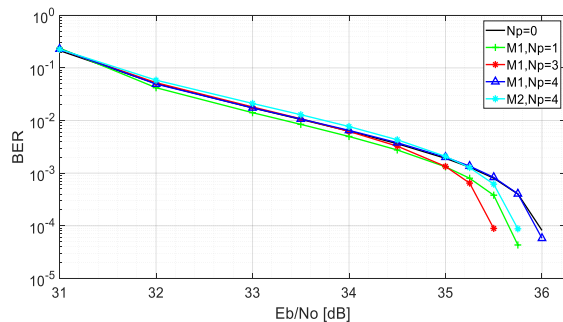


Figure 8. BER performance for PC(32,16) with  $L=16$  over fading channel

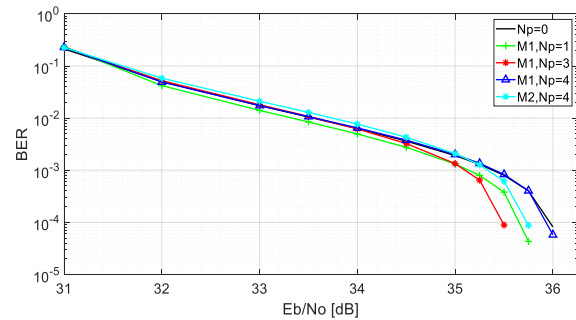


Figure 9. BER performance for PC(32,16) with  $L=32$  over fading channel

### 4.3. Rate flexibility of the modified polar codes

The pilot insertion approach presented in this work reduced the actual  $K$  input data bits of the polar code. This changed the coding rate of the modified polar code. As a result, the code rate was actually  $(K-N_p)/N$  instead of  $K/N$ . Thus, the coding rate for the modified polar codes could be varied from  $K/N$  (without any inserting pilot) to  $(K-N_p)/N$  (with  $N_p$  inserting pilots). The modified polar code can insert different numbers of pilot bits in order to control the coding rate or/and to improve the BER performance. The pilot insertion approach provides a rate control mechanism independent of the channel condition.

### 4.4. Discussion

In general, the design of polar codes relies on channel polarisation by selecting the code construction that guarantees a large coding gain. The bit-channels are divided into lower-reliability bit-channels carrying the frozen bits and higher-reliability bit-channels carrying the data bits. The data bit-channels are also ordered according to their reliability into high-index bit-channels (with high reliability and capacity) and low-index bit-channels [5], [7]. The proposed pilot insertion methods improved the BER performance in most of the considered arrangements related to the selected test parameters according to the obtained distance properties. In some of the test arrangements, the methods did not show any performance improvement when the selected pilot positions encountered the location of reliable bit-channels. This resulted in a violation of the polar code construction rules.

According to the weight distribution of the modified codes, the performance improved when the pilots were inserted in positions of the low-reliability information bit-channels with one of the following two conditions being satisfied (for most of the studied conditions of the number of pilots, as shown in Tables 3 and 4): There was a reasonable reduction in the number of codewords having a weight equivalent to  $h_{cent}$  combined with an increase in the minimum Hamming weight of the modified code; There was a large reduction in the number of codewords having a weight equivalent to  $h_{cent}$  combined with a considerable reduction in the number of codewords associated with the minimum weight of the modified code.

The pilot positions were determined before the transmission of the actual data. Their positions were known for both the encoder and the decoder. The rate control obtained was based on the number of inserted pilot bits. In addition, the possible improvement in the distance properties of the polar code could have resulted in improving the BER performance. The pilot insertion might not provide all the possible rates in the range of  $(K-N_p)/N$  to  $K/N$ , when a reliable bit-channel position was selected as the pilot position. This



resulted in the BER performance degradation. Finally, the following remarks regarding the comparison of the conventional polar code to the proposed one were deduced: Both the conventional and the proposed polar codes had the same decoding complexity and hence the same latency; Unlike the proposed polar code, the change in the coding rate of the conventional polar code required code reconstruction; The BER performance for the proposed polar code was better than that of the conventional code; The only drawback of the proposed polar code was the decrease in the bit rate due to the pilot insertion.

## 5. CONCLUSION

A new approach to improve the performance of short-length polar codes and to provide a mechanism for coding rate control was presented in this paper. The approach relied on internal pilot insertion by setting the binary '1' of the selected pilot position(s) among the input data blocks. Two methods were proposed to choose the pilot positions on the basis of an examination of the Hamming weight distribution of the codewords. The pilot positions were determined to improve the distance properties of the modified polar code and hence guaranteed an improvement in the BER performance. In addition to the minimum Hamming weight, this work emphasised on the importance of the centre Hamming weight in the polar code performance. The proposed pilot insertion methods introduced a new approach to change the nominal coding rate by varying the number of data bits in the input block according to the number of inserted pilots. The construction of the modified polar codes was not changed by the pilot insertion. The simulation results showed that the SNR gains of the modified code over that of the original polar code in tolerance of the system to AWGN and fading channels were in the range of 0.1 to 0.75 dB were achieved at a BER of  $10^{-4}$ . This was obtained under the preferred conditions regarding the number of pilots and the pilot insertion method. Thus, the use of internal pilot insertion with polar codes seems to be a promising approach where no added complexity is needed at both the encoder and the decoder during an actual transmission.

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