

## A Novel Optimization Means of Transfer Efficiency for Resonance Coupled Wireless Power Transfer

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### Abstract

When the transmitting and receiving coils are equivalent with or not significantly smaller than their separation distance between the two coils, the results of numerical simulations for transfer efficiency of system show that the maximum coupling of two coils or maximum transfer efficiency occur with a certain tilt angle between two coils, which means the two coils with parallel on the same axis is not able to obtain the maximum transmission efficiency. In view of this feature of the wireless power transfer system, we propose a novel means to optimize the transfer efficiency by designing rotatable transmitting coils to achieve the appropriate tile angle. The experimental results show that the presented means is efficient and greatly improves the transfer efficiency.

**Keywords:** Efficiency optimization, Misalignment, Mutual inductance.

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### 1. Introduction

Since Tesla had opened up the field of Wireless Power Transfer (WPT), it has been a dream for people. In recent years, the WPT technology has become a hot topic and some further breakthroughs have been made in electric vehicles, RFID systems, body implanted medical devices and portable devices [1-6].

Inductive coupling and resonant coupling are two major means, and in either inductive or magnetic coupled systems, power is transferred from a primary transmitter coil (Tx) to a secondary receiver coil (Rx) with the aid of an alternating magnetic field. The low power inductive links discussed in this study are loosely coupled and have extremely low coupling coefficients ( $\kappa \leq 0.02$ ) [7]. In this process of power transfer, the Rx coil is considered to be placed well within the reactive near-field of the Tx coil since the separation distance ( $d$ ) between the two coils and the wavelength of the system operating frequency ( $\lambda$ ) obey the inequality  $d \leq \lambda/2\pi$ . The energy is almost not radiated in space and stored in the near-field of the Tx coil.

To the general knowledge, the Tx and Rx coils would be ideally coaxially orientated so that maximum coupling such as the maximum mutual inductance and the optimal transfer efficiency results. However in practice, in electric vehicle wireless charging, misalignment of the two coils generally occurs due to the parking position. Although improved designs have emerged in the industry, the impact of coil misalignment on the coupling efficiency and optimal design of coil has received less attention by researchers.

Grover and Terman had demonstrated that the coupling coefficient is correlated to the mutual inductance [8]. The greater part of this work concentrated on the mutual inductance calculation. Any theoretical study of the mutual inductance in coil misalignment is extremely complex due to the tedious work required to solve the double integral in Neumann's formula. Fotopoulou et al. had introduced a novel analytical power transfer efficiency expressions which allowed the optimization of coil geometry for maximum power transfer and misalignment tolerance [9]. But these expressions were deduced based on electrically small system which means the distance of separation between the Tx and Rx coils is much greater than the dimensions of the Rx coil. As a result, these conclusions and expressions only are appropriate for some practical system, such as RFID and biomedical, but not in electric vehicle and portable devices.

The aim of this paper is to optimize the transfer efficiency of inductive coupled wireless power transfer (ICWPT) system. A novel means is presented based on the mutual inductance

analysis of coil misalignment. The results of numerical simulations show that the maximum coupling occurs with a certain tilt angle and no lateral misalignment between two coils. Then the removable and rotatable coil or coils could be designed for the maximum coupling to optimize the transfer efficiency. In our experiments, the primary Tx coil is removable and rotatable, and the transfer efficiency can be improved about from 38% to 60% by turning the Tx coil to make the angular misalignment varied.

## 2. Method Model and Power Splitting

Due to the system working in high frequency (HF), the generated parasitic parameters should not be ignored. Figure 1 represents the equivalent circuit of the ICWPT system [10]:

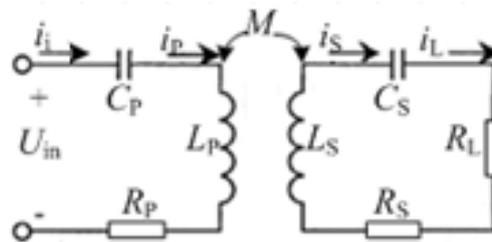


Figure 1. Equivalent circuit of ICWPT system

In Figure 1,  $M$  is the mutual induction of two coils,  $U_{in}$  is the induced potential source;  $R_p$ ,  $R_s$ ,  $C_p$  and  $C_s$  are parasitic parameters in HF and  $L_p$ ,  $L_s$  are the self-inductances of the two coils (the subscripts “P” and “S” stand for the primary and secondary respectively);  $R_L$  stands for the load resistance.

From Figure 1, it is easy to deduce the voltage KVL equations of the two circuits with the operating angular frequency  $\omega$ .

$$\begin{bmatrix} R_p + jX_p & j\omega M \\ j\omega M & R_s + R_L + jX_s \end{bmatrix} \begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} U_{in} \\ 0 \end{bmatrix} \quad (1)$$

$$\begin{cases} X_p = \omega L_p - 1 / (\omega C_p) \\ X_s = \omega L_s - 1 / (\omega C_s) \end{cases} \quad (2)$$

It is important to define the physical meaning of the power transfer efficiency studied in this paper. The power transfer efficiency, denoted  $\eta$ , is the ratio of the output power ( $P_{out}$ ) of the load to the input power ( $P_{in}$ ) generated by the primary coil, does not include the high frequency power supply system which is treated as a constant voltage source.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{(\omega M)^2 R_L}{Z_s [Z_p Z_s + (\omega M)^2]} \times 100\% \quad (3)$$

Under resonance, the received power by the Rx coil is maximal and delivered to the load  $R_L$ , and the reactive parts of the system cancel out [11]. Then eq. (3) can be rewrite as

$$\eta = \frac{(\omega M)^2 R_L}{(R_s + R_L)[R_p(R_s + R_L) + (\omega M)^2]} \times 100\% \quad (4)$$

It is evident that the bigger mutual inductance implies higher efficiency. The mutual inductance can be calculated with the Neumann formula [12].

$$M = \frac{N_{Tx} N_{Rx} \mu_0}{4\pi} \iint_{l_{Tx} l_{Rx}} \frac{dl_{Tx} \cdot dl_{Rx}}{R} \tag{5}$$

$N_{Tx}$ ,  $N_{Rx}$ ,  $l_{Tx}$ ,  $l_{Rx}$ ,  $dl_{Tx}$  and  $dl_{Rx}$  stand for the coil turns, the length of each turn and infinitesimal of  $l$  of the resonant  $T_x$  and  $R_x$  coils, respectively.  $R$  is the distance between  $dl_{Tx}$  and  $dl_{Rx}$ , and  $\mu_0$  is the magnetic permeability of free-space.

From eq. (5), the mutual inductance is correlated with the shapes and orientations of the two coils. Then the analysis of the mutual inductance with misalignment in many practical applications, such as wireless charging of electric vehicles, is carried out in Section 3.

**3. Mutual Inductance in Analysis and Transfer Efficiency Optimization**

Clearly, lateral and angular misalignments are main cases[9]:

- 1) Lateral misalignment: The coupled coils are situated in parallel planes with a vertical distance  $d$  and a lateral distance  $l$ , as shown in Figure 2 (a).

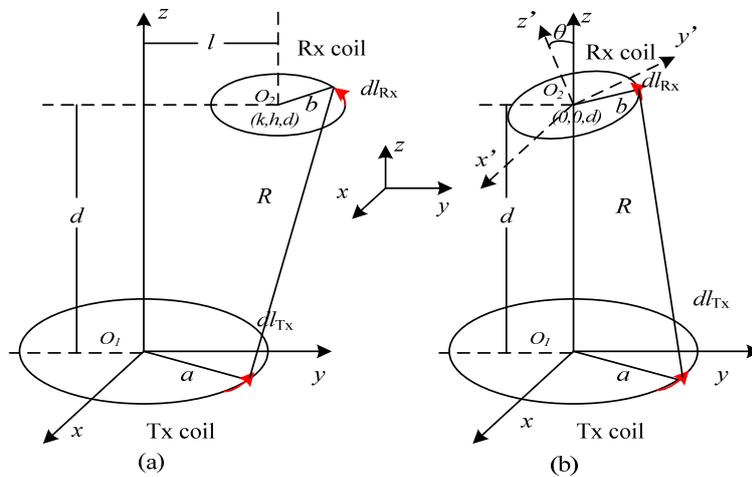


Figure 2. Misalignment configuration of the Tx and Rx coils: (a) lateral misalignment (b) angular misalignment.

- 2) Angular misalignment: There is a tilt angle  $\theta$  between two planes of the Tx and Rx coils and the axis of one coil passes through the center of the other coil, as shown in Figure 2 (b).

A general misalignment case incorporates both lateral displacement and angular tilt of the coupled coils. Due to the fact that there is no strong interaction between the two misalignment effects as demonstrated by Soma, the two displacement configurations are studied independently in this paper.

**3.1. Mutual Inductance Analysis of Lateral Misalignment**

In Figure 2 (a), the radiuses of the Tx and Rx coils are respectively  $a$  and  $b$ . And the center coordinates of the coupled coils are  $(0, 0, 0)$  and  $(k, h, d)$ . Then the lateral distance  $l$  is equal to  $\sqrt{k^2+h^2}$ , and the parametric equations of the two coils are given as

$$\begin{cases} x_{Tx} = a \cos \phi \\ y_{Tx} = a \sin \phi \\ z_{Tx} = 0 \end{cases} \quad \text{and} \quad \begin{cases} x_{Rx} = b \cos \varphi + k \\ y_{Rx} = b \sin \varphi + h \\ z_{Rx} = d \end{cases} \tag{6}$$

$dl_{Tx}$ ,  $dl_{Rx}$  and  $R$  are easily deduced as

$$\begin{aligned}
 dl_{Tx} &= (-a \sin \phi \bar{x} + a \cos \phi \bar{y}) d\phi, \\
 dl_{Rx} &= (-b \sin \phi \bar{x} + b \cos \phi \bar{y}) d\phi, \\
 dl_{Tx} dl_{Rx} &= ab \cos(\phi - \phi) d\phi d\phi, \\
 R &= \sqrt{(x_{Tx} - x_{Rx})^2 + (y_{Tx} - y_{Rx})^2 + (z_{Tx} - z_{Rx})^2}.
 \end{aligned} \tag{7}$$

Substituting eq. (7) into eq. (5), the mutual inductance with lateral displacement could be calculated. Due to the heavy work required to solve the double integral and the analytical solution of mutual inductance is almost impossible to be derived. Figure 3 shows the numerical solution when  $a = 0.2$  m,  $b = 0.16$  m,  $d = 0.15$  m, and  $N_{Tx} = N_{Rx} = 1$ .

In Figure 3, it is evident that the mutual inductance is concentric distribution and becomes smaller with the increase of the lateral distance  $l$ .

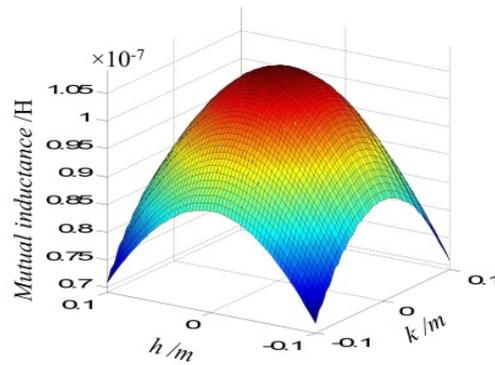


Figure 3. Mutual inductance with lateral misalignment.

### 3.2. Mutual Inductance Analysis of Angular Misalignment

In Figure 2 (b),  $\bar{z}$  and  $\bar{z}'$  represent the normal directions of the coupled coils and the angle  $\theta$  of them is the tilt angle between the two coil planes. Then the Cartesian coordinate system  $xyz$  could be established by drawing  $\bar{y}$  perpendicular to  $\bar{z}$  in the  $\bar{z}\bar{z}'$  plane. In the plane of the Rx coil, we could draw  $\bar{y}'$  perpendicular to  $\bar{x}$ .  $\bar{y}'$  is also in the  $yz$  plane, and another Cartesian coordinate system  $x'y'z'$  could be established, which is around the  $x$ -axis of rotation  $\theta$  angle to the coordinate system  $xyz$  and translation  $d$  of  $z$ -axis direction. The rotation matrix is deduced easily, given as

$$M_{ROT} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}. \tag{8}$$

Then

$$\begin{aligned}
 \begin{bmatrix} x_{Rx} \\ y_{Rx} \\ z_{Rx} \end{bmatrix} &= M_{ROT} \begin{bmatrix} b \cos \phi \\ b \sin \phi \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix} = \begin{bmatrix} b \cos \phi \\ b \cos \theta \sin \phi \\ b \sin \theta \sin \phi + d \end{bmatrix}, \\
 dl_{Rx} &= (-\sin \phi \bar{x} + \cos \theta \cos \phi \bar{y} + \sin \theta \cos \phi \bar{z}) b d \phi, \\
 dl_{Tx} \cdot dl_{Rx} &= ab \cos \alpha d \phi d \phi, \\
 \cos \alpha &= \frac{|dl_{Tx} \cdot dl_{Rx}|}{|dl_{Tx}| |dl_{Rx}|}.
 \end{aligned} \tag{9}$$

With the same parameters above, the numerical solutions of the mutual inductance with various tilt angles are shown in Figure 4.

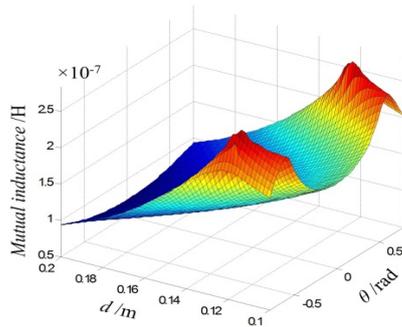


Figure 4. Mutual inductance with various tilt angle and vertical distance

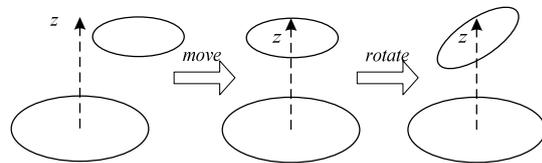


Figure 5. Schematic of efficiency optimization

Clearly, in Figure 4, there are partial optimal solutions of mutual inductance while the tilt angle changing. Due to no lateral misalignment, the tilt angles corresponding to the two partial optimal solutions are symmetrical about x-axis. It is also worth noting that the peaks of the mutual inductance do not occur at the shortest distance ( $d = 0.1$  m). It is related to the bifurcation phenomena of ICWPT system. In this paper, the bifurcation phenomena are not considered.

The bigger is mutual inductance, the higher transfer efficiency is. Then a novel means of optimizing efficiency is proposed to make the coil removable and rotatable for achieving maximum coupling effect, namely the peaks of the mutual inductance and transfer efficiency.

In the applications envisaged, such as electric vehicle wireless charging, the lateral misalignment shown in Figure 2 (a) occurs usually. Then a valid measure of optimizing the transfer efficiency is shown in Figure 5.

In Figure 5, the move is to make the two coupled coils coaxial and the rotation is to look for the partial optimal solution for achieving the maximum coupling. Of course, in practice, the ranges of move and rotation are limited.

**4. Experimental Results**

In order to verify the above analysis and the proposed means of optimizing the transfer efficiency, in experiments, we adjust the power supply to output the high-frequency signal of 0.58 MHz and measure the RF output power as the system input power  $P_{in}$ . Using the oscilloscope to detect the voltage signal of the load and reading the URMS, we can calculate the output power  $P_{out}$  of the load by  $P_{out} = U_{RMS}^2 / R_L$ . The ICWPT system, shown in Figure 6, is set up with the parameters listed in Table 1.

The measured values with the vertical distance of 0.15 m and a coil are shown in Figure 7. It is clear that in perfect alignment the system can only achieve about 38.6% efficiency, whereas the efficiency could improve up to 60.5% by turning one coil to make a tilt angle. Additionally, the efficiencies with lateral distances of 0.05 m and 0.1 m are measured and respectively equal to about 35% and 29%. The results of experiments are consistent with the theoretical analysis.

Table 1. Experiment Device Parameters

FACTORS		
Radius	Tx Coil	$R_{Tx} = 0.20m$
	Rx Coil	$R_{Rx} = 0.16m$
Turns	Tx Coil	$N_{Tx} = 13$
	Rx Coil	$N_{Rx} = 16$
Resonance Frequency		$f = 0.58MHz$
Load Resistance		$R_L = 7.2 \Omega$

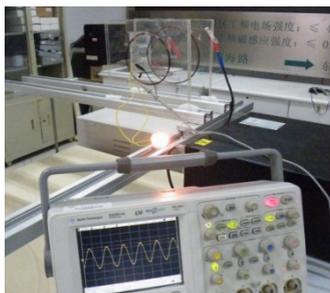


Figure 6. Photograph of the experiment setup

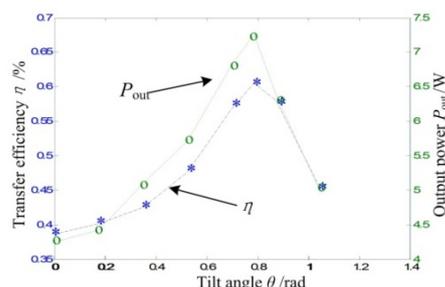


Figure 7. Measured transfer efficiencies and output powers

## 5. Conclusion

A novel means of optimizing transfer efficiency of the ICWPT system is presented based on the analysis of mutual inductance. The removable and rotatable coil or coils should be designed for searching the partial optimal solution of coupling effect. In experiments, the change of transfer efficiency is obvious and the maximum efficiency occurs with a tilt angle of  $\pi/4$ , which is consistent with the numerical simulation. The proposed means is very important for many practical applications to enhance their efficiency effectively.

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