

The Research of Digital Algorithm Based on Frequency-Dependent Transmission Lines

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Abstract

The algorithm for obtaining the discrete response of propagation function for frequency dependent parameter line is presented. Consider a minimum sampling period T_{sm} , that is, the highest frequency $f_H=1/(2T_{sm})$ in the signal is taken into account. The impedance $z(\omega)$ and the admittance $y(\omega)$ are obtained in the frequency range of $[0, f_H]$ by employing the Carson's formula. The propagation function at each frequency point is subsequently obtained, the impulse response in discrete time domain is then obtained using Poisson Sum Formula. In order to avoid the long length of impulse response under the higher sampling frequency, the poles and zeros of z transform of discrete propagation function are evaluated by the Prony's method. Subsequently, the coefficients of the discrete infinite impulse response of propagation function are obtained. Using these coefficients the wave transfer sources can be easily computed by discrete convolution operation. The simulation tests show that the results using the proposed method is accurate, the error is not more than 1% in contrast of the results generated by EMTP.

Keywords: Propagation Function Coefficient; Frequency Dependent Model; Propagating Traveling Waves

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1. Introduction

In the early 80's of the 20th century, J. Marti proposed a transient model for the frequency-dependent parameter lines, in which the distributed parameters vary with the frequency [1-5]. In Marti's model, the surge impedance $Z_c(\omega)$ is synthesized with an RC network, in which the constants R's and C's are determined by the poles and zeros of $Z_c(\omega)$, the equivalent model is shown in Figure 1.

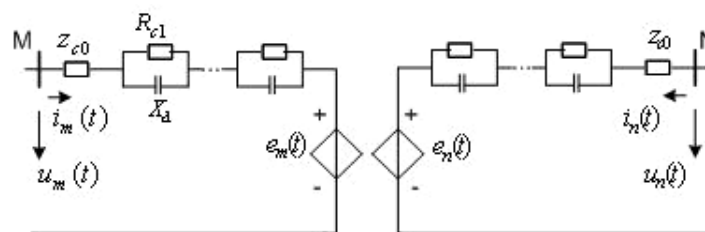


Figure 1. J Marti's Equivalent Model

In the equivalent circuit of Figure 1, the wave transfer sources are respectively the traveling waves propagate from the other terminal.

$$\begin{cases} e_M(t) = a_L(t) * b_N(t) = \int_{-\infty}^{\infty} a_L(t-\tau) b_N(\tau) d\tau \\ e_N(t) = a_L(t) * f_M(t) = \int_{-\infty}^{\infty} a_L(t-\tau) f_M(\tau) d\tau \end{cases} \quad (1)$$

Where: a_L is propagation function,

b_N is backward traveling wave,

f_M is forward traveling wave.

The propagation function is obtained by taking inverse Laplace transform of $A_L(\omega)$, where

$$A_L(\omega) = \exp(-\sqrt{z(\omega)y(\omega)l}) \quad (2)$$

The z and y are respectively the series impedance and shunt admittance per length of the transmission line, which are frequency-dependant [6-8]. In order to get the inverse Laplace transform of the propagation function, Marti proposed that the function in frequency domain can be approximated to be the synthesis by a sum of first order terms:

$$A_L(\omega) = e^{-j\omega T_{\min}} \left(\sum_{m=1}^P \frac{K_m}{s + p_m} \right) \quad (3)$$

Subsequently, the propagation function in time domain can be obtained by the following formula:

$$a_L(t) = \sum_{m=1}^P K_m e^{-p_m(t-T_{\min})} \quad (4)$$

Obviously, the discrete value of transfer source in equivalent circuit (Figure 1) requires the discrete computation of convolution integral with forward and backward traveling waves by employing formula (1). Suppose that if it directly get the discrete impulse response of the propagation function $a_L(k)$, the transfer sources can be directly obtained by the discrete convolution algorithm, that is,

$$\begin{cases} e_M(k) = \sum_{m=1}^n a_L(m)b_N(k-m) \\ e_N(k) = \sum_{m=1}^n a_L(m)f_M(k-m) \end{cases} \quad (5)$$

This could be simplify the computation of transient analysis. In this paper, the Poisson's Summation formula is employed to obtain the digital coefficients of propagation function, $a_L(k)$. However, the discrete propagation function can be regarded as an Infinite Impulse Response (IIR) digital filter [9-10], that is the length of $a_L(k)$ is infinite. The Prony's method is employed to calculate the zeros and poles of the z transform of $a_L(k)$.

Therefore the traveling waves propagating to another terminal (transfer sources) can be calculated in discrete time domain with less amount of computation. Compared with the results by the new technique with that by PSCAD, the results show that the proposed algorithm is efficiency and accurate even at a considerable large steps.

2. Basic Principle

2.1. Frequency Dependent Parameter Line

For a single phase transmission line MN which length is l , associated with the frequency dependent distributed parameters, the line equations in frequency domain at location x is shown as follows:

$$\begin{cases} -\frac{dU(x, j\omega)}{dx} = z(j\omega)I(x, j\omega) \\ -\frac{dI(x, j\omega)}{dx} = y(j\omega)U(x, j\omega) \end{cases} \quad (6)$$

Where, z and y are respectively series impedance and shunt admittance per length of line. The value of frequency-dependent impedance and admittance at each frequency can be obtained by Carson's formula [11-13].

One can get the propagation relations of forward and backward traveling waves in frequency domain:

$$\begin{cases} F_N(j\omega) = A_L(j\omega)F_M(j\omega) \\ B_M(j\omega) = A_L(j\omega)B_N(j\omega) \end{cases} \quad (7)$$

Where $F=U+ZcI$ is forward traveling wave and $B=U-ZcI$ is backward traveling wave. Zc is surge impedance, A_L is propagation function.

$$z_c(\omega) = \sqrt{\frac{z(\omega)}{y(\omega)}} \quad (8)$$

$$A_L(\omega) = \exp(-\sqrt{z(\omega)y(\omega)}l) \quad (9)$$

2.2. Poisson's Summation Formula

According to the theories of signal and systems, the discrete Fourier transform of a signal $x(n)$ is the periodic extension of the Fourier transform of its corresponding continuous signal $x(t)$. That is, if $x(n)$ is the discrete signal by sampling the signal $x(t)$ with the sampling frequency N , the Fourier transform of $x(t)$ is $X(\omega)$, then the Fourier transform of discrete signal $x(n)$ is shown as:

$$\tilde{X}(\omega) = \sum_k X(\omega + 2kN\pi) \quad (10)$$

Using Poisson's sum formula, one can get the discrete signals from the Fourier transform of continuous function:

$$\sum_k X(\omega + 2\pi kN) = \frac{1}{2\pi N} \sum_k \hat{X}\left(\frac{k}{N}\right) e^{-jk\frac{\omega}{N}} \quad (11)$$

Where:

$$\hat{X}\left(\frac{k}{N}\right) = \int_{-N\pi}^{N\pi} X(\omega) e^{-j\omega k / a} d\omega \quad (12)$$

Then the z transform of discrete signal can be obtained as the following formula.

$$X(z) = \frac{1}{2\pi N} \sum_k \hat{X}\left(\frac{k}{N}\right) z^{-k} \quad (13)$$

Therefore the discrete signal is obtained:

$$x(n) = \frac{1}{2\pi N} \hat{X}\left(\frac{k}{N}\right) \quad (14)$$

Certainly using Poisson's Sum Formula, one can easily get the discrete propagation function and discrete surge impedance.

2.3. The Discrete Propagation Function and Surge Impedance

The discrete Fourier transform of propagation function can be written as:

$$\tilde{A}_L(\omega) = \sum_k A_L(\omega + 2kN\pi) \quad (15)$$

By employing Poisson's sum formula, propagation function will be written as:

$$A_L(z) = \sum_k a_L(k) e^{-j\omega k/a} = \sum_k a_L(k) z^{-k} \quad (16)$$

Similar procedure for surge impedance:

$$Z_c(z) = \sum_k Z_c(k) e^{-j\omega k/a} = \sum_k Z_c(k) z^{-k} \quad (17)$$

In order to simplify the computation, the minimum traveling time N_{min} is separated from A_L :

$$A_L = Z^{-N_{min}} A_{L0}(z) \quad (18)$$

where N_{min} is the minimum delayed time of propagation traveling waves. It can be determined by the light velocity:

$$N_{min} = \text{floor}(L / cT_s) \quad (19)$$

By employing Poisson sum formula, recursive discrete sequence of propagation function and surge impedance will be obtained.

$$\begin{cases} a_{L0}(n) = \frac{1}{2\pi N_s} \sum_{k=0}^N A_{L0}(k\Delta f) \exp(j2\pi k\Delta f \frac{n}{N_s}) \Delta f \\ z_c(n) = \frac{1}{2\pi N_s} \sum_{k=0}^N Z_c(k\Delta f) \exp(j2\pi k\Delta f \frac{n}{N_s}) \Delta f \end{cases} \quad (20)$$

However, the propagation function is an Infinite Impulse Low Pass filter, that is, its discrete coefficient is infinite long. In order for simplified computation, the z transform of A_L and Z_c require to be expressed as recursive form:

$$A_L(z) = Z^{-N_{min}} \frac{\beta_0 + \beta_1 z^{-1} + \dots + \beta_M z^{-M}}{1 + \alpha_1 z^{-1} + \dots + \alpha_N z^{-N}} \quad (21)$$

So does the surge impedance. It need to determine the coefficients of a and b from the discrete propagation function and surge impedance. The Prony's method is employed to estimate these coefficients.

2.4. The Recursive Coefficients by Prony's Method

The coefficients α_k can be estimated by solving homogeneous difference equation.

$$\begin{bmatrix} a_{L0}(n) \\ a_{L0}(n-1) \\ \dots \\ a_{L0}(n-p) \end{bmatrix} = \begin{bmatrix} -a_{L0}(n-1) & -a_{L0}(n-2) & \dots & -a_{L0}(n-N) \\ -a_{L0}(n-2) & -a_{L0}(n-3) & \dots & -a_{L0}(n-1-N) \\ \dots & \dots & \dots & \dots \\ -a_{L0}(n-p-1) & -a_{L0}(n-p-2) & \dots & -a_{L0}(n-p-N) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \dots \\ \alpha_N \end{bmatrix} \quad (22)$$

By means of Least Square Method, coefficient α can be estimated as:

$$\alpha = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{A}_{L0} \quad (23)$$

The z transform of A_{L0} can be synthesized by a sum of first order terms.

$$A_{L0}(z) = \frac{\beta_0 + \beta_1 z^{-1} + \dots + \beta_M z^{-M}}{1 + \alpha_1 z^{-1} + \dots + \alpha_N z^{-N}} = \sum_k \frac{b_k}{1 - z_k z^{-1}} \quad (24)$$

Where z_k is the poles determined by the following formula.

$$z_k = \text{roots}([1, \alpha_1, \dots, \alpha_N]) \quad (25)$$

B_k can be calculated by using the following formula.

$$\begin{bmatrix} a_{L0}(n) \\ a_{L0}(n-1) \\ \dots \\ a_{L0}(n-p) \end{bmatrix} = \begin{bmatrix} z_1^n & z_2^n & \dots & z_N^n \\ z_1^{n-1} & z_2^{n-1} & \dots & z_N^{n-1} \\ \dots & \dots & \dots & \dots \\ z_1^{n-p} & z_2^{n-p} & \dots & z_N^{n-p} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{bmatrix} \quad (26)$$

By employing Least Square Method, coefficient b can be solved as:

$$b = (\mathbf{Z}^H \mathbf{Z})^{-1} \mathbf{Z}^H \mathbf{A}_{L0} \quad (27)$$

At finally, z_k 、 b_k have been obtained, then we can get propagation function $A_L(z)$.

$$A_L(z) = Z^{-N_{\min}} \sum_{k=1}^N \frac{b_k}{1 - z_k z^{-1}} = Z^{-N_{\min}} \frac{\beta_0 + \beta_1 z^{-1} + \dots + \beta_M z^{-M}}{1 + \alpha_1 z^{-1} + \dots + \alpha_N z^{-N}} \quad (28)$$

The procedure to obtaining the recursive coefficients of propagation function and surge impedance is shown in Figure 2.

2.5. Propagation Function Verification

On the basis of the frequency dependent model, propagation function will be verified. when Sampling interval $T_s = 5 \times 10^{-4} ms$, propagation function and surge impedance coefficient are shown in Figure 3. The comparison result between the actual simulation results and Fourier algorithm, shown in Figure 4.

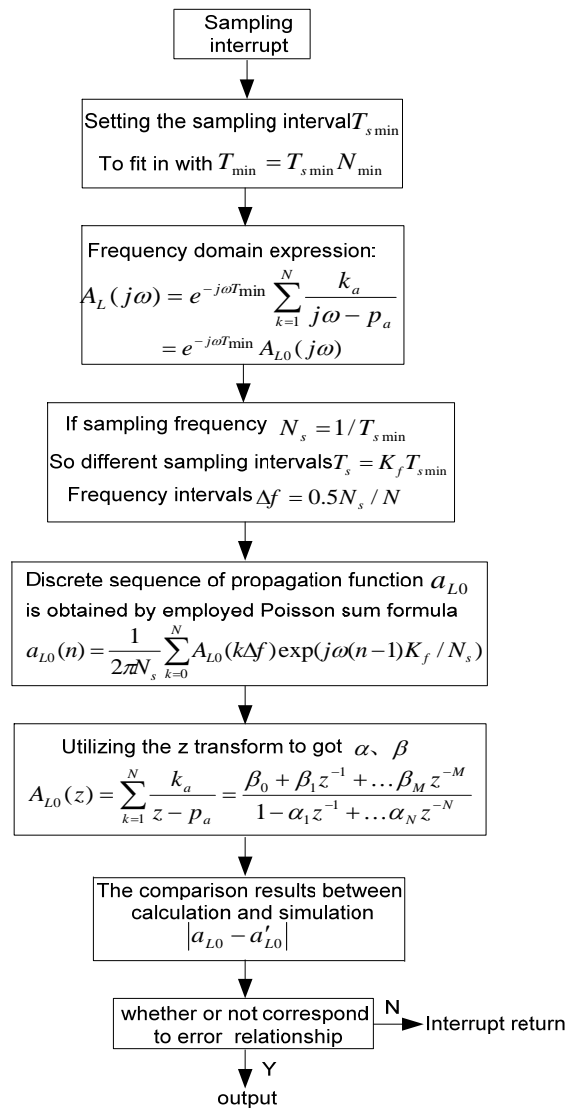


Figure 2. Procedure of Obtaining The Recursive Coefficients

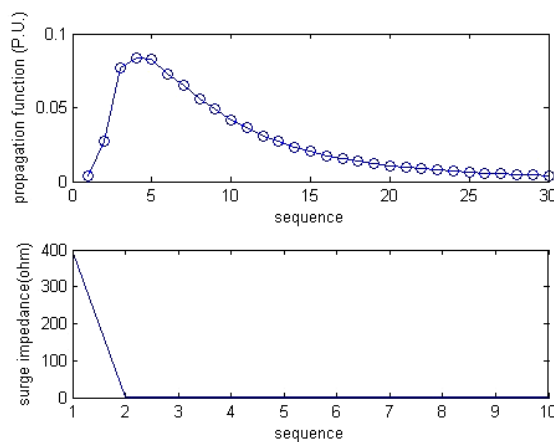


Figure 3. Propagation Function and Surge Impedance Coefficient

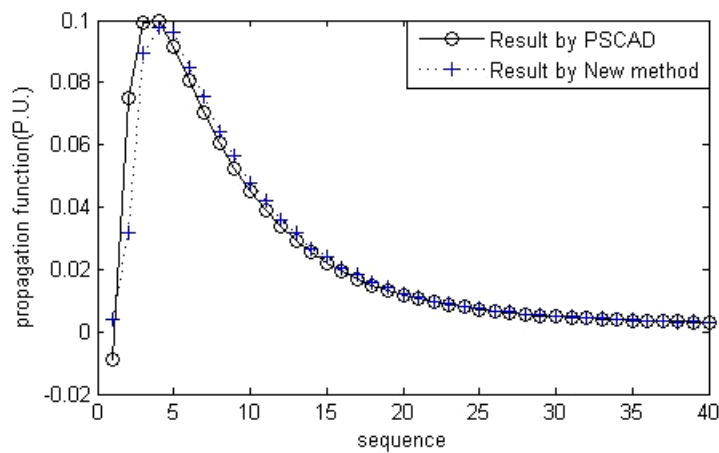


Figure 4. The comparison results between discrete sequence propagation function with the simulation results

From Figure 4, one can see that propagation coefficient have 40 points non-zero elements, if line is longer, sampling frequency is higher, then propagation function will be longer delay time, Lead to increased computation. By employing Prony algorithm to reasonably limit the length of sequence, compared with propagation function given by PSCAD, are shown in Figure 5.

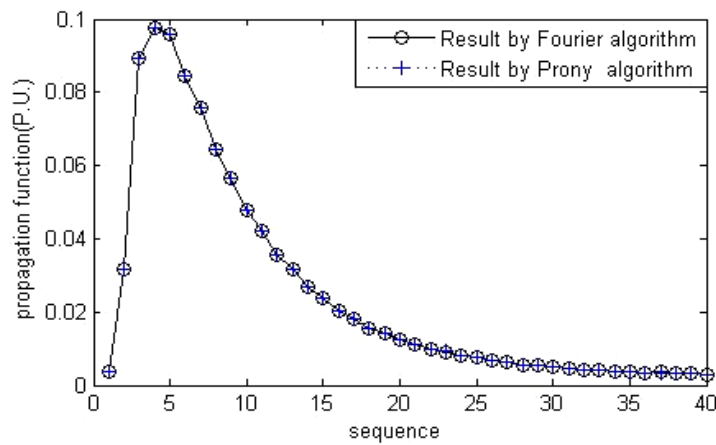


Figure 5. The comparison results Prony algorithm

From figure 5, it is concluded that Prony algorithm is correct and accurate, and have less computation quantities.

3. The Propagating Traveling Waves

Suppose that, at the location $x=0$, the traveling wave can be established by measured voltage and current at original terminal M, forward and backward traveling wave digital expression are written as:

$$\begin{cases} f_m(k) = i_m(k) + Y_c * u_m(k) \\ b_m(k) = i_m(k) - Y_c * u_m(k) \end{cases} \tag{28}$$

Where, $Y_C(k)$ is admittance coefficient, Symbol “*” means revolution operator. Therefore the solution of differential traveling wave equation is shown in the follows.

$$\begin{cases} e_n(k) = f_m(k) * a_L(k) \\ e_m(k) = b_n(k) * a_L(k) \end{cases} \quad (29)$$

The propagating relations in discrete time domain between the two terminals M and N are shown in the following:

$$\begin{cases} e_n(k) = \sum_{l=0}^M \beta_l f_m(k-l-N_{\min}) - \sum_{l=1}^N \alpha_l e_n(k-l) \\ e_m(k) = \sum_{l=0}^M \beta_l b_n(k-l-N_{\min}) - \sum_{l=1}^N \alpha_l e_m(k-l) \end{cases} \quad (30)$$

4. Simulation Results

This section presents simulation results of the new algorithm compared with the results of EMTP, associated with real 500kV transmission network in ShanDong power system, shown in Figure 6. The interested transmission line is that of ZiBo Station to ZouXian Plant, the line length is 328km, the line with frequency dependent parameters.

Suppose that the voltages and currents are measured at Zibo Station, the sampling period is 0.1ms, that is, 200 samples per cycle. The forward traveling waves at ZouXian terminal and p backward traveling waves at Zibo Station are required to calculate.

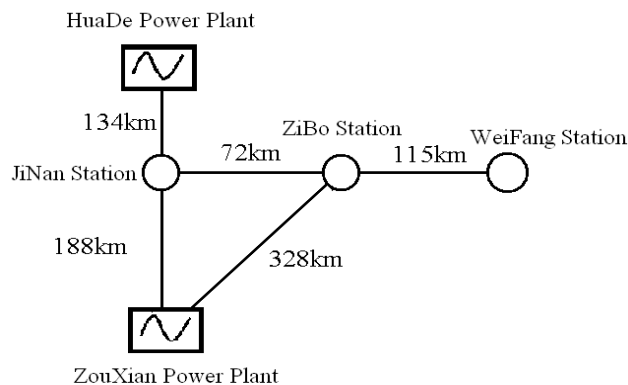


Figure 6. The associated 500kV power network

4.1. Compared with the Calculated Results

Suppose that a single phase to earth fault is taken place in the line of ZouXian Station, the forward traveling waves at ZouXian terminal and backward traveling waves, calculated by the data of Zibo Station using the new algorithm, compared with the waveforms given by PSCAD, are shown in Figure 7 and 8.

From Figure 8 and 9, one can see that the results calculated by new algorithm have relatively accuracy, compared with the results by PSCAD the maximum error is no more than 1.04% under the sampling period 0.1ms, at the same time, with less amount of computation, only have 10 times sum and multiply.

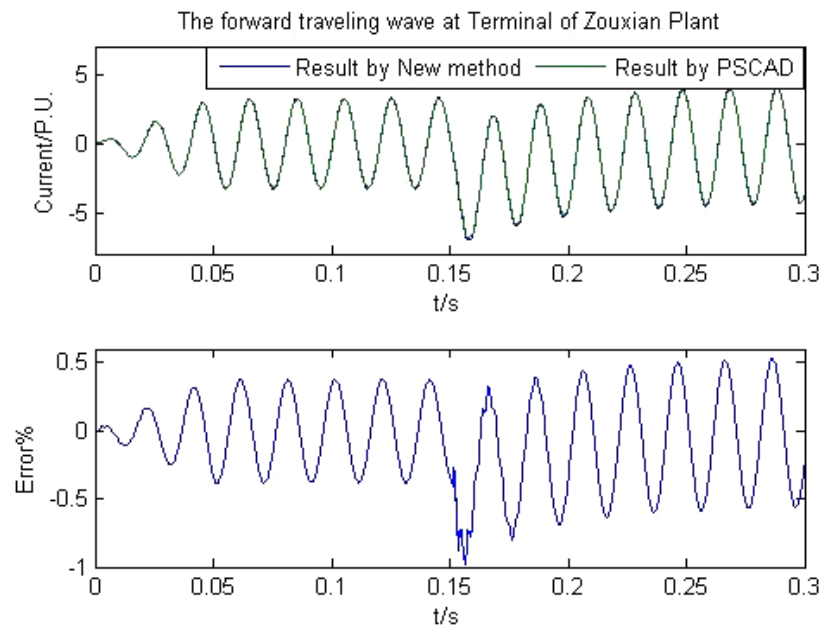


Figure 7. The waveform of the calculated forward traveling wave at terminal ZouXian Plant compared with the result

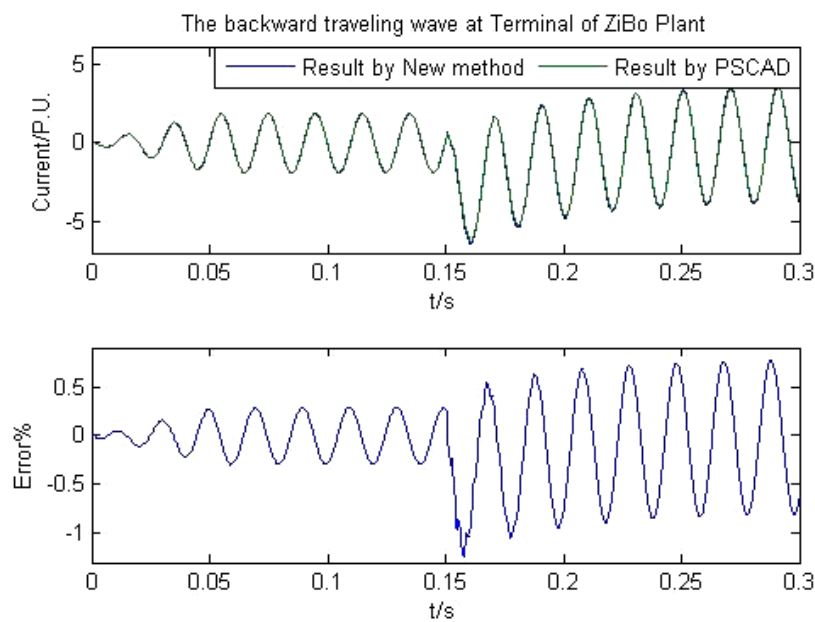


Figure 8. The waveform of the calculated backward traveling wave at terminal Zibo Plant compared with the result

4.2. The Results under Various Sampling Periods

The waveforms of traveling waves computed by the proposed method, compared with waveforms given by PSCAD, under the sampling period of 0.01ms, are shown in Figure 9.

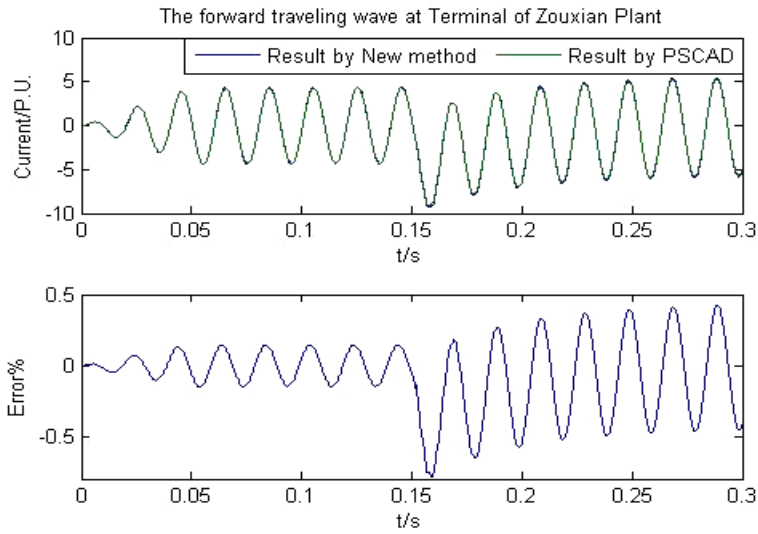


Figure 9. The forward traveling wave comparison results at terminal ZouXian station when $T_s = 0.01ms$

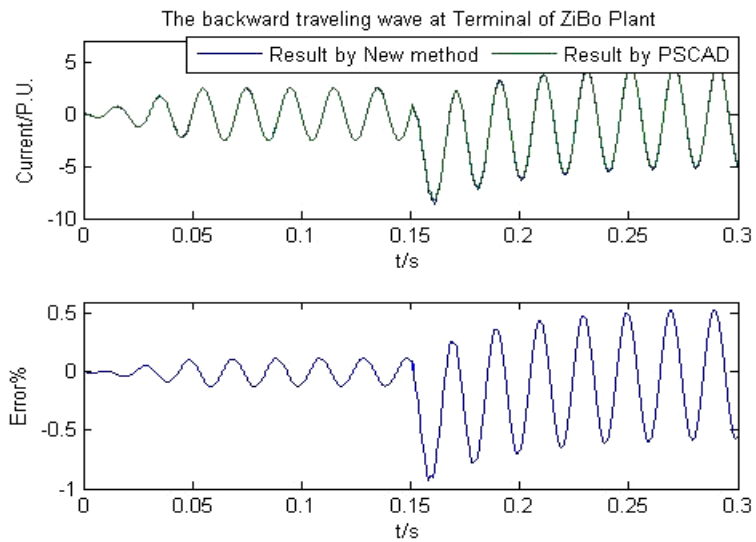


Figure 10. The backward traveling wave comparison results at terminal Zibo Station when $T_s = 0.01ms$

The maximum errors compared with the results given by PSCAD at various sampling periods are shown in Table 1.

Table 1. Maximum errors under various sampling periods

Sampling period (ms)	The forward traveling wave Error (%)	The backward traveling wave Error (%)
0.20	3.14	2.85
0.10	0.90	1.14
0.05	0.74	0.84
0.01	0.73	0.89

From Table 1, one can see that the higher sampling frequency, the higher accuracy of the results, and less computation quantities.

5. Conclusion

The paper presents a new algorithm for precisely calculating the propagating traveling waves from one terminal to another with less amount of computation. The digital model of frequency dependent parameter is proposed based on the projection theorem in function space, then traveling waves propagation equations is established by means of the new algorithm. The surge impedance and the propagating coefficients are then obtained by employing Poisson sum formula and Prony algorithm, at the same time, using these coefficients; the propagating traveling waves are calculated with efficiency and accuracy. Compared with the results given by PSCAD, the simulation tests associated with real power network show that the results are accuracy with less computation

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